MIXING THROUGH TURBULENT CASCADES

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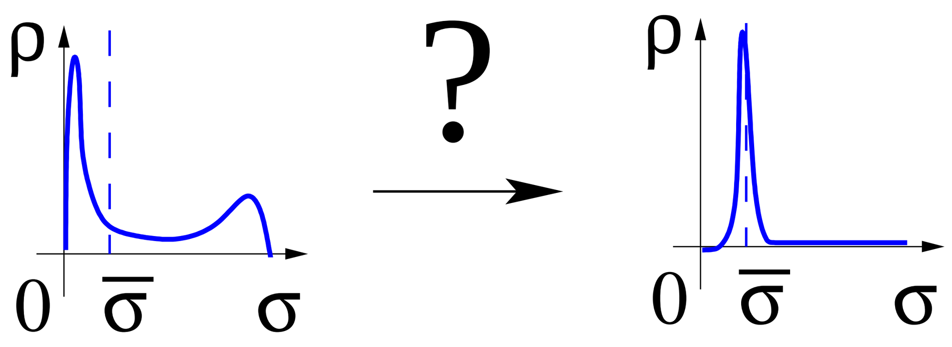
Mixing of a passive tracer

What is the temporal evolution of the tracer distribution ?



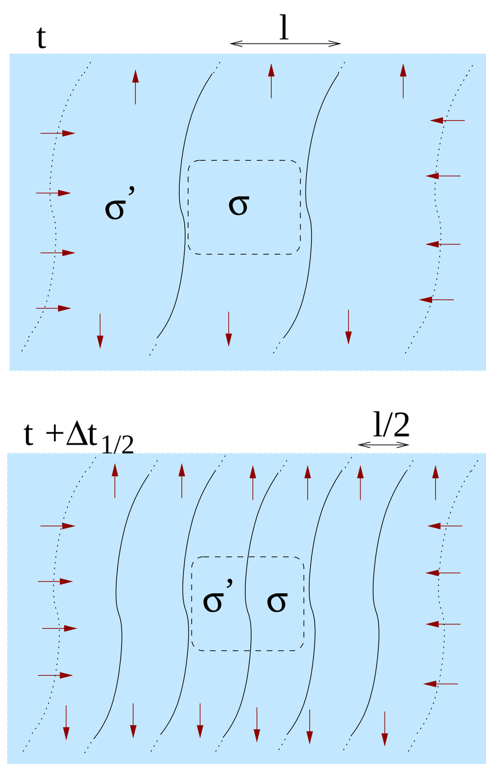
We propose a phenomenological approach, taking into account a single parameter for the turbulent flow, i.e a global straining rate s.

Mixing as a self-convolution process

The key idea of the model is that a probe of width l_0 sees structures coming from larger and larger scales, because of the turbulent cascade. This coarse-grained process dissipates scalar fluctuations.

- A succession of scalar sheets of width l are **stretched**, with $\mathrm{d}l = -sl\mathrm{d}t$.
- After time $\Delta t_{1/2}$, the scalar field filtered at scale l becomes the average of two realisations of the field at the previous time (and $l_{1/2} = l/2$).
- The probabilities of scalar values in adjacent strips can be assumed independent, as they come from regions initially far apart (through **folding**).

$$p(\sigma, t+\Delta t_{1/2}) = 2\int \mathrm{d}\sigma'\, p(\sigma', t)p(2\sigma-\sigma', t)\ .$$



A dynamical equation for a scalar advected by turbulence

Laplace transform : $\widetilde{p}(2\kappa, t+\Delta t_{1/2}) = [\widetilde{p}(\kappa, t)]^2$.

Similarly, after $\Delta t_{1/n}$ ($l_{1/n} = l/n$), the PDF is : $\widetilde{p}(n\kappa, t+\Delta t_{1/n}) = [\widetilde{p}(\kappa, t)]^n$.

