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A stylized model for the anomalous impact of meta-orders

Journées MAS 2014:
"Phénomènes de grand dimension"

Toulouse,
August 28th 2014

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B. Tóth

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Motivation

"Buy trades move prices up and sell trades move prices down"

Why and how trades move prices?

Is this trivial?

Not at all! The details about **how** this happens are still unknown, and there is no consensus so far about which model should describe the effect of trades on prices.

Why is this relevant?

For practitioners and regulators:

- Control the effect of their actions on the market (trading costs, stability)

For theorists:

- Knowing how information is incorporated into prices

Outline

- Response to trades: empirical evidence and theoretical implications
- The microstructure of financial markets
- A stylized model for market impact
- A more empirically grounded generalization

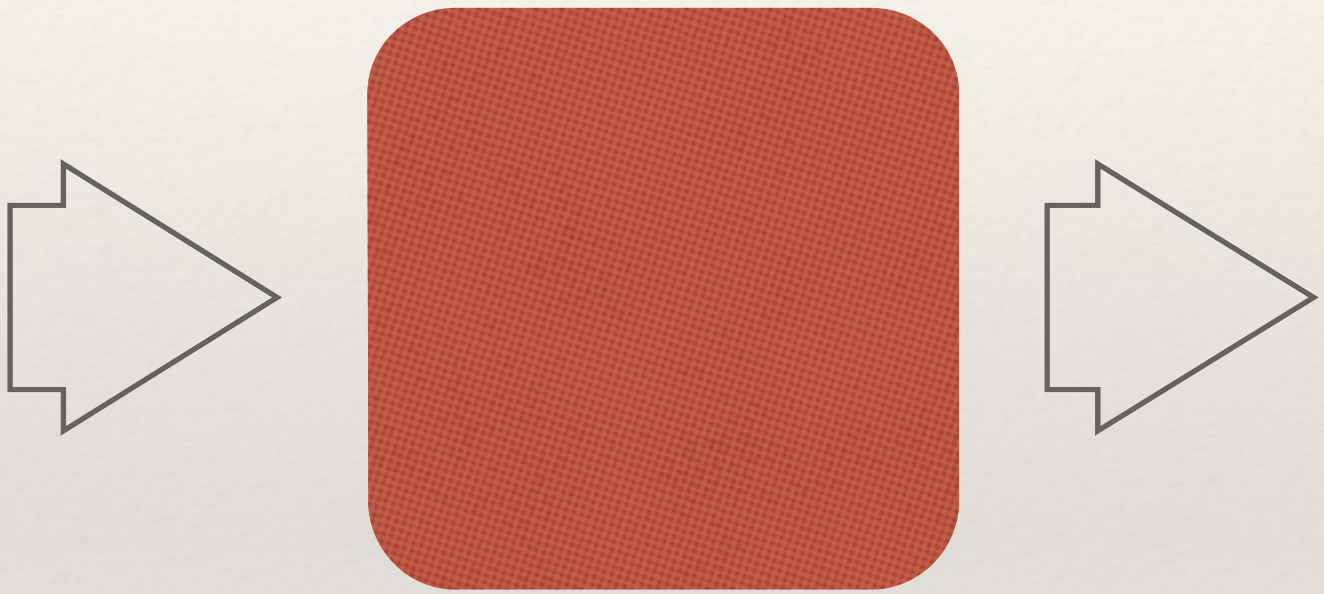
Markets as oracles

Markets can be seen as **large information processing devices**

Predictable

ϵ_t

traded signs



Market

Unpredictable

p_t

prices

Long-range correlated sequence of +1 and -1 variables.
It is the incoming flux of all orders from financial actors

Statistically efficient (**martingale**) process encoding all the information contained in trades. It contains no information whatsoever

