## A BoostingTutorial

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## Example: "How May I Help You?"

- goal: automatically categorize type of call requested by phone customer (Collect, CallingCard, PersonToPerson, etc.)
- yes I'd like to place a collect call long distance please (Collect)
- operator I need to make a call but I need to bill it to my office (ThirdNumber)
- yes I'd like to place a call on my master card please (CallingCard)
- I just called a number in sioux city and I musta rang the wrong number because I got the wrong party and I would like to have that taken off of my bill (BillingCredit)
- observation:
- easy to find "rules of thumb" that are "often" correct
- e.g.: "IF 'card' occurs in utterance

THEN predict 'CallingCard' "

- $\underline{\text { hard }}$ to find single highly accurate prediction rule


## The Boosting Approach

- devise computer program for deriving rough rules of thumb
- apply procedure to subset of examples
- obtain rule of thumb
- apply to 2nd subset of examples
- obtain 2nd rule of thumb
- repeat $T$ times


## Details

- how to choose examples on each round?
- concentrate on "hardest" examples (those most often misclassified by previous rules of thumb)
- how to combine rules of thumb into single prediction rule?
- take (weighted) majority vote of rules of thumb


## Boosting

- boosting = general method of converting rough rules of thumb into highly accurate prediction rule
- technically:
- assume given "weak" learning algorithm that can consistently find classifiers ("rules of thumb") at least slightly better than random, say, accuracy $\geq 55 \%$ (in two-class setting)
- given sufficient data, a boosting algorithm can provably construct single classifier with very high accuracy, say, $99 \%$


## Outline of Tutorial

- brief background
- basic algorithm and core theory
- other ways of understanding boosting
- experiments, applications and extensions


## Brief Background

## The Boosting Problem

- "strong" PAC algorithm
- for any distribution
- $\forall \epsilon>0, \delta>0$
- given polynomially many random examples
- finds classifier with error $\leq \epsilon$ with probability $\geq 1-\delta$
- "weak" PAC algorithm
- same, but only for $\epsilon \geq \frac{1}{2}-\gamma$
- [Kearns \& Valiant '88]:
- does weak learnability imply strong learnability?


## Early Boosting Algorithms

- [Schapire '89]:
- first provable boosting algorithm
- call weak learner three times on three modified distributions
- get slight boost in accuracy
- apply recursively
- [Freund '90]:
- "optimal" algorithm that "boosts by majority"
- [Drucker, Schapire \& Simard '92]:
- first experiments using boosting
- limited by practical drawbacks


## AdaBoost

- [Freund \& Schapire '95]:
- introduced "AdaBoost" algorithm
- strong practical advantages over previous boosting algorithms
- experiments and applications using AdaBoost:
[Drucker \& Cortes '96]
[Jackson \& Craven '96]
[Freund \& Schapire '96]
[Quinlan '96]
[Breiman '96]
[Maclin \& Opitz '97]
[Bauer \& Kohavi '97]
[Schwenk \& Bengio '98]
[Schapire, Singer \& Singhal '98]
[Abney, Schapire \& Singer '99]
[Haruno, Shirai \& Ooyama '99]
[Cohen \& Singer' 99]
[Dietterich '00]
[Schapire \& Singer '00]
[Collins '00]
[Escudero, Màrquez \& Rigau '00]
[Iyer, Lewis, Schapire et al. '00]
[Onoda, Rätsch \& Müller '00]
[Tieu \& Viola '00]
[Walker, Rambow \& Rogati '01]
[Rochery, Schapire, Rahim \& Gupta '01]
[Merler, Furlanello, Larcher \& Sboner '01]
[Di Fabbrizio, Dutton, Gupta et al. '02]
[Qu, Adam, Yasui et al. '02]
[Tur, Schapire \& Hakkani-Tür '03]
[Viola \& Jones '04]
[Middendorf, Kundaje, Wiggins et al. '04]
- continuing development of theory and algorithms:
[Breiman '98, '99]
[Schapire, Freund, Bartlett \& Lee '98]
[Grove \& Schuurmans '98]
[Mason, Bartlett \& Baxter '98]
[Schapire \& Singer '99]
[Cohen \& Singer '99]
[Freund \& Mason '99]
[Domingo \& Watanabe '99]
[Mason, Baxter, Bartlett \& Frean '99, '00]
[Duffy \& Helmbold '99, '02]
[Freund \& Mason '99]
[Ridgeway, Madigan \& Richardson '99]
[Kivinen \& Warmuth '99]
[Friedman, Hastie \& Tibshirani '00]
[Rätsch, Onoda \& Müller '00]
[Rätsch, Warmuth, Mika et al. '00]
[Allwein, Schapire \& Singer '00]
[Friedman '01]
[Koltchinskii, Panchenko \& Lozano '01]
[Collins, Schapire \& Singer '02]
[Demiriz, Bennett \& Shawe-Taylor '02]
[Lebanon \& Lafferty '02]
[Wyner '02]
[Rudin, Daubechies \& Schapire '03]
[Jiang '04]
[Lugosi \& Vayatis '04]
[Zhang '04]


