EQUILIBRIUM STATISTICAL MECHANICS EXPLANATION OF OCEANIC JETS AND RINGS

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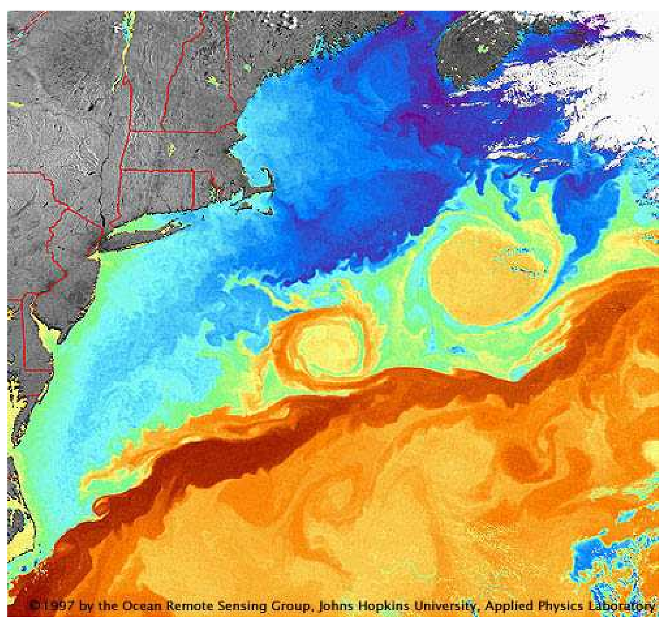
Statistical Mechanics of Large Scale Geophysical Flows

Large scale statistics of turbulent flows

In many applications of fluid dynamics, one of the most important problem is the prediction of the very high Reynolds' large-scale flows. The highly turbulent nature of such flows, for instance ocean circulation or atmosphere dynamics, renders a probabilistic description desirable. A statistical mechanics explanation of the self-organization of geophysical flows has been proposed by Robert-Sommeria and Miller (RSM).

Toward ocean applications



Left panel: robust coherent structures in the ocean: rings and midlatitude eastward jets (here the Gulf Stream), Reynolds number Re $\approx 10^{10}$.

We show in the following that oceanic rings and midlatitude eastward jets can actually be explained in the framework of RSM theory.

Statistical Mechanics of the Quasi-Geostrophic Model

1-1/2 layer quasi-geostrophic equations.

$$\frac{\partial q}{\partial t} + \mathbf{u}.\nabla q = 0 \quad \text{with} \quad q = \Delta\psi - \frac{\psi}{R^2} + \beta y \quad \text{and} \quad \mathbf{u} = (\partial_y\psi, \partial_x\psi),$$

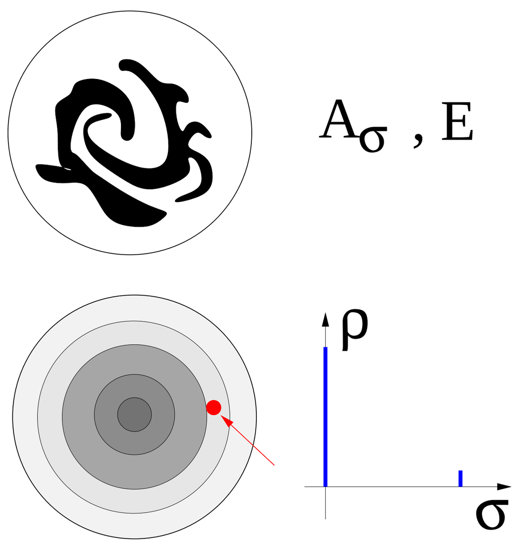
where q is the potential vorticity (PV), $\zeta = \Delta\psi$ is the relationve vorticity, βy is the planetary vorticity (beta effect) and ψ/R^2 a stretching term with R is the Rossby radius of deformation, a screening length scale.

