On the statistical leak of the GGH13 multilinear map and its variants

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Introduction

In this talk:

- Focus on the GGH13 multilinear map
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- **Classical attacks:** zeroizing attacks
  ⇒ main application of GGH today: obfuscators
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In this talk:

- Focus on the GGH13 multilinear map
- Classical attacks: zeroizing attacks
  ⇒ main application of GGH today: obfuscators
- Contribution: analyze averaging attacks
  - In some case, we have a complete attack against GGH.
  - In some other cases, we get some leaked information.
Table of Contents

1 The GGH13 multilinear map

2 Zeroizing attacks and consequences

3 Averaging attacks
History of multilinear maps (until February 2015)

2000 Joux introduces bilinear maps (pairings) for cryptographic uses.

2003 Boneh and Silverberg introduce the concept of multilinear maps.

≥ 2003 Many applications.

2013 Garg, Gentry and Halevi publish the first candidate multilinear map (GGH13 map).

2013 Garg et al. publish the first candidate obfuscator, using the GGH13 map.

2013 Coron, Lepoint and Tibouchi propose another candidate multilinear map, relying on integers (CLT map).

2015 Gentry, Gorbunov and Halevi propose a graph-induced multilinear map (GGH15 map).
Cryptographic multilinear maps

Definition: \( \kappa \)-multilinear map

Different levels of encodings, from 0 to \( \kappa \).
Denote by \( C(a, i) \) a level-\( i \) encoding of the message \( a \).

- **Level-0 encoding**: a plaintext (message not encoded).
- **Addition**: \( \text{Add}(C(a_1, i), C(a_2, i)) = C(a_1 + a_2, i) \).
- **Multiplication**: \( \text{Mult}(C(a_1, i), C(a_2, j)) = C(a_1 \cdot a_2, i + j) \).
- **Zero-test**: \( \text{Zero-test}(C(a, \kappa)) = \text{True} \text{ iff } a = 0 \).
Cryptographic multilinear maps

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**Security:** What should be hard for a cryptographic multilinear map?
Application to multipartite key-exchange

**Objective:** $\kappa + 1$ users want to agree on a shared secret $s$. Let $D$ be a distribution over the message space.
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\begin{align*}
    a_1 &\leftarrow D \\
    C(a_1, 1) &\leftarrow D \\
    a_2 &\leftarrow D \\
    C(a_2, 1) &\leftarrow D \\
    s &= C(a_1 a_2 a_3 a_4, 3) \\
    C(a_3, 1) &\leftarrow D \\
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    C(a_4, 1) &\leftarrow D \\
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\end{align*}
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Application to multipartite key-exchange

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\text{User 1:} & \quad a_1 \leftarrow D \\
& \quad C(a_1, 1) \\
\text{User 2:} & \quad a_2 \leftarrow D \\
& \quad C(a_2, 1) \\
\text{Users 1-4 agree on:} & \quad s = C(a_1, 0)C(a_2, 1)C(a_3, 1)C(a_4, 1) \\
\text{User 3:} & \quad a_3 \leftarrow D \\
& \quad C(a_3, 1) \\
\text{User 4:} & \quad a_4 \leftarrow D \\
& \quad C(a_4, 1) \\
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The GGH13 multilinear map

Define $R = \mathbb{Z}[X]/(X^n + 1)$ with $n = 2^k$. 

Sample $g$ a "small" element in $R$. ⇒ the plaintext space is $P = R/\langle g \rangle$.

Sample $q$ a "large" integer. ⇒ the encoding space is $R_q = R/(qR) = \mathbb{Z}_q[X]/(X^n + 1)$. 

Notation We write $[r]_q$ or $[r]$ the elements in $R_q$, and $r$ (without $\cdot$) the elements in $R$. 

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The GGH13 multilinear map

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- Sample \( g \) a “small” element in \( R \).
  \( \Rightarrow \) the plaintext space is \( \mathcal{P} = R/\langle g \rangle \).
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**Notation**

We write \([r]_q\) or \([r]\) the elements in \( R_q \), and \( r \) (without \([\cdot]\)) the elements in \( R \).
The GGH13 multilinear map: encodings

- Sample $z$ uniformly in $R_q$.
- **Encoding**: An encoding of $a$ at level $i$ is

$$u = [(a + rg)z^{-i}]_q$$

where $a + rg$ is a small element in $a + \langle g \rangle$. 
The GGH13 multilinear map: encodings

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**Addition and multiplication**

**Addition:**

$$[(a_1 + r_1g)z^{-i}]_q + [(a_2 + r_2g)z^{-i}]_q = [(a_1 + a_2 + r'g)z^{-i}]_q.$$ 

**Multiplication:**

$$[(a_1 + r_1g)z^{-i}]_q \cdot [(a_2 + r_2g)z^{-j}]_q = [(a_1 \cdot a_2 + r'g)z^{-(i+j)}]_q.$$
The GGH13 multilinear map: zero-test

- Sample $h$ in $R$ of the order of $q^{1/2}$.
- Define

$$p_{zt} = [z^\kappa h g^{-1}]_q.$$
The GGH13 multilinear map: zero-test

- Sample \(h \in R\) of the order of \(q^{1/2}\).
- Define
  \[
p_{zt} = [z^\kappa hg^{-1}]_q.
  \]

**Zero-test**

To test if \(u = [cz^{-\kappa}]\) is an encoding of zero (i.e. \(c = 0 \mod g\)), compute

\[
[u \cdot p_{zt}]_q = [chg^{-1}]_q.
\]

This is small iff \(c\) is a small multiple of \(g\).
Question

How to compute an encoding of $a$ at level 1 when we only have the public parameters $R$, $q$ and $p_{zt}$?

