The vertical structure of quasi-geostrophic turbulence

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Barotropization in the oceans?

Injection of the energy at the surface: where does it go ?

The primary source is baroclinic instability, involving surface intensified modes

Barotropization : tendency to spread the energy on the vertical

Continuously stratified quasi-geostrophic flows

$$\partial_t q + J(\psi, q) = 0\,, \quad J(\psi, q) = \partial_x \psi \partial_y q - \partial_y \psi \partial_x q$$

Potential vorticity $q(x, y, z, t)$**, streamfunction** $\psi(x, y, z, t)$

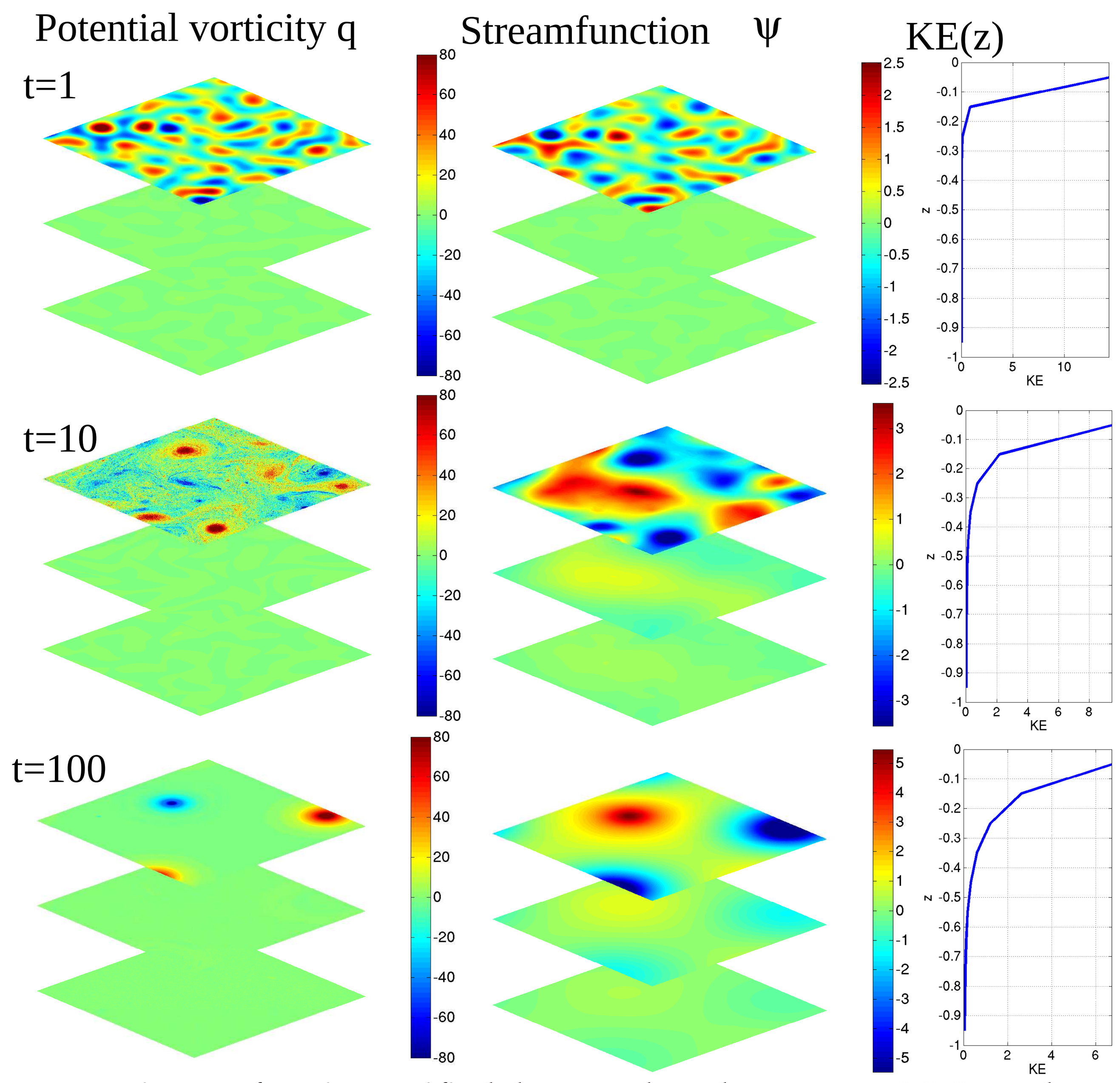
$$q = \Delta \psi + \frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial}{\partial z}\psi\right) + \beta_c y$$