## Basic Algorithm and Core Theory

## A Formal Description of Boosting

- given training set $\left(x_{1}, y_{1}\right), \ldots,\left(x_{m}, y_{m}\right)$
- $y_{i} \in\{-1,+1\}$ correct label of instance $x_{i} \in X$
- for $t=1, \ldots, T$ :
- construct distribution $D_{t}$ on $\{1, \ldots, m\}$
- find weak classifier ("rule of thumb")

$$
h_{t}: X \rightarrow\{-1,+1\}
$$

with small error $\epsilon_{t}$ on $D_{t}$ :

$$
\epsilon_{t}=\operatorname{Pr}_{D_{t}}\left[h_{t}\left(x_{i}\right) \neq y_{i}\right]
$$

- output final classifier $H_{\text {final }}$


## AdaBoost

- constructing $D_{t}$ :
- $D_{1}(i)=1 / m$
- given $D_{t}$ and $h_{t}$ :

$$
\begin{aligned}
D_{t+1}(i) & =\frac{D_{t}(i)}{Z_{t}} \times \begin{cases}e^{-\alpha_{t}} & \text { if } y_{i}=h_{t}\left(x_{i}\right) \\
e^{\alpha_{t}} & \text { if } y_{i} \neq h_{t}\left(x_{i}\right)\end{cases} \\
& =\frac{D_{t}(i)}{Z_{t}} \exp \left(-\alpha_{t} y_{i} h_{t}\left(x_{i}\right)\right)
\end{aligned}
$$

where $Z_{t}=$ normalization constant

$$
\alpha_{t}=\frac{1}{2} \ln \left(\frac{1-\epsilon_{t}}{\epsilon_{t}}\right)>0
$$

- final classifier:
- $H_{\text {final }}(x)=\operatorname{sign}\left(\sum_{t} \alpha_{t} h_{t}(x)\right)$


## Toy Example


weak classifiers = vertical or horizontal half-planes

Round 1

$$
\begin{aligned}
& \varepsilon_{1}=0.30 \\
& \alpha_{1}=0.42
\end{aligned}
$$



## Round 2



$$
\begin{aligned}
& \varepsilon_{2}=0.21 \\
& \alpha_{2}=0.65
\end{aligned}
$$

## Round 3



## Final Classifier



$$
=\quad \begin{array}{|c|cc|c}
\hline & & & \\
& + & & \\
+ & - & - & \\
+ & & & - \\
& - & & \\
\hline
\end{array}
$$

## Analyzing the training error

- Theorem:
- write $\epsilon_{t}$ as $1 / 2-\gamma_{t}$
- then

$$
\begin{aligned}
\operatorname{training} \operatorname{error}\left(H_{\text {final }}\right) & \leq \prod_{t}\left[2 \sqrt{\epsilon_{t}\left(1-\epsilon_{t}\right)}\right] \\
& =\prod_{t} \sqrt{1-4 \gamma_{t}^{2}} \\
& \leq \exp \left(-2 \sum_{t} \gamma_{t}^{2}\right)
\end{aligned}
$$

