Cryptanalysis of branching program obfuscators

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What is this talk about

Two partial attacks against some candidate obfuscators built upon the GGH13 multilinear map [GGH13a]

- an attack for specific choices of parameters
- a quantum attack

Main idea of the two attacks

Transform known weaknesses of the GGH13 map into concrete attacks against the candidate obfuscators

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An obfuscator O for a class of circuits $\mathcal{C}$ is an efficiently computable function over $\mathcal{C}$ such that

$$\forall C \in \mathcal{C}, \forall x, C(x) = O(C)(x)$$

In this talk, $\mathcal{C} =$ polynomial size circuits

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Many cryptographic constructions from iO: functional encryption, deniable encryption, NIZKs, oblivious transfer, ...

Multilinear maps (mmaps) and iO

Observation

Almost all iO constructions for all circuits rely on multilinear maps (mmap).

Three main candidate multilinear maps: GGH13, CLT13, GGH15

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$\Rightarrow$ all current attacks against iO rely on the underlying mmap

In this talk: we exploit known weaknesses of GGH13 to mount concrete attacks against some iO using it.

History (branching program obfuscators based on GGH13)

Some candidate iO for all circuits and attacks:

2013: [GGH$^+$13b], first candidate

2014-2016: [AGIS14, BGK$^+$14, BR14, MSW14, PST14, BMSZ16], with proofs in idealized models (the mmap is supposed to be somehow ideal)

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2017: [CGH17], attack against [GGH$^+$13b] (in input-partitionable case)

2017: [FRS17], prevent [CGH17] attack

State of the art and contributions

iO (using GGH13) / Attacks	Branching program obfuscators				Circuit obfuscators
	[GGH+13b]	[BR14]	[AGIS14, MSW14] [PST14, BGK+14] [BMSZ16]	[GMM+16]	[Zim15, AB15] [DGG+16]
[MSZ16]		✓	✓		
[CGH17]*	✓				
This work 1† [CHKL18]	✓	✓	✓	✓	
This work 2‡ [Pel18]			✓	✓	✓

* for input-partitionable branching programs

† for specific choices of parameters

‡ in the quantum setting

Outline

1. Simple obfuscator
2. GGH13 multilinear map
3. Contributions

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- 2ℓ matrices $A_{i,b}$ (for $i \in \{1,\ldots,\ell\}$ and $b \in \{0,1\}$),
- two vectors A_0 and $A_{\ell+1}$,
- a function $\mathrm{inp} : \{1,\ldots,\ell\} \to \{1,\ldots,r\}$ (where r is the size of the input).

i	1	2	3	4	5	6
$\mathrm{inp}(i)$	1	1	2	1	3	2

$x = 0 \quad 1 \quad 1$

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Cryptographic multilinear maps

Definition: κ-multilinear map

Different levels of encodings, from 1 to κ.
Denote by $\mathrm{Enc}(a, i)$ a level-i encoding of the message a.

Addition: $\mathrm{Add}(\mathrm{Enc}(a_1, i), \mathrm{Enc}(a_2, i)) = \mathrm{Enc}(a_1 + a_2, i)$.

Multiplication: $\mathrm{Mult}(\mathrm{Enc}(a_1, i), \mathrm{Enc}(a_2, j)) = \mathrm{Enc}(a_1 \cdot a_2, i + j)$.

Zero-test: $\text{Zero-test}(\mathrm{Enc}(a, \kappa)) = \text{True iff } a = 0$.