Taking $n = 1 + s(t)dt$ leads to

$$\boldsymbol{\partial_t \widetilde{p} = s(t)[\widetilde{p}\ln\widetilde{p} - \kappa\partial_\kappa \widetilde{p}]}$$

whose solution is

$$\widetilde{p}(\kappa, t) = \left[\widetilde{p}\left(\frac{\kappa}{f}, 0\right)\right]^f \quad \text{with} \quad f = \exp\left(\int_0^t \mathrm{d}t'\, s(t')\right)$$

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Other mixing models

- Agregation process : $\partial_t \widetilde{p} = s(t)\left[f(t)\left[\widetilde{p}^{1+1/f} - \widetilde{p}\right] - \kappa\partial_\kappa \widetilde{p}\right]$
  Tends to a γ-distribution
  Villermaux and Duplat PRL, 2003
- Model LMSE, IEM: $\partial_t p = s(t)\partial_\sigma\left[p\left(\sigma - \overline{\sigma}\right)\right]$
  Contraction of the distribution around the mean
  Dopazo et O'Brien and *Villermaux and Duvillon*
- Curl model: $\partial_t \widetilde{p}(\kappa, t) = s(t)\left[\widetilde{p}^2(\kappa/2, t) - \widetilde{p}(\kappa, t)\right]$

Experimental test

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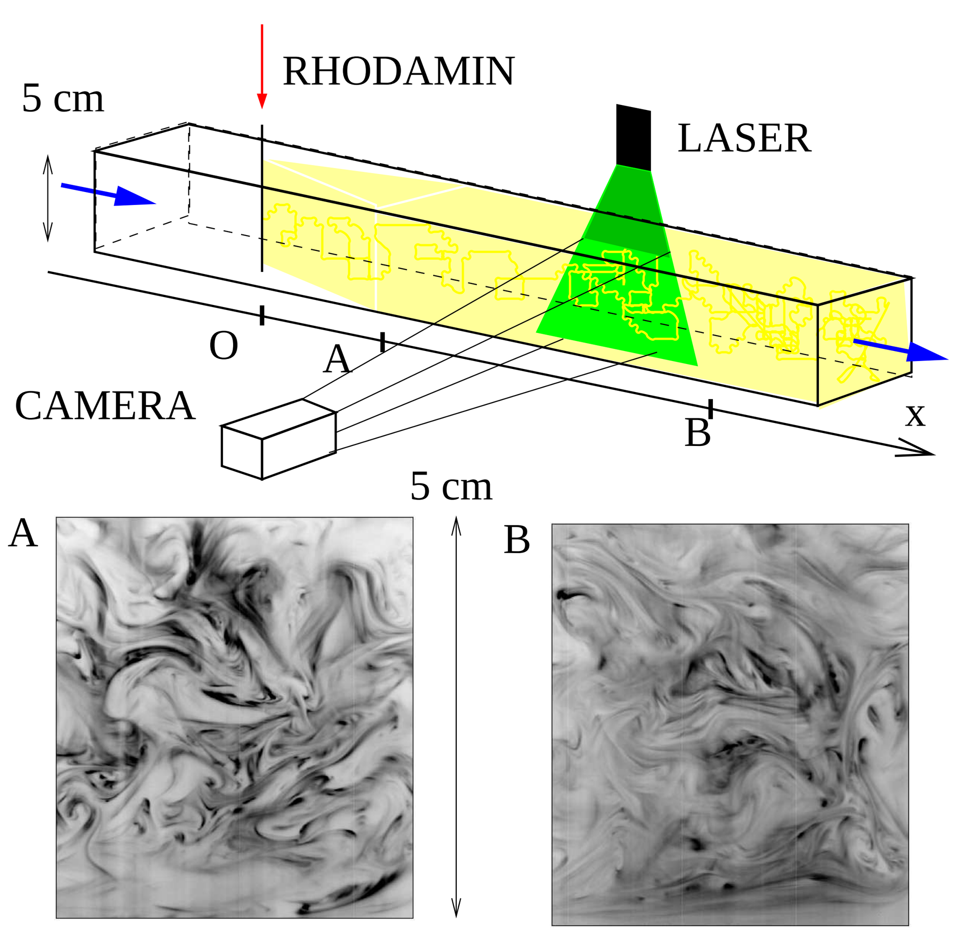
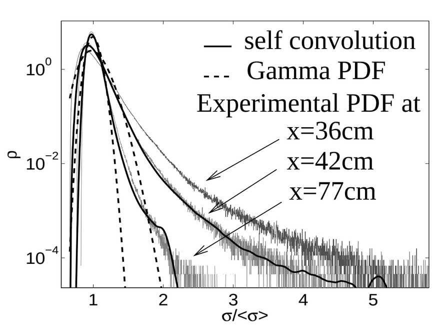
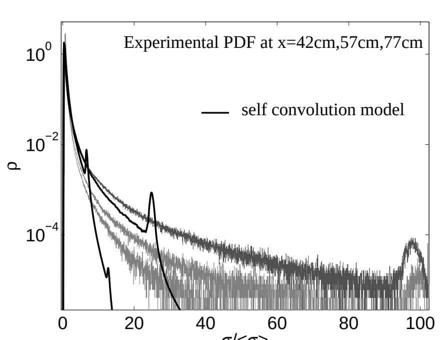