Solution.

We add to the public parameters $-y$ an encoding of 1 at level 1 and $-x$ an encoding of 0 at level 1. To compute $C(a, 1)$:

Sample $r$ in $R$ and output $u = [ay + rx]_q$. 

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The GGH13 multilinear map: other public parameters

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Conclusion on the GGH13 map

- We have a mathematical object, that satisfies some properties (addition, multiplication, zero-test).
- What about its security?
Table of contents: 2 - Zeroizing attacks and consequences

1. The GGH13 multilinear map
2. Zeroizing attacks and consequences
3. Averaging attacks
Zeroizing attacks

Idea

When \( u = [cz^{-\kappa}]_q \) with \( c = bg \) a small multiple of \( g \), we have

\[
[u \cdot p_{zt}]_q = [chg^{-1}]_q = bh
\]

because \( bh \) is smaller than \( q \) so \([bh]_q = bh \in R\).

Example of attack (from GGH13)

Compute

\[
[x^2y^{\kappa-2}p_{zt}]_q = [g^2 \cdot r \cdot g^{-1}]_q = g \cdot r
\]

\(\Rightarrow\) recover multiples of \( g \), and then \( \langle g \rangle \).
Hu and Jia’s attack

Hu and Jia, 2015

An attacker can recover the shared secret $s$ in the multipartite key exchange protocol, when using the GGH13 multilinear map.

For this attack, we need $x$, the level 1 encoding of zero.

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An attacker can recover the shared secret $s$ in the multipartite key exchange protocol, when using the GGH13 multilinear map.

For this attack, we need $x$, the level 1 encoding of zero.

Question

Maybe the GGH13 map is still safe if we do not have low level encodings of zero?

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Not all obfuscators are broken yet

**Good news for obfuscators**

We do not need the public parameters $x$ and $y$ in the GGH13 map when used for obfuscators.

$\Rightarrow$ the attack of Hu and Jia does not apply.
Not all obfuscators are broken yet

Good news for obfuscators

We do not need the public parameters $x$ and $y$ in the GGH13 map when used for obfuscators.

$\Rightarrow$ the attack of Hu and Jia does not apply.

Yes but...

Still, many obfuscators using the GGH13 map were proven insecure using zeroizing techniques.
Table of contents: 3 - Averaging attacks

1. The GGH13 multilinear map

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Another approach: averaging

**Idea**

Instead of looking at the arithmetic properties of $R$, we use statistical properties.

This kind of attacks was already mentioned in the original article of GGH13.
Another approach: averaging

Idea

Instead of looking at the arithmetic properties of $R$, we use statistical properties.

This kind of attacks was already mentioned in the original article of GGH13.

Property: If $D$ is a distribution over $R$ and $x_1, \ldots, x_\ell$ are independent elements sampled from $D$, then

$$
\frac{1}{\ell} \sum_{i=1}^{\ell} x_i \xrightarrow{\ell \to +\infty} \mathbb{E}(x_1).
$$

With $\ell$ samples, we expect to get $\log(\ell)$ bits of precision for $\mathbb{E}(x_1)$. 
Notations and definitions (1)

Definitions

A distribution is said **centered** if its mean is zero.

A distribution is said **isotropic** if no direction is privileged.

**Notation:** We write in red the centered isotropic variables.
Notations and definitions (1)

**Definitions**

A distribution is said **centered** if its mean is zero.

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**Notation:** We write in **red** the centered isotropic variables.

**Gaussian distribution**

We denote by $D_\sigma$ the (discrete) Gaussian distribution centered in 0 and of variance $\sigma^2$.

**Remark.** $D_\sigma$ is a centered isotropic distribution (if $\sigma$ is large enough).
Definitions / Notation

- For $r \in R$, we denote $A(r) = r\bar{r}$ the **auto-correlation** of $r$, where $\bar{r}$ is the complex conjugate of $r$ when seen in $\mathbb{C}$.
- The **variance** of a centered variable $r$ is $\text{Var}(r) := \mathbb{E}(r\bar{r})$. 
Definitions and properties (2)

Definitions / Notation

- For $r \in R$, we denote $A(r) = r \bar{r}$ the **auto-correlation** of $r$, where $\bar{r}$ is the complex conjugate of $r$ when seen in $\mathbb{C}$.
- The **variance** of a centered variable $r$ is $\text{Var}(r) := \mathbb{E}(r \bar{r})$.

**Proposition:** If $r$ is sampled in $R$ according to a centered isotropic distribution, then

- $\mathbb{E}(r) = 0$
- $\text{Var}(r) = \mu \in \mathbb{R}$
Back to the attack: what do we know?