Simple obfuscator

- **Input:** A branching program
- Randomize the branching program
  - Add random diagonal blocks
  - Killian's randomization
  - Multiply by random (non zero) bundling scalars
- Encode the matrices using GGH13
- **Output:** The encoded matrices and vectors

$$\begin{array}{ccccc} & \boxed{A_{1,1}} & \boxed{A_{2,1}} & \boxed{A_{3,1}} & \\ \underline{A_0} & & & & \big| A_4 \\ & \boxed{A_{1,0}} & \boxed{A_{2,0}} & \boxed{A_{3,0}} & \end{array}$$

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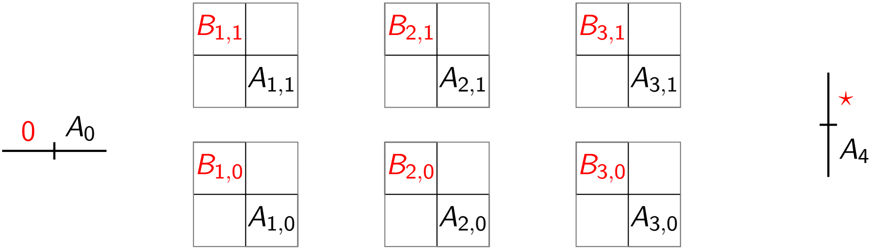

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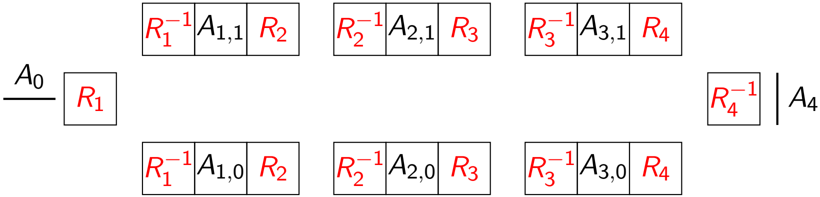

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$$\widetilde{A_0} \qquad \begin{array}{ccc} \boxed{\widetilde{A_{1,1}}} & \boxed{\widetilde{A_{2,0}}} & \boxed{\widetilde{A_{2,0}}} \\ \\ \boxed{\widetilde{A_{1,0}}} & \boxed{\widetilde{A_{2,0}}} & \boxed{\widetilde{A_{2,0}}} \end{array} \qquad \widetilde{A_4}$$

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Outline

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- The plaintext space is $\mathcal{P} = R/\langle g \rangle$ for a “small” element g in R.
- The encoding space is $R_q = R/(qR) = \mathbb{Z}_q[X]/(X^n + 1)$ for a “large” integer q.

Notation

We write $[x]_q$ the elements in R_q for $x \in R$.

The GGH13 multilinear map: encodings and zero-test

- Sample z uniformly in R_q and h in R of the order of $q^{1/2}$.
- **Encoding:** An encoding of a at level i is

$$u = [(a + rg)z^{-i}]_q$$

where $a + rg$ is a small element in $a + \langle g \rangle$.

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Zero-test

To test if $u = [cz^{-\kappa}]_q$ is an encoding of zero (i.e. $c = 0 \mod g$), compute

$$[u \cdot p_{zt}]_q = [chg^{-1}]_q.$$

This is small iff c is a small multiple of g.

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  - recover multiple of sensitive elements
  - classical polynomial time, for specific choices of parameters
- Attack 2 [Pel18]:
  - short principal ideal solver [CDPR16]
  - recover a sensitive element
  - quantum polynomial time [BS16] (or classical sub-exponential time [BEF^{+}17] for specific (unused) choices of parameters)

Attack 1: Starting point = NTRU

For two encodings $[a_1 \cdot z^{-1}]_q, [a_2 \cdot z^{-1}]_q$ for small a_1, a_2, we can compute

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NTRU problem [ABD16, CJL16, KF17]

Let a_1, a_2 be *sufficiently small* elements of R. For a given NTRU instance $[a_1/a_2]_q$, we can efficiently recover

$$(c \cdot a_1, c \cdot a_2) \in R^2$$

for some small c for a given NTRU instance.