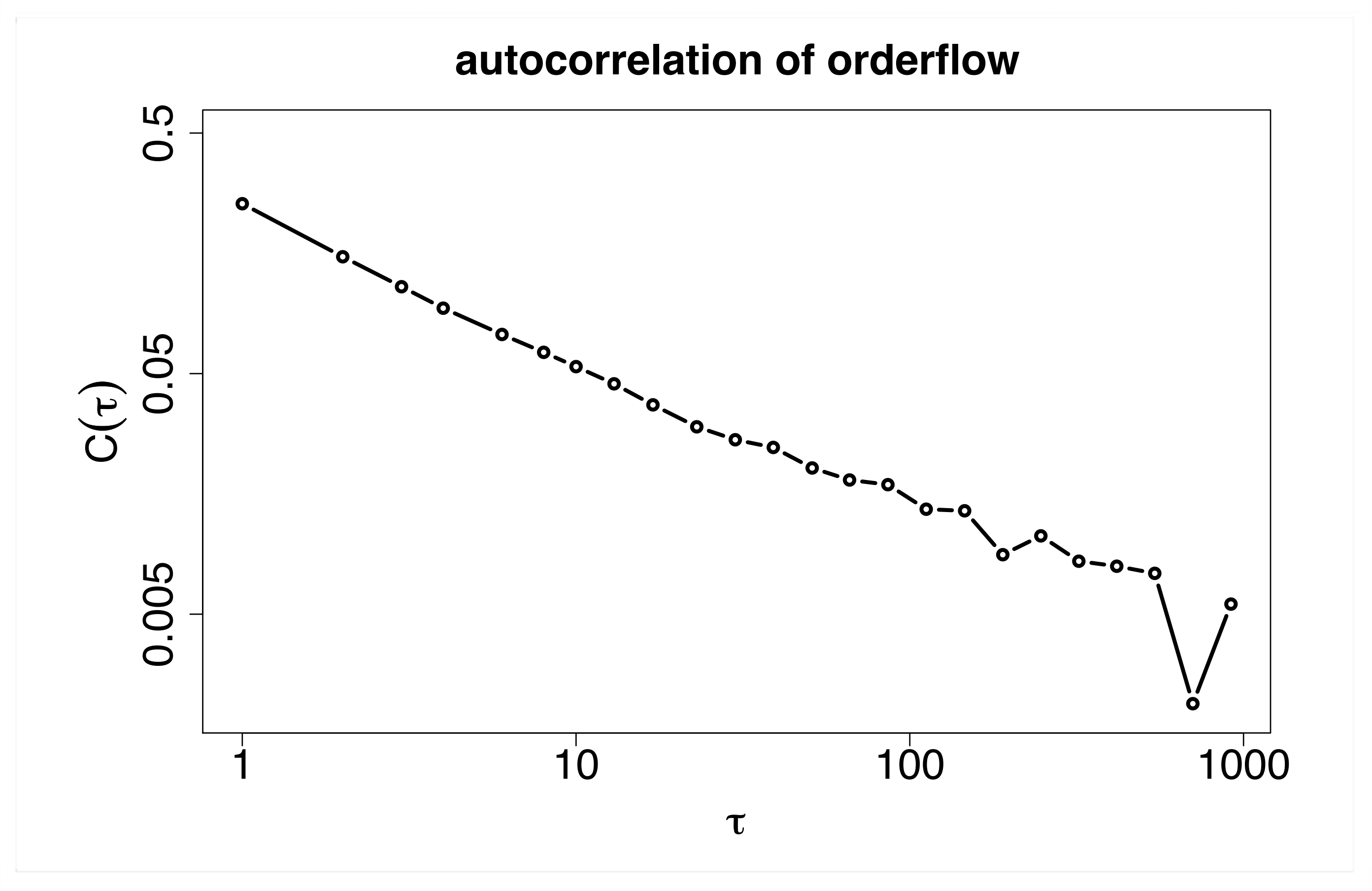
The input process

Empirically, the sign process is strongly autocorrelated!