- so: if $\forall t: \gamma_{t} \geq \gamma>0$
then training error $\left(H_{\text {final }}\right) \leq e^{-2 \gamma^{2} T}$
- AdaBoost is adaptive:
- does not need to know $\gamma$ or $T$ a priori
- can exploit $\gamma_{t} \gg \gamma$


## Proof

- let $f(x)=\sum_{t} \alpha_{t} h_{t}(x) \Rightarrow H_{\text {final }}(x)=\operatorname{sign}(f(x))$
- Step 1: unwrapping recurrence:

$$
\begin{aligned}
D_{\text {final }}(i) & =\frac{1}{m} \frac{\exp \left(-y_{i} \sum_{t} \alpha_{t} h_{t}\left(x_{i}\right)\right)}{\prod_{t} Z_{t}} \\
& =\frac{1}{m} \frac{\exp \left(-y_{i} f\left(x_{i}\right)\right)}{\prod_{t} Z_{t}}
\end{aligned}
$$

## Proof (cont.)

- Step 2: training $\operatorname{error}\left(H_{\text {final }}\right) \leq \prod_{t} Z_{t}$
- Proof:

$$
\begin{aligned}
\operatorname{training} \operatorname{error}\left(H_{\text {final }}\right) & =\frac{1}{m} \sum_{i}\left\{\begin{array}{l}
1 \text { if } y_{i} \neq H_{\text {final }}\left(x_{i}\right) \\
0 \\
\text { else }
\end{array}\right. \\
& =\frac{1}{m} \sum_{i}\left\{\begin{array}{l}
1 \text { if } y_{i} f\left(x_{i}\right) \leq 0 \\
0 \text { else }
\end{array}\right. \\
& \leq \frac{1}{m} \sum_{i} \exp \left(-y_{i} f\left(x_{i}\right)\right) \\
& =\sum_{i} D_{\text {final }}(i) \prod_{t} Z_{t} \\
& =\prod_{t} Z_{t}
\end{aligned}
$$

## Proof (cont.)

- Step 3: $Z_{t}=2 \sqrt{\epsilon_{t}\left(1-\epsilon_{t}\right)}$
- Proof:

$$
\begin{aligned}
Z_{t} & =\sum_{i} D_{t}(i) \exp \left(-\alpha_{t} y_{i} h_{t}\left(x_{i}\right)\right) \\
& =\sum_{i: y_{i} \neq h_{t}\left(x_{i}\right)} D_{t}(i) e^{\alpha_{t}}+\sum_{i: y_{i}=h_{t}\left(x_{i}\right)} D_{t}(i) e^{-\alpha_{t}} \\
& =\epsilon_{t} e^{\alpha_{t}}+\left(1-\epsilon_{t}\right) e^{-\alpha_{t}} \\
& =2 \sqrt{\epsilon_{t}\left(1-\epsilon_{t}\right)}
\end{aligned}
$$

## How Will Test Error Behave? (A First Guess)


expect:

- training error to continue to drop (or reach zero)
- test error to increase when $H_{\text {final }}$ becomes "too complex"
- "Occam's razor"
- overfitting
- hard to know when to stop training


## Actual Typical Run



- test error does not increase, even after 1000 rounds
- (total size > 2,000,000 nodes)
- test error continues to drop even after training error is zero!

|  | \# rounds |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | 5 | 100 | 1000 |
| train error | 0.0 | 0.0 | 0.0 |  |
| test error | 8.4 | 3.3 | 3.1 |  |

- Occam's razor wrongly predicts "simpler" rule is better


## A Better Story: Theory of Margins

[with Freund, Bartlett \& Lee]

- key idea:
- training error only measures whether classifications are right or wrong
- should also consider confidence of classifications
- recall: $H_{\text {final }}$ is weighted majority vote of weak classifiers
- measure confidence by margin $=$ strength of the vote
$=($ fraction voting correctly $)-($ fraction voting incorrectly $)$