Vertical injector



Ponctual injector

Intermittency

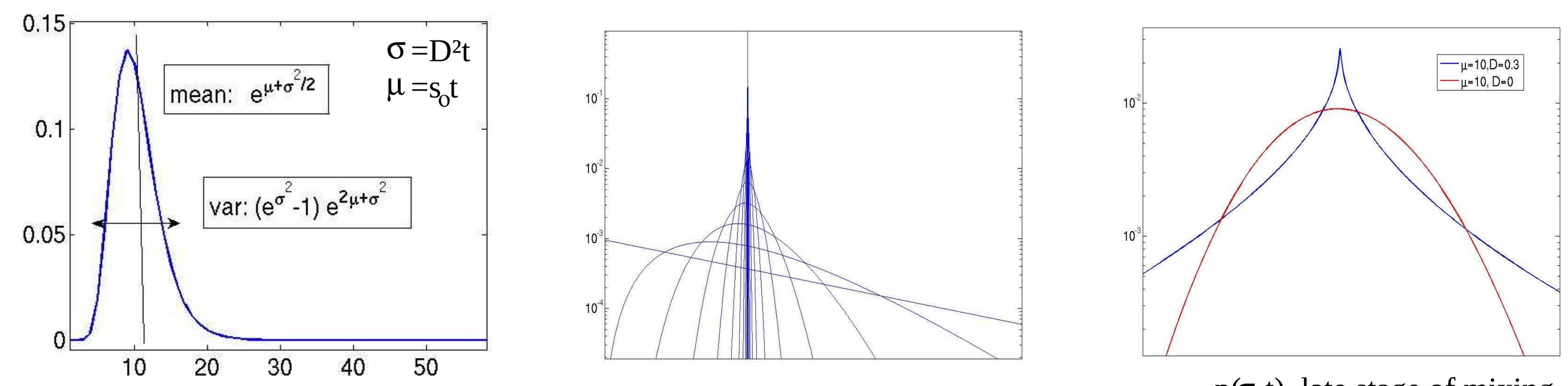
Fluctuations of "stretching history" f: $\mathcal{P}(f, t)$

$$\widetilde{p}(\kappa, t) = \int \mathrm{d}f\ \mathcal{P}(f, t)\widetilde{p}(\kappa, f) = \int \mathrm{d}f\ \mathcal{P}(f, t)\left[\widetilde{p}\left(\frac{\kappa}{f}, 0\right)\right]^f$$

Case of a log-normal distribution

Assuming that $(\int_0^t \mathrm{d}t' s(t') - s_0 t)$ is a random walk, that $P(\int_0^t \mathrm{d}t'\ s)$ is a gaussian with variance $Dt^{1/2}$ and mean $\mu = s_0 t$, then $\mathcal{P}\left(f = e^{\int_0^t \mathrm{d}t' s(t')}\right)$ is a log-normal distribution:

$$\mathcal{P}(f, t) = \frac{1}{fDt^{1/2}\sqrt{2\pi}}\exp\left(-\frac{(\ln f - s_0 t)^2}{2D^2 t}\right)\ ,$$