Conservation laws. Energy: $E = \frac{1}{2}\int_D \mathrm{d}x\mathrm{d}y \, (\nabla\psi)^2 + \frac{\psi^2}{R^2}$, Casimirs: $C_f(q) = \int_D \mathrm{d}x\mathrm{d}y \, f(q)$

The Casimirs conservation are equivalent to an "incompressibility" constraint: conservation of the area A_σ associated with each level of PV between σ and $\sigma + \mathrm{d}\sigma$.

Robert-Miller-Sommeria theory

The observed flow is the most probable one among all the states that satisfy the constraints of the dynamics (given by conservation laws). Assuming that the dynamics explores evenly the phase space, it is possible to count the number of microstate q associated with a given macrostate ρ:



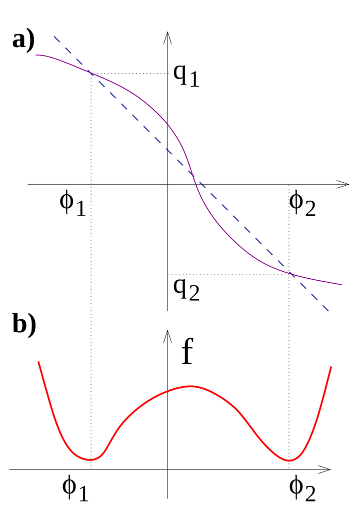
RSM Variational problem:

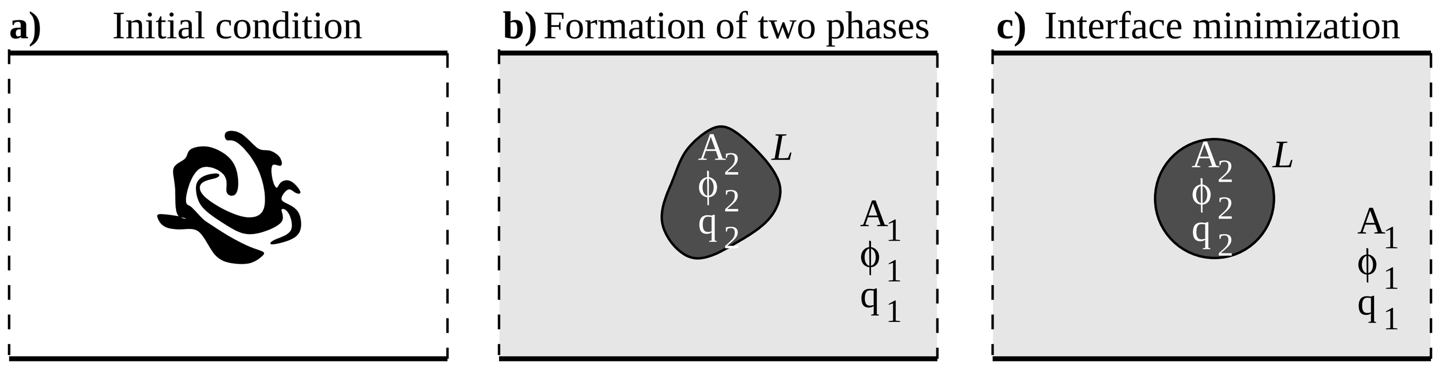
- To find $\overline{q} = \int \mathrm{d}\sigma \, \sigma\rho$ (coarse-grained PV field)
- Maximize $\mathcal{S} = -\int \mathrm{d}x\mathrm{d}y\mathrm{d}\sigma \, \rho \ln \rho$
- With constraints expressed in term of ρ.

Critical points are dynamical equilibria:
$\delta\mathcal{S} - \beta_t \delta E + \int \mathrm{d}\sigma \, \alpha \delta A_\sigma = 0$
gives a relation $\rho(\psi)$ and then $\overline{q} = g(\overline{\psi})$
Which of them are entropy maxima ?