Boundary conditions $\partial_z \psi|_{z=0} = 0, \ \left.\frac{f_0}{N^2}\partial_z \psi\right|_{z=-H} = -h_b\,,$

Parameters ratio rotation/stratification $f_0/N(z)$, beta effect β_c, topography h_b

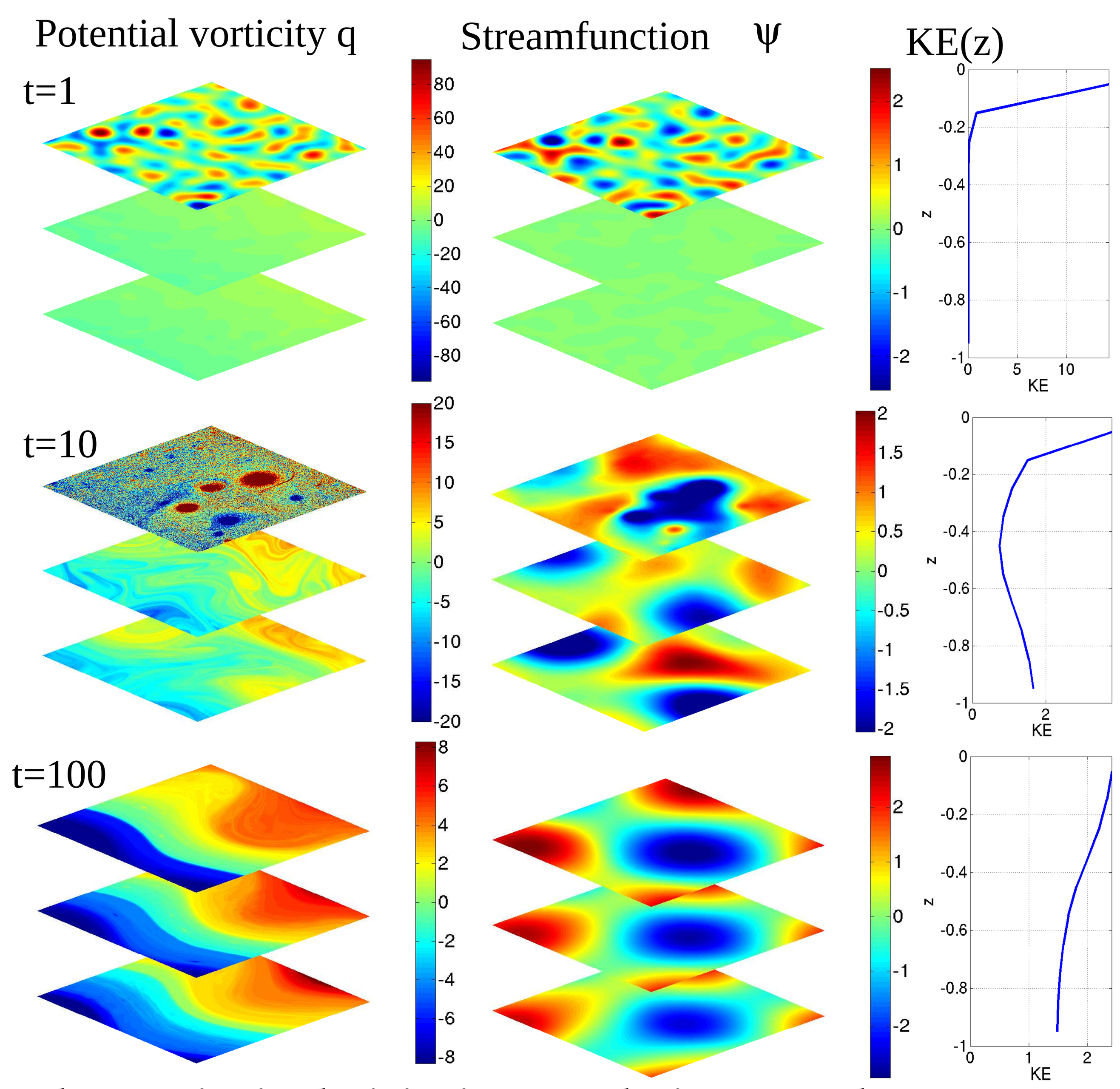
For an initial unstable velocity field $\psi_0(x, y, z)$**, what is the effect of** $\beta_c y$ **and** h_b **on the final state organization ?**