$$C(\tau) = \langle \epsilon_t \epsilon_{t+\tau} \rangle - \langle \epsilon_t \rangle \langle \epsilon_{t+\tau} \rangle$$

$$C(\tau) \sim \tau^{-\gamma}$$
$$\gamma \in [0.4, 0.8]$$

(for different market venues, epochs, products)

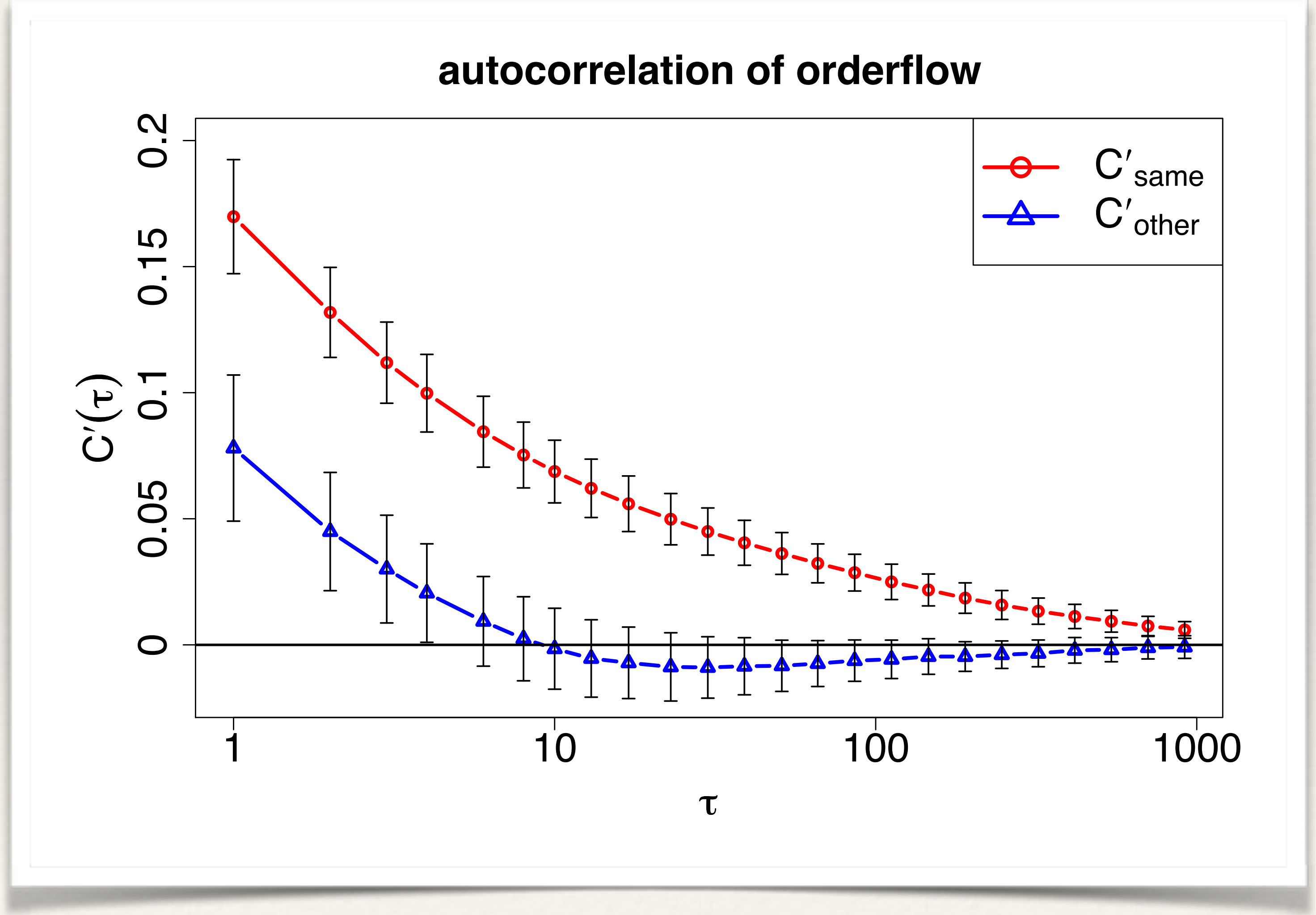


Response to trades is fine tuned!