Thermodynamical analogy with bubble formation

- Analytical computations in the limit R small.
- Minimize the free energy (with $\phi = \psi/R^2$):
  $\mathcal{F} = \int_D \mathrm{d}\mathbf{r} \left[\frac{1}{2}R^2(\nabla\phi)^2 + f(\phi) - \phi\beta y\right]$
- with constraint $\mathcal{A} = \int_D \mathrm{d}x\mathrm{d}y \, \phi$
- Critical points satisfy
  $f'(\overline{\phi}) = \overline{q}(\phi) + 2\phi - \alpha$





Conclusion: when $\beta = 0$ (no beta effect), statistical equilibria are regions of homogenized coarse-grained potential vorticity separated by interfaces of minimal length. These interfaces, corresponding to strong jets of width R, cost free energy.

Dynamics: cascade processes and ribbon turbulence

Scales smaller than R
$\partial_t\zeta + J(\psi, \zeta) = 0$

- Kinetic energy $E_c = -\int \mathrm{d}r \, \zeta\psi$ cascades towards **large** scales.
- Enstrophy $Z = \int \mathrm{d}r \, \zeta^2$ cascades towards **small** scales.

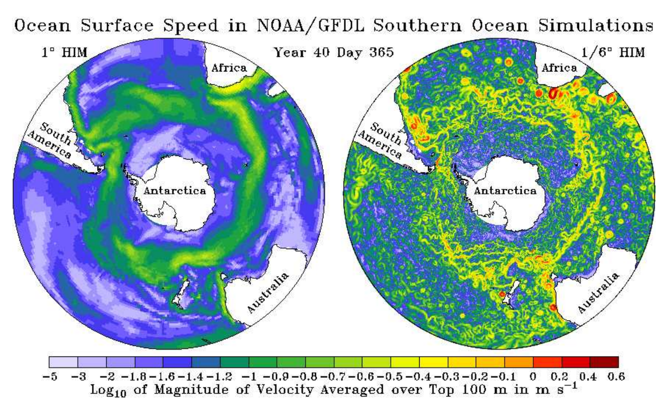
Scales larger than R
$\partial_\tau\psi + J(\zeta, \psi) = 0 \quad \tau = R^2 t$

- Kinetic energy $E_c = -\int \mathrm{d}r \, \zeta\psi$ cascades towards **small** scales.
- Potential energy $E_p = \int \mathrm{d}r \, \psi^2$ cascades toward **large** scales.

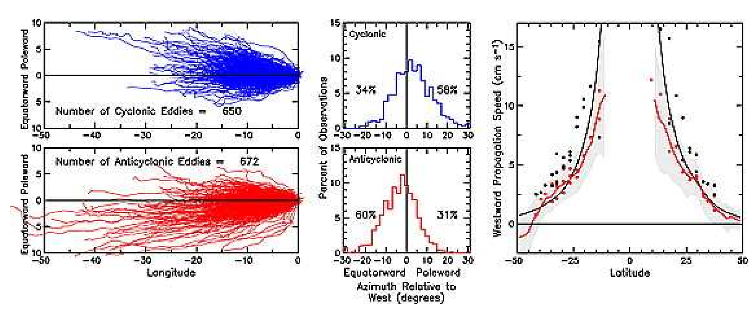
The inverse cascade of potential energy, and the condensation of kinetic energy around R leads to ribbons (jets) separating regions of homogenized potential vorticity.