Surface-quasi-geostrophic dynamics



The energy remains surface intensified, but reaches the gravest SQG mode.

Add beta effect $\beta_c y$ or other large scale PV gradients



Beta favors barotropization by injecting perturbation enstrophy

Equilibrium statistical mechanics

Robert-Sommeria-Miller theory

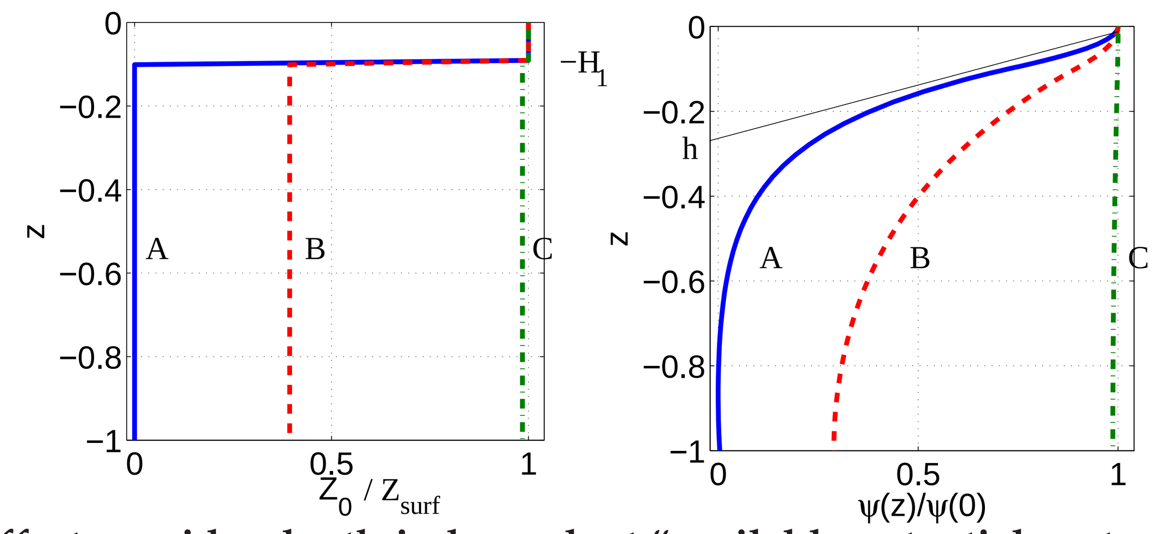
- **The observed flow is the most probable one among all the states that satisfy the constraints of the dynamics** *Onsager 49, ... , Robert-Sommeria-Miller early 90's , ...*
- Stratified case: *Merryfield 98, Schecter 05*
- **In a low energy limit**, the computation of statistical equilibria amounts to solve

$$\min_{\overline{q}}\left\{\mathcal{Z} = \frac{1}{2}\int \mathrm{d}x\mathrm{d}y\mathrm{d}z\ \frac{\overline{q}^2}{Z_0(z)} \mid \mathcal{E}\left[\overline{q}\right] = E_0\right\}$$

$$\mathcal{E}[\overline{q}] = -\frac{1}{2}\int \mathrm{d}x\mathrm{d}y\mathrm{d}z\ \overline{q}\psi,$$