## Empirical Evidence: The Margin Distribution

- margin distribution
= cumulative distribution of margins of training examples



## Theoretical Evidence: Analyzing Boosting UsingMargins

- Theorem: large margins $\Rightarrow$ better bound on generalization error (independent of number of rounds)
- proof idea: if all margins are large, then can approximate final classifier by a much smaller classifier (just as polls can predict not-too-close election)
- Theorem: boosting tends to increase margins of training examples (given weak learning assumption)
- proof idea: similar to training error proof
- so:
although final classifier is getting larger, margins are likely to be increasing, so final classifier actually getting close to a simpler classifier, driving down the test error


## More Technically...

- with high probability, $\forall \theta>0$ :

$$
\text { generalization error } \leq \hat{\operatorname{Pr}}[\text { margin } \leq \theta]+\tilde{O}\left(\frac{\sqrt{d / m}}{\theta}\right)
$$

where

- $m=$ \# training examples
- $d=$ "complexity" of weak classifiers
- $\hat{\operatorname{Pr}}[$ margin $\leq \theta] \rightarrow 0$ exponentially fast (in $T)$ if $\gamma_{t}>\theta(\forall t)$


## Other Ways of Understanding AdaBoost

## Game Theory

- game defined by matrix M:

|  | Rock | Paper | Scissors |
| ---: | :---: | :---: | :---: |
| Rock | $1 / 2$ | 1 | 0 |
| Paper | 0 | $1 / 2$ | 1 |
| Scissors | 1 | 0 | $1 / 2$ |

- row player chooses row $i$
- column player chooses column $j$ (simultaneously)
- row player's goal: minimize loss $\mathbf{M}(i, j)$
- usually allow randomized play:
- players choose distributions $\mathbf{P}$ and $\mathbf{Q}$ over rows and columns
- learner's (expected) loss

$$
\begin{aligned}
& =\sum_{i, j} \mathbf{P}(i) \mathbf{M}(i, j) \mathbf{Q}(j) \\
& =\mathbf{P}^{\mathrm{T}} \mathbf{M} \mathbf{Q} \equiv \mathbf{M}(\mathbf{P}, \mathbf{Q})
\end{aligned}
$$

## The Minmax Theorem

- von Neumann's minmax theorem:

$$
\begin{aligned}
\min _{\mathbf{P}} \max _{\mathbf{Q}} \mathbf{M}(\mathbf{P}, \mathbf{Q}) & =\max _{\mathbf{Q}} \min _{\mathbf{P}} \mathbf{M}(\mathbf{P}, \mathbf{Q}) \\
& =v \\
& =\text { "value" of game } \mathbf{M}
\end{aligned}
$$

- in words:
- $v=$ min max means:
- row player has strategy $\mathbf{P}^{*}$ such that $\forall$ column strategy $\mathbf{Q}$ loss $\mathbf{M}\left(\mathbf{P}^{*}, \mathbf{Q}\right) \leq v$
- $v=$ max min means:
- this is optimal in sense that column player has strategy $\mathbf{Q}^{*}$ such that $\forall$ row strategy $\mathbf{P}$ $\operatorname{loss} \mathbf{M}\left(\mathbf{P}, \mathbf{Q}^{*}\right) \geq v$


## The Boosting Game

- let $\left\{\bar{h}_{1}, \ldots, \bar{h}_{N}\right\}=$ space of all weak classifiers
- row player $\leftrightarrow$ booster
- column player $\leftrightarrow$ weak learner
- matrix M:
- row $\leftrightarrow$ example $\left(x_{i}, y_{i}\right)$
- column $\leftrightarrow$ weak classifier $\bar{\hbar}$
- $\mathbf{M}(i, j)=\left\{\begin{array}{l}1 \text { if } y_{i}=h_{j}\left(x_{i}\right) \\ 0 \text { else }\end{array}\right.$
weak learner