Applications to the ocean

Ocean Mesoscale Vortices as Statistical Equilibria



Rings are everywhere
Hallberg-Gnanadesikan - JPO 2006



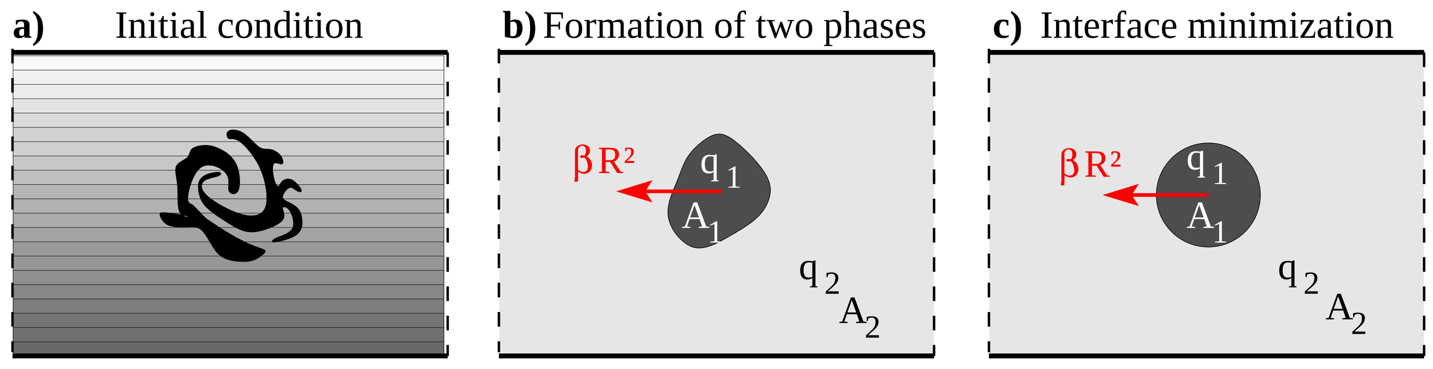
Westward drift observations
Chelton and co. - GRL 2007

- Both cyclonic and anticyclonic rings drift westward with a velocity $\tilde{\beta}R^2$.
- Tendency of anticyclonic rings to drift equatorward, of cyclonic rings to drift poleward.

On a beta plane, with translational invariance in x direction, the beta effect can be cancelled by considering a referential drifting tpward the west at speed βR^2. The results wihtout beta plane apply, and:

- **Zonal drift βR^2: consequence of the homogenization of PV**
- **Meridional drift: consequence of the conservation of linear monentum**

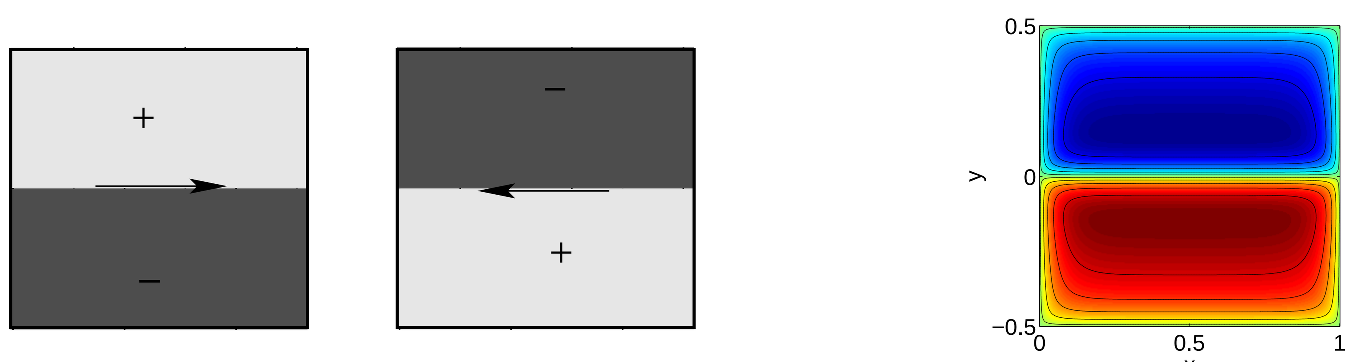
$$\mathcal{L} = \int_D \mathrm{d}x\mathrm{d}y \, qy$$



Conclusion: oceanic rings are close to an equilibrium state. They are bubbles of homogenized potential vorticity.

Ocean Midlatitude Jets as Statistical Equilibria

- With western boundaries, drifting structures can not be equilibria
- Without beta plane, eastward and westward jets are degenerated equilibrium states
- The beta effect ($\beta > 0$) acts like a magnetic field and favors westward jets
- Eastward jets remains metastables for $\beta < \pi^2 U/L_x^2$.



Left: PV fields for the eastward jet and the westward jet configurations. Right: equilibrium streamfunction in the eastward jet case on a beta plane.

Conclusion: oceanic eastward jets are marginally unstable statistical equilibria.

References

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