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For another encoding $[a_3 \cdot z^{-1}]_q$, compute

$$[a_3 \cdot z^{-1}]_q / [a_1 \cdot z^{-1}]_q \cdot (c \cdot a_1) = c \cdot a_3 \in R.$$

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$$[a_1 \cdot z^{-1}]_q \cdot [a_2 \cdot z^{-1}]_q^{-1} = [a_1/a_2]_q$$

NTRU problem [ABD16, CJL16, KF17]

Let a_1, a_2 be *sufficiently small* elements of R. For a given NTRU instance $[a_1/a_2]_q$, we can efficiently recover

$$(c \cdot a_1, c \cdot a_2) \in R^2$$

for some small c for a given NTRU instance.

For another encoding $[a_3 \cdot z^{-1}]_q$, compute

$$[a_3 \cdot z^{-1}]_q / [a_1 \cdot z^{-1}]_q \cdot (c \cdot a_1) = c \cdot a_3 \in R.$$

Thus we can compute $(ca_i \in R)_i$ using $([a_i \cdot z^{-1}]_q)_i$.

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- **Input:** An obfuscated program $\mathcal{O}(P)$ and plain program Q
- De-randomize the branching program
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  - Distinguish by Matrix Zeroizing Attack
- **Result:** Distinguishing Attack: $P = Q$?

$$\boxed{\mathrm{Enc}(\widetilde{A_{1,1}})} \quad \boxed{\mathrm{Enc}(\widetilde{A_{2,1}})} \quad \boxed{\mathrm{Enc}(\widetilde{A_{3,1}})}$$

$$\mathrm{Enc}(\widetilde{A_0}) \qquad\qquad\qquad\qquad \mathrm{Enc}(\widetilde{A_4})$$

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	$c_{1,1}(\widetilde{A_{1,1}} + R_{1,1}g)$	$c_{2,1}(\widetilde{A_{2,1}} + R_{2,1}g)$	$c_{3,1}(\widetilde{A_{3,1}} + R_{3,1}g)$	
$c_0(\widetilde{A_0} + R_0 g)$				$c_4(\widetilde{A_4} + R_4 g)$
	$c_{1,0}(\widetilde{A_{1,0}} + R_{1,0}g)$	$c_{2,0}(\widetilde{A_{2,0}} + R_{2,0}g)$	$c_{3,0}(\widetilde{A_{3,0}} + R_{3,0}g)$	

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These matrices $\in R$ rather that R_q

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BP matrix	$\mathrm{Enc}(\widetilde{A})$
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$$\begin{array}{ccccc} \text{BP matrix} & \text{Enc}(\widetilde{A}) & & c(\widetilde{A} + Rg) \\ & & \Rightarrow & \\ \text{Enc}(0) & [rg/z^{\kappa}]_q & & c'rg \end{array}$$

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BP matrix	$\mathrm{Enc}(\widetilde{A})$	$\Rightarrow$	$c(\widetilde{A} + Rg)$	$\Rightarrow$	$c\widetilde{A} \bmod g$
$\mathrm{Enc}(0)$	$[rg/z^{\kappa}]_q$		$c'rg$		

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$c\widetilde{A}$ mod g do not contain the randomness r and level parameter z

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We remove the effects of scalar bundlings using algebraic ways

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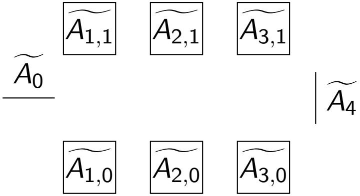

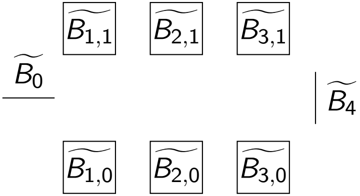

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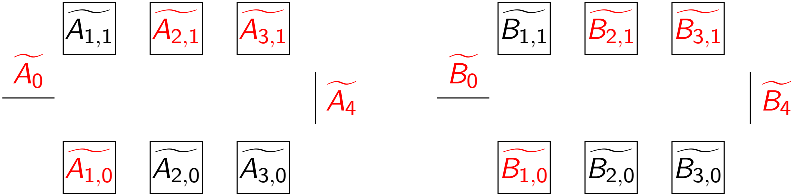