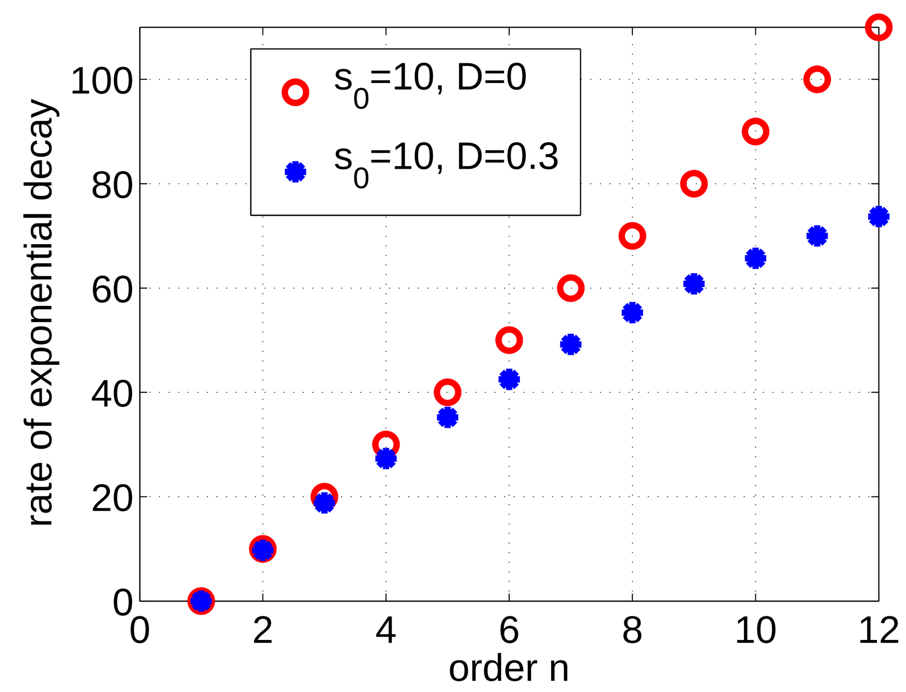
Distribution of stretching parameter f — p(σ,f) for increasing f — p(σ,t), late stage of mixing

Cumulants

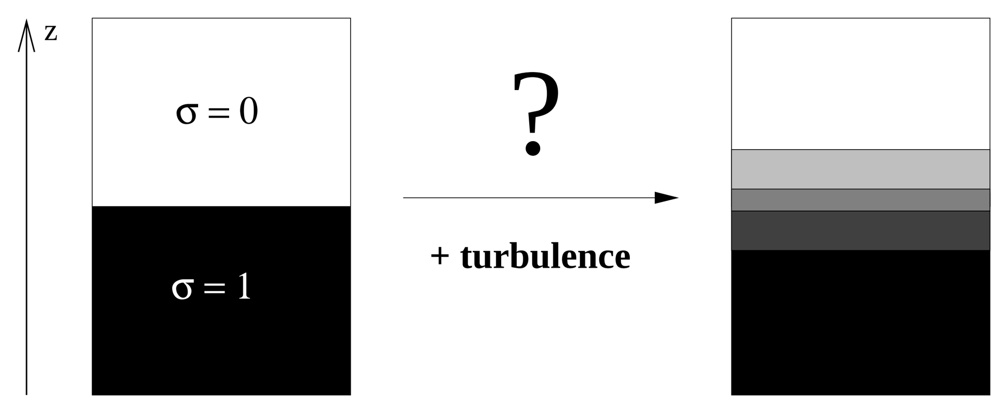
$$c_n(t) = (-\partial_\kappa)^n \ln(\widetilde{p}(\kappa, t))\big|_{\kappa=0}$$

In the case of a log-normal distribution for f,

$$c_{n+1}(t) = c_{n+1}(0)\exp\left(-\left[ns_0 - n^2D^2\right]t\right)$$



Mixing in stratified flows



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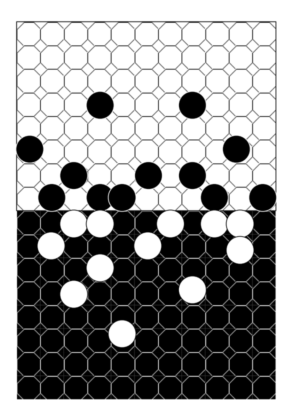
Temporal evolution of $p(\sigma, z, t)$

1. Turbulent transport
2. Buoyancy (sedimentation)
3. Irreversible mixing through turbulent cascade

$$\partial_t p = \partial_z(D\partial_z p) + \partial_z(D\beta p(\sigma - \overline{\sigma})) + s\ \mathcal{D}issipation$$



Mixing of a stable interface