- It is shown by applying a method proposed by *Bouchet 08*, and **it generalizes the minimum (coarse-graines) enstrophy variational problem of** *Bretherton Haidvogel JFM 76*. This
- **Critical points** : $\delta\mathcal{Z} + \beta\delta\mathcal{E} = 0$, which gives $\overline{q} = \beta Z_0 \psi$.

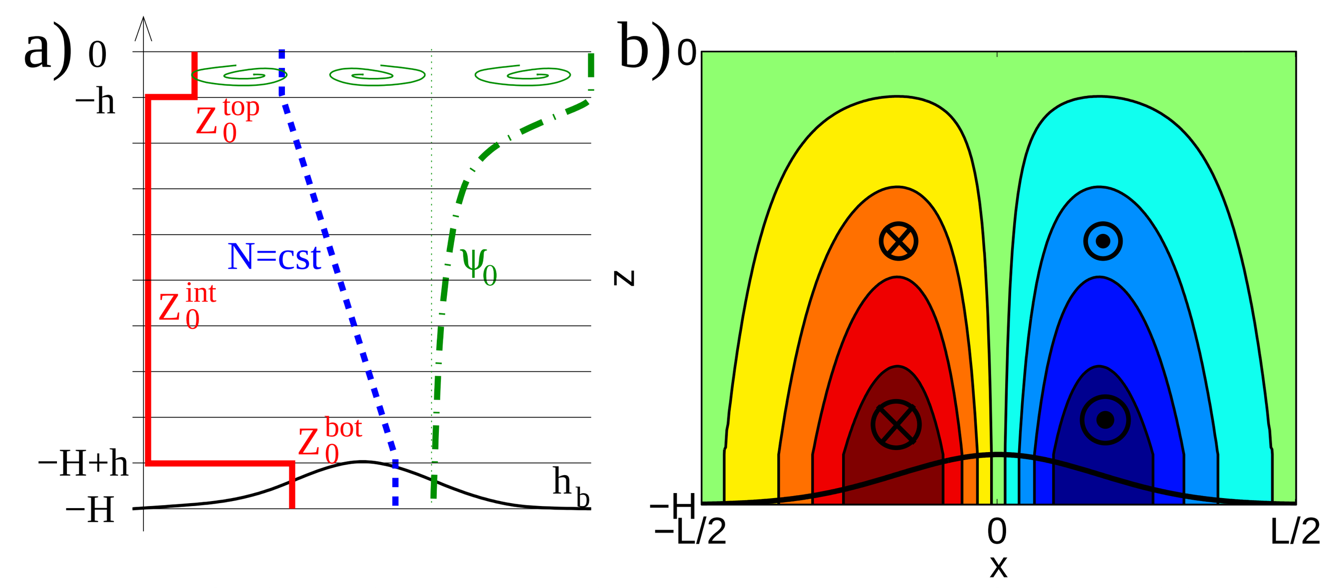
the crucial role of the fine-grained enstrophy profile $Z_0(z)$



The beta effect provides depth-independant "available potential enstrophy".
Z_0 **depth independant ⇒ depth independant equilibrium state.**