(stock AZN traded in LSE, from B.Tóth *et al.*, "Why is the order flow so persistent?")

Herding vs splitting

In some cases, the **ID of the brokers are available**. This allows to decompose correlations in *same broker / other brokers* contributions



from B.Tóth *et al.*, "Why is the order flow so persistent?" arXiv:1108.1632 (2011)

$$C(\tau) = C_{same}(\tau) + C_{other}(\tau)$$

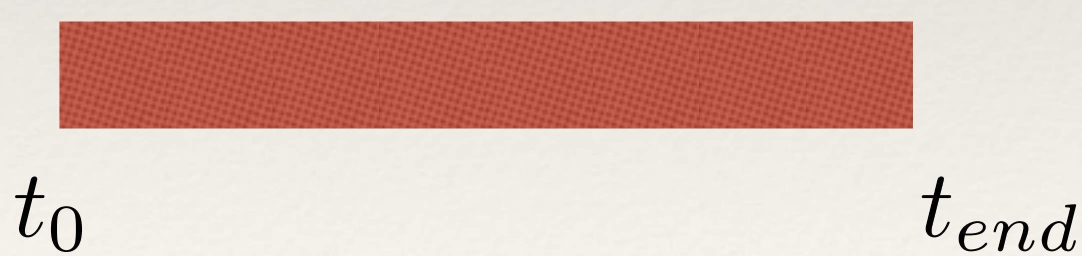
Meta-orders

Autocorrelation is dominated by splitting: why is this?

Information: As soon as you trade, you are **giving away private information** to others. You should better hide it!

Costs: The more you trade, the more you move price by reducing quantity available at best price: **trading fast is expensive**!

Hence traders **hide their orders into the noise** (of the regular order flow)!