## Boosting and the Minmax Theorem

- if:
- $\forall$ distributions over examples
$\exists h$ with accuracy $\geq \frac{1}{2}-\gamma$
- then:
- $\min _{\mathbf{P}} \max _{h} \mathbf{M}(\mathbf{P}, h) \geq \frac{1}{2}-\gamma$
- by minmax theorem:
- $\max _{\mathbf{Q}} \min _{i} \mathbf{M}(i, \mathbf{Q}) \geq \frac{1}{2}-\gamma>\frac{1}{2}$
- which means:
- $\exists$ weighted majority of classifiers which correctly classifies all examples with positive margin (2 $\gamma$ )
- optimal margin $\leftrightarrow$ "value" of game


## AdaBoost and Game Theory

[Freund \& Schapire]

- AdaBoost is special case of general algorithm for solving games through repeated play
- can show
- distribution over examples converges to (approximate) minmax strategy for boosting game
- weights on weak classifiers converge to (approximate) maxmin strategy
- different instantiation of game-playing algorithm gives on-line learning algorithms (such as weighted majority algorithm)


## AdaBoost and Exponential Loss

- many (most?) learning algorithms minimize a "loss" function
- e.g. least squares regression
- training error proof shows AdaBoost actually minimizes

$$
\prod_{t} Z_{t}=\frac{1}{m} \sum_{i} \exp \left(-y_{i} f\left(x_{i}\right)\right)
$$

where $f(x)=\sum_{t} \alpha_{t} h_{t}(x)$

- on each round, AdaBoost greedily chooses $\alpha_{t}$ and $h_{t}$ to minimize loss
- exponential loss is an upper bound on 0-1 (classification) loss
- AdaBoost provably minimizes exponential loss



## Coordinate Descent

[Breiman]

- $\left\{\bar{h}_{1}, \ldots, \bar{h}_{N}\right\}=$ space of all weak classifiers
- want to find $\lambda_{1}, \ldots, \lambda_{N}$ to minimize

$$
L\left(\lambda_{1}, \ldots, \lambda_{N}\right)=\sum_{i} \exp \left(-y_{i} \sum_{j} \lambda_{j} \bar{h}_{j}\left(x_{i}\right)\right)
$$

- AdaBoost is actually doing coordinate descent on this optimization problem:
- initially, all $\lambda_{j}=0$
- each round: choose one coordinate $\lambda_{j}$ (corresponding to $h_{t}$ ) and update (increment by $\alpha_{t}$ )
- choose update causing biggest decrease in loss
- powerful technique for minimizing over huge space of functions


## Functional Gradient Descent

[Friedman][Mason et al.]

- want to minimize

$$
L(f)=L\left(f\left(x_{1}\right), \ldots, f\left(x_{m}\right)\right)=\sum_{i} \exp \left(-y_{i} f\left(x_{i}\right)\right)
$$

- say have current estimate $\bar{f}$ and want to improve
- to do gradient descent, would like update

$$
\bar{f} \leftarrow \bar{f}-\eta \nabla_{f} L(\bar{f})
$$

- but update restricted in class of weak classifiers
- so choose $h_{t}$ "closest" to $\nabla_{f} L(\bar{f})$
- equivalent to AdaBoost


## Benefits of Model Fitting View

- immediate generalization to other loss functions
- e.g. squared error for regression
- e.g. logistic regression (by only changing one line of AdaBoost)
- sensible approach for converting output of boosting into conditional probability estimates
- caveat: wrong to view AdaBoost as just an algorithm for minimizing exponential loss
- other algorithms for minimizing same loss will (provably) give very poor performance
- thus, this loss function cannot explain why AdaBoost "works"


## Estimating Conditional Probabilities

[Friedman, Hastie \& Tibshirani]

