

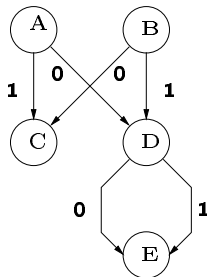
# Outline

- 1 Introduction au cours
  - Compilation et optimisations de codes
  - Des p'tites boucles, toujours des p'tites boucles
  - Exemples de spécificités architecturales
- 2 Pipeline logiciel
  - Sans contraintes de ressources
  - Compaction de boucles et retiming
  - Optimisations des durées de vie
- 3 Fusion de boucles
  - Intérêts et problèmes
  - Fusion de boucles simple : variantes
  - Fusion avec décalage

# Fusion with loop shifting

```

DO i=2, n
  a(i) = f(i)
  b(i) = g(i)
  c(i) = a(i-1) + b(i)
  d(i) = a(i) + b(i-1)
  e(i) = d(i-1) + d(i)
ENDDO
    
```



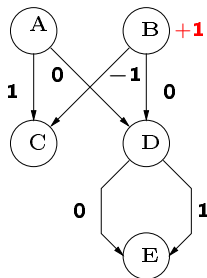
```

DOPAR i=2, n
  a(i) = f(i)
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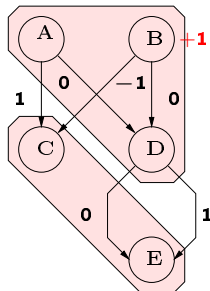
```

a(2) = f(2)
DOPAR i=3, n
  a(i) = f(i)
  b(i-1) = g(i-1)
ENDDOPAR
b(n) = g(n)
DOPAR i=2, n
  c(i) = a(i-1) + b(i)
  d(i) = a(i) + b(i-1)
ENDDOPAR
DOPAR i=2, n
  e(i) = d(i-1) + d(i)
ENDDO
    
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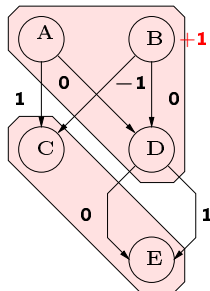
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b(n) = g(n)
DOPAR i=2, n
  c(i) = a(i-1) + b(i)
  e(i) = d(i-1) + d(i)
ENDDOPAR

```

➡ Yet another retiming problem.

But **strongly NP-complete** ( $\sim$  scheduling with communications).

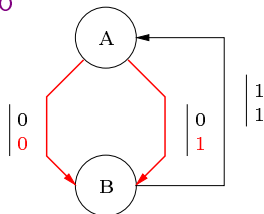
# Multi-dimensional loop shifting and parallel loops

In dimension one :

- Minimal number of parallel loops with shifting : NP-complete.
- Complete fusion possible  $\Leftrightarrow$  any (undirected) cycle weight is 0.

In 2D, find an outer shift that enables the complete inner fusion.

```
DO i=1, n-1
  DO j=1, n-1
    a(i,j) = b(i-1,j-1)
    b(i,j) = a(i,j) + a(i,j-1)
  ENDDO
ENDDO
```



```
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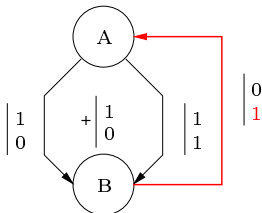
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  ENDDO
ENDDO
```



```
DO i=1, n
  DOPAR j=1, n-1
    IF (i>1) THEN
      b(i-1,j) = a(i-1,j) + a(i-1,j-1)
    ENDDOPAR
  DOPAR j=1, n-1
    IF (i<n) THEN
      a(i,j) = b(i-1,j-1)
    ENDDOPAR
  ENDDO
```

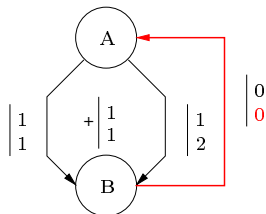
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    b(i,j) = a(i,j) + a(i,j-1)
  ENDDO
ENDDO
```



```
DO i=1, n
  DOPAR j=1, n
    IF (i>1) and (j>1) THEN
      b(i-1,j-1) = a(i-1,j-1) + a(i-1,j-2)
    IF (i<n) and (j<n) THEN
      a(i,j) = b(i-1,j-1)
    ENDDOPAR
  ENDDO
```



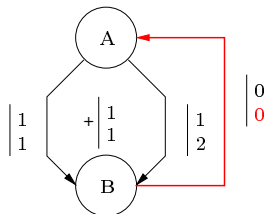
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```



```
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    IF (i>1) and (j>1) THEN
      b(i-1,j-1) = a(i-1,j-1) + a(i-1,j-2)
    IF (i<n) and (j<n) THEN
      a(i,j) = b(i-1,j-1)
    ENDDOPAR
  ENDDO
```

➡ Strongly NP-complete.

# Parallelism detection : more loop transformations needed

Is there some **loop parallelism** (i.e., parallel loop iterations) in the following two codes? What is their **degree of parallelism**?

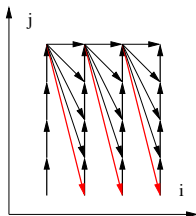
```
DO i=1, N
  DO j=1, N
    a(i,j) = c(i,j-1)
    c(i,j) = a(i,j) + a(i-1,N)
  ENDDO
ENDDO
```

```
DO i=1, N
  DO j=1, N
    a(i,j) = c(i,j-1)
    c(i,j) = a(i,j) + a(i-1,j)
  ENDDO
ENDDO
```

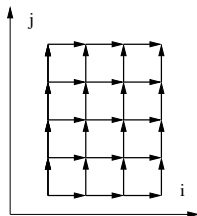
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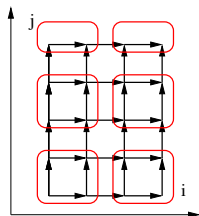
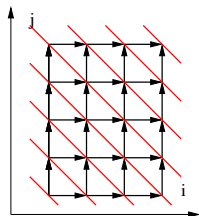
```
DO i=1, N
  DO j=1, N
    a(i,j) = c(i,j-1)
    c(i,j) = a(i,j) + a(i-1,j)
  ENDDO
ENDDO
```



# Loop tiling, blocked algorithms

```
DO t=2, 2N
  DOPAR j=max(1,t-N), min(N,t-1)
    a(t-j,j) = c(t-j,j-1)
    c(t-j,j) = a(t-j,j) + a(t-j-1,j)
  ENDDO
ENDDO
```

```
DO I=1, N, B
  DO J=1, N, B
    DO i=I, min(I+B-1,N)
      DO j=J, min(J+B-1,N)
        a(i,j) = c(i,j-1)
        c(i,j) = a(i,j) + a(i-1,j)
      ENDDO
    ENDDO
  ENDDO
ENDDO
```



➡ Tiling and parallelism detection : similar problem.