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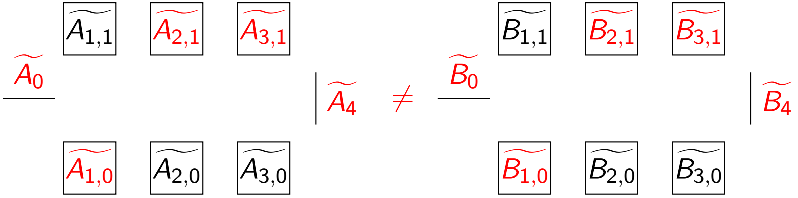

outputs zero

outputs non-zero

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Matrix-zeroizing attack: extended mixed-input attack

- *Invalid* inputs can induce the different outputs of equivalent BPs

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inp(i)	1	1	2

invalid input
indices 0 1 1

$$\frac{\widetilde{A_0}}{}\begin{array}{ccc} \boxed{\widetilde{A_{1,1}}} & \boxed{\widetilde{A_{2,1}}} & \boxed{\widetilde{A_{3,1}}} \\ & & \\ \boxed{\widetilde{A_{1,0}}} & \boxed{\widetilde{A_{2,0}}} & \boxed{\widetilde{A_{3,0}}} \end{array} \Big| \widetilde{A_4} \neq \frac{\widetilde{B_0}}{}\begin{array}{ccc} \boxed{\widetilde{B_{1,1}}} & \boxed{\widetilde{B_{2,1}}} & \boxed{\widetilde{B_{3,1}}} \\ & & \\ \boxed{\widetilde{B_{1,0}}} & \boxed{\widetilde{B_{2,0}}} & \boxed{\widetilde{B_{3,0}}} \end{array} \Big| \widetilde{B_4}$$

outputs zero

outputs non-zero

Attack 1: Matrix Zeroizing Attack

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Matrix-zeroizing attack: extended mixed-input attack

- *Invalid* inputs can induce the different outputs of equivalent BPs
- Summation of mixed-input can yield the different outputs of BPs

i	1	2	3
inp(i)	1	1	2

invalid input
010, 011, 100, 101, $\cdots$

$$\sum \; \widetilde{A_0} \; \begin{array}{ccc} \boxed{\widetilde{A_{1,1}}} & \boxed{\widetilde{A_{2,1}}} & \boxed{\widetilde{A_{3,1}}} \\ \boxed{\widetilde{A_{1,0}}} & \boxed{\widetilde{A_{2,0}}} & \boxed{\widetilde{A_{3,0}}} \end{array} \; \widetilde{A_4} \neq \widetilde{B_0} \; \begin{array}{ccc} \boxed{\widetilde{B_{1,1}}} & \boxed{\widetilde{B_{2,1}}} & \boxed{\widetilde{B_{3,1}}} \\ \boxed{\widetilde{B_{1,0}}} & \boxed{\widetilde{B_{2,0}}} & \boxed{\widetilde{B_{3,0}}} \end{array} \; \widetilde{B_4}$$

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Attack 2: Starting point = Principal Ideal Problem

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Short Principal Ideal Problem [BS16, CDPR16]

Given many multiples of h, we can recover

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in *quantum polynomial time*.

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Remark: every computations works correctly.