- often want to estimate probability that $y=+1$ given $x$
- AdaBoost minimizes (empirical version of):

$$
\mathrm{E}_{x, y}\left[e^{-y f(x)}\right]=\mathrm{E}_{x}\left[\mathrm{P}[y=+1 \mid x] e^{-f(x)}+\mathrm{P}[y=-1 \mid x] e^{-f(x)}\right]
$$

where $x, y$ random from true distribution

- over all $f$, minimized when

$$
f(x)=\frac{1}{2} \cdot \ln \left(\frac{\mathrm{P}[y=+1 \mid x]}{\mathrm{P}[y=-1 \mid x]}\right)
$$

or

$$
\mathrm{P}[y=+1 \mid x]=\frac{1}{1+e^{-2 f(x)}}
$$

- so, to convert $f$ output by AdaBoost to probability estimate, use same formula


## Calibration Curve



- order examples by $f$ value output by AdaBoost
- break into bins of size $r$
- for each bin, plot a point:
- $x$-value: average estimated probability of examples in bin
- $y$-value: actual fraction of positive examples in bin


## Other Ways to Think about AdaBoost

- dynamical systems
- statistical consistency
- maximum entropy


## Experiments, Applications and Extensions

## Practical Advantages of AdaBoost

- fast
- simple and easy to program
- no parameters to tune (except $T$ )
- flexible - can combine with any learning algorithm
- no prior knowledge needed about weak learner
- provably effective, provided can consistently find rough rules of thumb
$\rightarrow$ shift in mind set - goal now is merely to find classifiers barely better than random guessing
- versatile
- can use with data that is textual, numeric, discrete, etc. - has been extended to learning problems well beyond binary classification


## Caveats

- performance of AdaBoost depends on data and weak learner
- consistent with theory, AdaBoost can fail if
- weak classifiers too complex
$\rightarrow$ overfitting
- weak classifiers too weak ( $\gamma_{t} \rightarrow 0$ too quickly)
$\rightarrow$ underfitting
$\rightarrow$ low margins $\rightarrow$ overfitting
- empirically, AdaBoost seems especially susceptible to uniform noise


## UCI Experiments

- tested AdaBoost on UCI benchmarks
- used:
- C4.5 (Quinlan's decision tree algorithm)
- "decision stumps": very simple rules of thumb that test on single attributes



## UCI Results




## Multiclass Problems

[with Freund]

- say $y \in Y=\{1, \ldots, k\}$
- direct approach (AdaBoost.M1):

$$
\begin{gathered}
h_{t}: X \rightarrow Y \\
D_{t+1}(i)=\frac{D_{t}(i)}{Z_{t}} \cdot \begin{cases}e^{-\alpha_{t}} & \text { if } y_{i}=h_{t}\left(x_{i}\right) \\
e^{\alpha} & \text { if } y_{i} \neq h_{t}\left(x_{i}\right)\end{cases} \\
H_{\text {final }}(x)=\arg \max _{y \in Y} \sum_{t: h_{t}(x)=y} \alpha_{t}
\end{gathered}
$$

- can prove same bound on error $\underline{\text { if }} \forall t: \epsilon_{t} \leq 1 / 2$
- in practice, not usually a problem for "strong" weak learners (e.g., C4.5)
- significant problem for "weak" weak learners (e.g., decision stumps)
- instead, reduce to binary


## Reducing Multiclass to Binary

- say possible labels are $\{a, b, c, d, e\}$
- each training example replaced by five $\{-1,+1\}$-labeled examples:

$$
x, c \rightarrow \begin{cases}(x, a), & -1 \\ (x, b), & -1 \\ (x, c), & +1 \\ (x, d), & -1 \\ (x, e), & -1\end{cases}
$$

- predict with label receiving most (weighted) votes


## AdaBoost.MH

- can prove:

$$
\text { training error }\left(H_{\text {final }}\right) \leq \frac{k}{2} \cdot \Pi Z_{t}
$$

- reflects fact that small number of errors in binary predictors can cause overall prediction to be incorrect
- extends immediately to multi-label case (more than one correct label per example)


## Using Output Codes

[with Allwein \& Singer]