**Reminder:** We do not want to publicly give $x$ and $y$ anymore. So what is public?
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**Toy model inspired by obfuscators**
- we are given $R$, $q$ and $p_{zt}$ as before.
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**Toy model inspired by obfuscators**

- we are given \( R, q \) and \( p_{zt} \) as before.
- we are given \( u_i = [c_i z^{-i}] \) for \( 1 \leq i < \kappa \) and \( c_i \leftarrow D_\sigma \).
- such that \( u_i u_{\kappa-i} \) is an encoding of 0 at level \( \kappa \).

\[
\begin{array}{cccccccc}
\kappa - 2 & \kappa - 1 & \kappa - 2 & \kappa - 3 & \kappa - 4 & \kappa - 5 & \kappa - 6 & \kappa - 7 \\
0 & 1 & 2 & 3 & 4 & 5 & 6 & \kappa - 3 & \kappa - 1 \\
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levels of encodings
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- such that $u_i u_{\kappa-i}$ is an encoding of 0 at level $\kappa$.
Idea of the attack

Recall our model

- we are given $u_i = [c_iz^{-i}]$ for $1 \leq i \leq \kappa - 1$ and $c_i \leftarrow D_\sigma$.
- such that $u_i u_{\kappa-i}$ is an encoding of $0$ at level $\kappa$.

Observation:

$$[u_i u_{\kappa-i} \cdot p_{zt}] = [c_i c_{\kappa-i} \cdot h/g]$$
$$= c_i c_{\kappa-i} \cdot h/g$$
$$= c_i^* \cdot h/g$$
Idea of the attack (2)

Recall

We know

\[ c_i^* \cdot h/g \]

for \( 1 \leq i \leq \kappa \), with \( c_i^* \) centered and isotropic.
Idea of the attack (2)

Recall

We know

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\[ \mathbb{E}(c_i^*) = 0 \Rightarrow \text{we do not learn anything with } \mathbb{E}(c_i^* \cdot h/g). \]
Idea of the attack (2)

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We know

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- \( \mathbb{E}(c_i^*) = 0 \Rightarrow \) we do not learn anything with \( \mathbb{E}(c_i^* \cdot h/g) \).
- \( \text{Var}(c_i^*) = \mathbb{E}(A(c_i^*)) = \mu \in \mathbb{R} \) is some scalar \( \Rightarrow \) we obtain

\[
\frac{1}{\kappa} \sum_{i=1}^{\kappa} A(c_i^* \cdot h/g) \xrightarrow{\kappa \to +\infty} \mu A(h/g).
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\frac{1}{\kappa} \sum_{i=1}^{\kappa} A(c_i^* \cdot h/g) \xrightarrow{\kappa \to +\infty} \mu A(h/g).
\]

We get an approximation of \( A(h/g) \) with \( \log(\kappa) \) bits of precision.
GGH13 counter-measure

GGH13’s authors noticed that their scheme was subject to averaging attacks \(\Rightarrow\) they proposed a countermeasure.

**Definition**

Let \(z_i\) be the representative of \([z^i]\) in \(R\) with coefficients in \([-q/2, q/2]\).

**Idea:** choose \(c_i\) such that \(c_i/z_i\) is isotropic.

**Counter-measure**

- Sample \(\tilde{c}_i \leftarrow D_\sigma\).
- Define \(c_i = \tilde{c}_i \cdot z_i\).
- And \(u_i = [c_iz^{-i}]\) as before.
Adapting the attack to the counter-measure

Recall

- \( c_i = \tilde{c}_i \cdot z_i \).
- \( u_i = [c_i z^{-i}] \).
- \( u_i u_{\kappa-i} \) is an encoding of 0 at level \( \kappa \).

Observation:

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[u_i u_{\kappa-i} \cdot pzt] = \tilde{c}_i \tilde{c}_{\kappa-i} \cdot z_i z_{\kappa-i} \cdot h/g
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But: the \( z_i \) are isotropic and independent.
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But: the \( z_i \) are isotropic and independent.

Averaging: we get an approx of \( \mu A(h/g) \), for some constant \( \mu \).
Conclude the attack

Lemma

If we have

- an approximation of $A(h/g)$ with $\log \ell$ bits of precision,
- a guarantee that for any encoding $[cz^{-i}]$, the coefficients of $c$ are less than $\ell/2$.

Then, we can recover $A(h/g)$ exactly and attack the GGH13 map.
Conclude the attack

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Then, we can recover $A(h/g)$ exactly and attack the GGH13 map.

Do we get enough samples for recovering $A(h/g)$ exactly?
- Without the counter-measure $\Rightarrow$ yes.
- With the counter-measure $\Rightarrow$ no, but this is because of constraints in the sampling procedure.
Conclusion

In the case where $q$ is polynomial:

- complete attack without the counter-measure (if $\kappa$ is large enough).
- leaked information with the counter-measure.
- other variants (adapted from [DGG+16]$^2$): leaked information but no complete attack.

$^2$Döttling, N. et al. “Obfuscation from Low Noise Multilinear Maps”.
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*Thank you for your attention.*

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