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- Run mixed-input attack on obfuscated program at level κ

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$$\boxed{\widetilde{A_{1,1}}} \quad \boxed{\widetilde{A_{2,1}}} \quad \boxed{\widetilde{A_{3,1}}}$$

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invalid input
indices 0 1 1

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$$\widetilde{A_0} \quad \boxed{\widetilde{A_{1,1}}} \quad \boxed{\widetilde{A_{2,1}}} \quad \boxed{\widetilde{A_{3,1}}} \quad \widetilde{A_4}$$

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[1] level parameters, scalar bundlings

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- Construct 2κ-level obfuscated program
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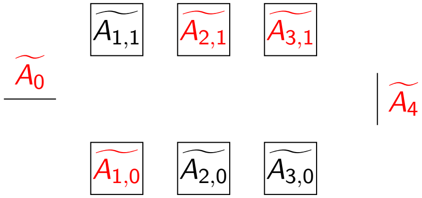

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invalid input

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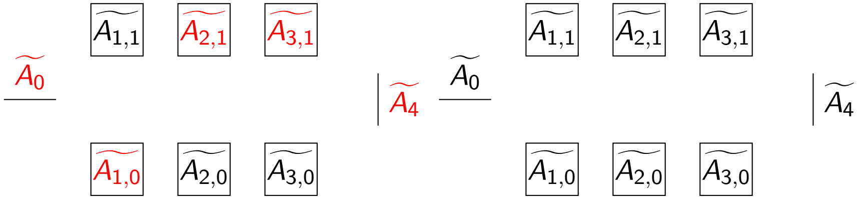

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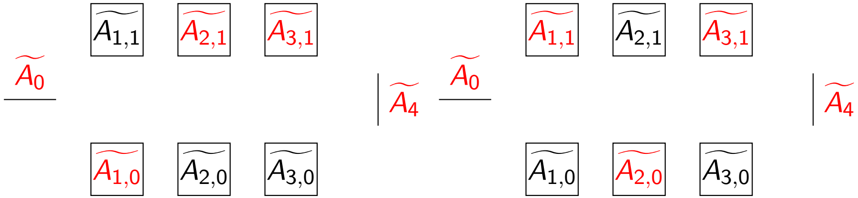

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Summary and work in progress

iO (using GGH13) / Attacks	Branching program obfuscators				Circuit obfuscators
	[GGH+13b]	[BR14]	[AGIS14, MSW14] [PST14, BGK+14] [BMSZ16]	[GMM+16]	[Zim15, AB15] [DGG+16]
[MSZ16]		✓	✓		
[CGH17]*	✓				
This work 1† [CHKL18]	✓	✓	✓	✓	
This work 2‡ [Pel18]			✓	✓	✓

* for input-partitionable branching programs

† for specific choices of parameters

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[MSZ16]		✓	✓		
[CGH17]⋆	✓				
This work 1† [CHKL18]	✓	✓	✓	✓	?
This work 2‡ [Pel18]	?		✓	✓	✓

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  - Extending the NTRU attack!
  - We also try to find a countermeasure on the attack

Perspectives / Open problems

- Obfuscation for evasive functions
- Countermeasure on the attacks
- Parameter constraints to prevent our classical attack[2]: $n = \tilde{\Omega}(\kappa^2\lambda)$
  - This constraint agrees to the current best algorithms to solve the overstretched NTRU problem

[2] n: dimension of space, κ: multilinearity level, λ: security parameter
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$\Rightarrow$ This gap can cause the significant weakness of concrete scheme!

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References I

Benny Applebaum and Zvika Brakerski.
Obfuscating circuits via composite-order graded encoding.
In TCC 2015, pages 528–556, 2015.

Martin R. Albrecht, Shi Bai, and Léo Ducas.
A subfield lattice attack on overstretched NTRU assumptions - cryptanalysis of some FHE and graded encoding schemes.
In Crypto 2016, pages 153–178, 2016.

Prabhanjan Ananth, Divya Gupta, Yuval Ishai, and Amit Sahai.
Optimizing obfuscation: Avoiding barrington's theorem.
In CCS 2014, pages 646–658. ACM, 2014.