- alternative: choose "code word" for each label

|  | $\pi_{1}$ | $\pi_{2}$ | $\pi_{3}$ | $\pi_{4}$ |
| :--- | :--- | :--- | :--- | :--- |
| a | - | + | - | + |
| b | - | + | + | - |
| c | + | - | - | + |
| d | + | - | + | + |
| e | - | + | - | - |

- each training example mapped to one example per column

$$
x, c \rightarrow \begin{cases}\left(x, \pi_{1}\right), & +1 \\ \left(x, \pi_{2}\right), & -1 \\ \left(x, \pi_{3}\right), & -1 \\ \left(x, \pi_{4}\right), & +1\end{cases}
$$

- to classify new example $x$ :
- evaluate classifier on $\left(x, \pi_{1}\right), \ldots,\left(x, \pi_{4}\right)$
. choose label "most consistent" with results


## Output Codes (cont.)

- training error bounds independent of \# of classes
- overall prediction robust to large number of errors in binary predictors
- but: binary problems may be harder


## Ranking Problems

[with Freund, Iyer \& Singer]

- other problems can also be handled by reducing to binary
- e.g.: want to learn to rank objects (say, movies) from examples
- can reduce to multiple binary questions of form:
"is or is not object A preferred to object B?"
- now apply (binary) AdaBoost


## Problem with "Hard" Predictions

- ideally, want weak classifier that says:

$$
h(x)= \begin{cases}+1 & \text { if } x \in R \\ \text { "don't know" else }\end{cases}
$$

- problem: cannot express using "hard" predictions
- if must predict $\pm 1$ outside $R$, will introduce many "bad" predictions
- need to "clean up" on later rounds
- dramatically increases time to convergence


## Confidence-rated Predictions

- useful to allow weak classifiers to assign confidences to predictions
- formally, allow $h_{t}: X \rightarrow \mathbb{R}$

$$
\begin{aligned}
\operatorname{sign}\left(h_{t}(x)\right) & =\text { prediction } \\
\left|h_{t}(x)\right| & =\text { "confidence" }
\end{aligned}
$$

- use identical update:

$$
D_{t+1}(i)=\frac{D_{t}(i)}{Z_{t}} \cdot \exp \left(-\alpha_{t} y_{i} h_{t}\left(x_{i}\right)\right)
$$

and identical rule for combining weak classifiers

- question: how to choose $\alpha_{t}$ and $h_{t}$ on each round


## Confidence-rated Predictions (cont.)

- saw earlier:

$$
\text { training error }\left(H_{\text {final }}\right) \leq \prod_{t} Z_{t}=\sum_{i} \exp \left(-y_{i} \sum_{t} \alpha_{t} h_{t}\left(x_{i}\right)\right)
$$

- therefore, on each round $t$, should choose $\alpha_{t} h_{t}$ to minimize:

$$
Z_{t}=\sum_{i} D_{t}(i) \exp \left(-\alpha_{t} y_{i} h_{t}\left(x_{i}\right)\right)
$$

- in many cases (e.g., decision stumps), best confidence-rated weak classifier has simple form that can be found efficiently


## Confidence-rated Predictions Help a Lot



## Application: Boosting for Text Categorization

- weak classifiers: very simple weak classifiers that test on simple patterns, namely, (sparse) $n$-grams
- find parameter $\alpha_{t}$ and rule $h_{t}$ of given form which minimize $Z_{t}$
- use efficiently implemented exhaustive search
- "How may I help you" data:
- 7844 training examples
- 1000 test examples
- categories: AreaCode, AttService, BillingCredit, CallingCard, Collect, Competitor, DialForMe, Directory, HowToDial, PersonToPerson, Rate, ThirdNumber, Time, TimeCharge, Other.