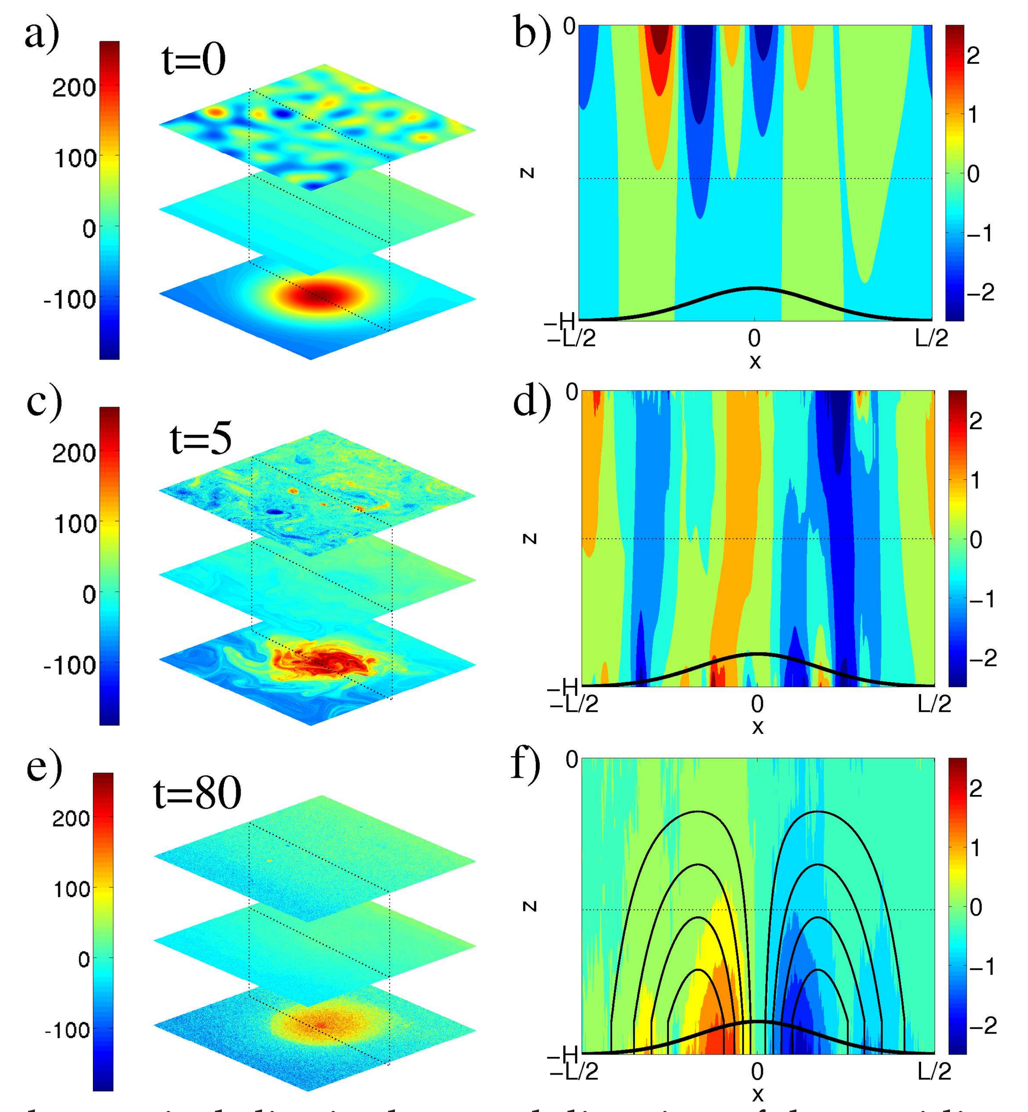
The role of bottom topography h_b

Statistical equilibria above a large topographic anomaly



Z_0 **bottom intensified and low energy limit: bottom intensified equilibrium state.**

Formation of bottom-trapped currents



Left: PV field. Right: vertical slice in the zonal direction of the meridional velocity field.

References

- **A. Venaille 2012,** *J. Fluid Mechanics*, **(in press)**
- **A. Venaille; G. Vallis, S. Griffies 2012** *J. Fluid Mechanics*, **(in revision)**
- **F. Bouchet, A. Venaille, 2012** *Physics Reports* **(in press).**