Jean-François Biasse, Thomas Espitau, Pierre-Alain Fouque, Alexandre Gélin, and Paul Kirchner.
Computing generator in cyclotomic integer rings.
In Eurocrypt 2017, pages 60–88. Springer, 2017.

Boaz Barak, Oded Goldreich, Rusell Impagliazzo, Steven Rudich, Amit Sahai, Salil Vadhan, and Ke Yang.
On the (im) possibility of obfuscating programs.
In Crypto 2001, pages 1–18. Springer, 2001.

Boaz Barak, Sanjam Garg, Yael Tauman Kalai, Omer Paneth, and Amit Sahai.
Protecting obfuscation against algebraic attacks.
In Eurocrypt 2014, pages 221–238, 2014.

Saikrishna Badrinarayanan, Eric Miles, Amit Sahai, and Mark Zhandry.
Post-zeroizing obfuscation: New mathematical tools, and the case of evasive circuits.
In Eurocrypt 2016, pages 764–791, 2016.

References II

Zvika Brakerski and Guy N Rothblum.
Obfuscating conjunctions.
Crypto 2014, 2014.

Jean-François Biasse and Fang Song.
Efficient quantum algorithms for computing class groups and solving the principal ideal problem in arbitrary degree number fields.
In SODA 2016, pages 893–902. Society for Industrial and Applied Mathematics, 2016.

Ronald Cramer, Léo Ducas, Chris Peikert, and Oded Regev.
Recovering short generators of principal ideals in cyclotomic rings.
In Eurocrypt 2016, pages 559–585, 2016.

Yilei Chen, Craig Gentry, and Shai Halevi.
Cryptanalyses of candidate branching program obfuscators.
In Eurocrypt 2017, pages 278–307. Springer, 2017.

Jung Hee Cheon, Jinhyuck Jeong, and Changmin Lee.
An algorithm for ntru problems and cryptanalysis of the ggh multilinear map without a low-level encoding of zero.
LMS Journal of Computation and Mathematics, 19(A):255–266, 2016.

Nico Döttling, Sanjam Garg, Divya Gupta, Peihan Miao, and Pratyay Mukherjee.
Obfuscation from low noise multilinear maps.
ePrint, Report 2016/599, 2016.

Rex Fernando, Peter Rasmussen, and Amit Sahai.
Preventing CLT attacks on obfuscation with linear overhead.
In Asiacrypt 2017, pages 242–271, 2017.

References III

Sanjam Garg, Craig Gentry, and Shai Halevi.
Candidate multilinear maps from ideal lattices.
In Eurocrypt 2017, pages 1–17. Springer, 2013.

Sanjam Garg, Craig Gentry, Shai Halevi, Mariana Raykova, Amit Sahai, and Brent Waters.
Candidate indistinguishability obfuscation and functional encryption for all circuits.
FOCS 2013, 2013.

Sanjam Garg, Eric Miles, Pratyay Mukherjee, Amit Sahai, Akshayaram Srinivasan, and Mark Zhandry.
Secure obfuscation in a weak multilinear map model.
In TCC 2016, pages 241–268, 2016.

Paul Kirchner and Pierre-Alain Fouque.
Revisiting lattice attacks on overstretched ntru parameters.
In Eurocrypt 2017, pages 3–26. Springer, 2017.

Eric Miles, Amit Sahai, and Mor Weiss.
Protecting obfuscation against arithmetic attacks.
ePrint, Report 2014/878, 2014.

Eric Miles, Amit Sahai, and Mark Zhandry.
Annihilation attacks for multilinear maps: Cryptanalysis of indistinguishability obfuscation over GGH13.
In Crypto 2016, pages 629–658, 2016.

Rafael Pass, Karn Seth, and Sidharth Telang.
Indistinguishability obfuscation from semantically-secure multilinear encodings.
In Crypto 2014, pages 500–517, 2014.

References IV

Joe Zimmerman.
How to obfuscate programs directly.
In Eurocrypt 2015, pages 439–467, 2015.