## Weak Classifiers



## More Weak Classifiers

| rnd term | AC | AS | BC | CC | CO | CM | DM | DI | HO | PP | RA | 3N | TI | TC | OT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 time | - | - | - | ■ | - | - | - | - | - |  | - | ㅍ |  |  | - |
|  |  | - | - | - | - | - | - | - | - | - | - | - | 巨 | ㄷ | - |
| 8 wrong number |  |  |  |  |  |  |  |  | E |  |  | $\square$ |  |  | $\square$ |
|  |  | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 9 how |  |  |  |  |  |  |  |  |  |  |  |  |  |  | - |
|  |  | - | - | - | - | - | - | - | - | - | = | - | - | - | - |
| 10 call |  |  |  | - |  |  |  | - | - | - | - | - | - | - | - |
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| 11 seven |  |  |  |  |  |  |  |  | ■ |  |  |  |  |  | - |
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| 12 trying to | - |  | - | $=$ | - | - | - | - | - | ■ | - | [ | - |  | - |
|  | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 13 and | - |  | - | - | - | - | - | - | - | - | - | - | - | - | - |
|  | - | - | = | - | - | - | - | - | - | - | - | - | - | - | - |

## More Weak Classifiers



## Finding Outliers

## examples with most weight are often outliers (mislabeled and/or ambiguous)

- I'm trying to make a credit card call (Collect)
- hello (Rate)
- yes I'd like to make a long distance collect call please (CallingCard)
- calling card please (Collect)
- yeah I'd like to use my calling card number (Collect)
- can I get a collect call (CallingCard)
- yes I would like to make a long distant telephone call and have the charges billed to another number (CallingCard DialForMe)
- yeah I can not stand it this morning I did oversea call is so bad (BillingCredit)
- yeah special offers going on for long distance (AttService Rate)
- mister allen please william allen (PersonToPerson)
- yes ma'am I I'm trying to make a long distance call to a non dialable point in san miguel philippines (AttService Other)
- yes I like to make a long distance call and charge it to my home phone that's where I'm calling at my home (DialForMe)


## Application: Human-computer Spoken Dialogue

 [with Rahim, Di Fabbrizio, Dutton, Gupta, Hollister \& Riccardi]- application: automatic "store front" or "help desk" for AT\&T Labs' Natural Voices business
- caller can request demo, pricing information, technical support, sales agent, etc.
- interactive dialogue
- naturalvoices.att.com, 1-877-741-4321


## How It Works



- NLU's job: classify caller utterances into 24 categories (demo, sales rep, pricing info, yes, no, etc.)


## Need for Prior, Human Knowledge

[with Rochery, Rahim \& Gupta]

- building NLU: standard text categorization problem
- need lots of data, but for cheap, rapid deployment, can't wait for it
- bootstrapping problem:
- need labeled data to deploy
- need to deploy to get labeled data
- idea: use human knowledge to compensate for insufficient data
- modify loss function to balance fit to data against fit to prior model


## Results: AP-Titles



## Results: Helpdesk



## Problem: Labels are Expensive

- for spoken-dialogue task
- getting examples is cheap
- getting labels is expensive
- must be annotated by humans
- how to reduce number of labels needed?


## Active Learning

- idea:
- use selective sampling to choose which examples to label
- focus on least confident examples [Lewis \& Gale]
- for boosting, use (absolute) margin $|f(x)|$ as natural confidence measure
[Abe \& Mamitsuka]


## Labeling Scheme

- start with pool of unlabeled examples
- choose (say) 500 examples at random for labeling
- run boosting on all labeled examples
- get combined classifier $f$
- pick (say) 250 additional examples from pool for labeling
- choose examples with minimum $|f(x)|$
- repeat


## Results: How-May-I-Help-You?



## Results: Letter



## Application: Detecting Faces

[Viola \& Jones]

- problem: find faces in photograph or movie
- weak classifiers: detect light/dark rectangles in image

- many clever tricks to make extremely fast and accurate


## Conclusions

- boosting is a practical tool for classification and other learning problems
- grounded in rich theory
- performs well experimentally
- often (but not always!) resistant to overfitting
- many applications and extensions
- many ways to think about boosting
- none is entirely satisfactory by itself, but each useful in its own way
- considerable room for further theoretical and experimental work


## References

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