the collective order is usually referred to as **meta-order**

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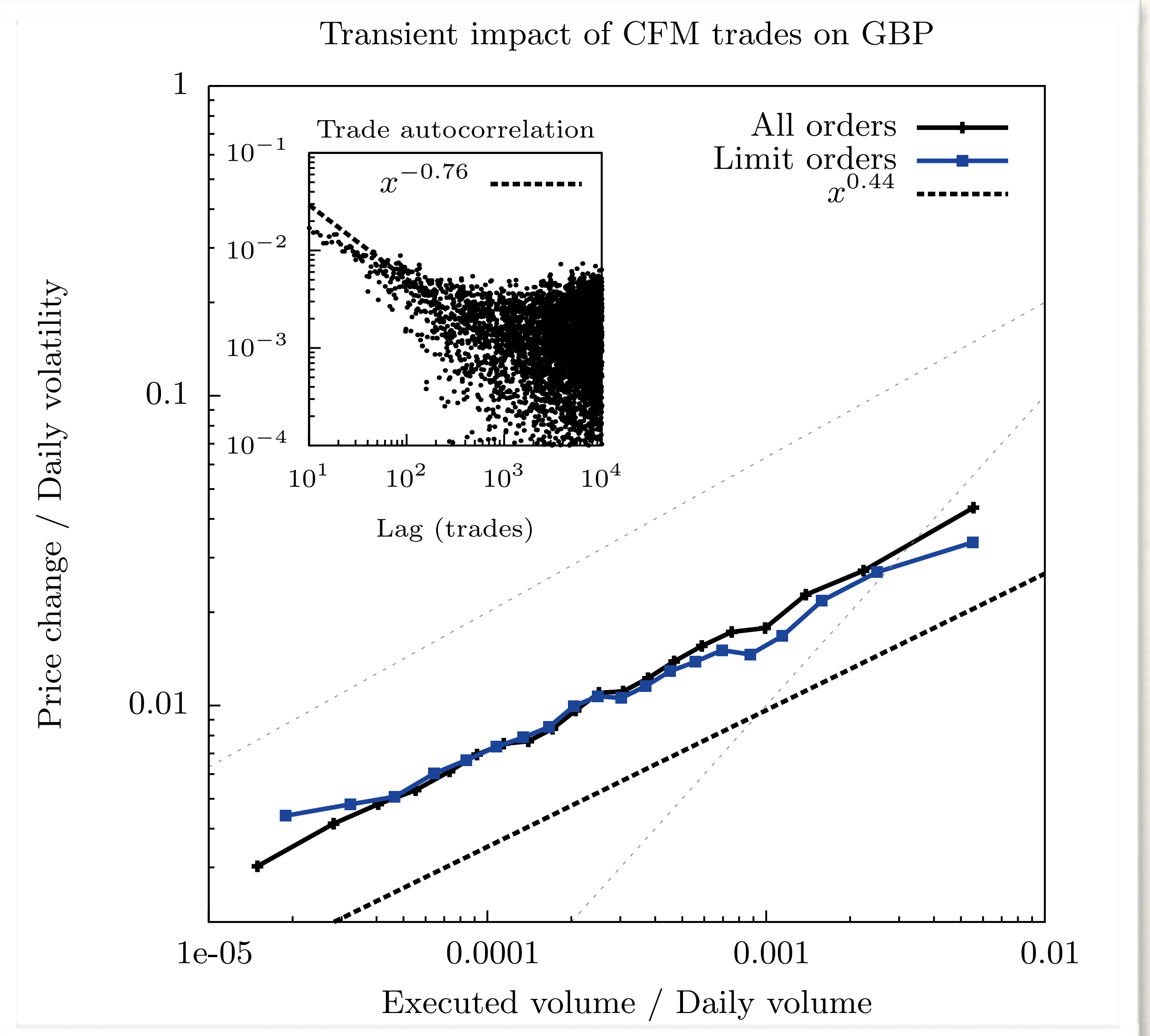
Market impact for 8 000 000 CFM trades



Impact of meta-orders:

empirical results

The response of price to a set of sequential trades has a concave shape:

$$\langle \Delta p \rangle = Y \sigma_D \left(\frac{Q}{V_D} \right)^{1/2}$$

Δp	price change
Y	dimensionless, **remarkably stable** (1995-2013)
σ_D	daily fluctuations
V_D	daily traded volume
Q	executed volume

Notes:

- **Signal is very weak**: you need to average in order to catch it (SNR ~ 10^{-2})
- **Fragility of markets**: Impact diverges at the origin
- **Non-additivity:** The impact of two consecutive trades is not the sum of the separate impacts

Strategy for the model

What are the causes of impact?

- **Trades forecast prices:** trades cause price changes because they add information to the price process
- **Prices forecast trades:** people trade because they discover how prices are going to change in the future
- **Trades mechanically impact prices:** while buying, I reduce offer and when selling I reduce demand

Order book (I)

What is the mechanics of trading?

Traded **contract**

Buy orders (***bid***)

Sell orders (***ask***)

QQQ

GET STOCK

QQQ go

Symbol Search

LAST MATCH		TODAY'S ACTIVITY	
Price	25.1290	Orders	67,212
Time	11:42:15.597	Volume	12,778,400

BUY ORDERS		SELL ORDERS	
SHARES	PRICE	SHARES	PRICE
600	25.1240	500	25.1470
3,200	25.1230	400	25.1470
3,200	25.1220	600	25.1480
4,000	25.1220	100	25.1500
100	25.1210	3,200	25.1520
100	25.1200	4,000	25.1520
3,200	25.1200	4,000	25.1530
9,600	25.1130	7,200	25.1530
4,000	25.1130	3,200	25.1550
400	25.1130	4,000	25.1570
4,000	25.1130	4,000	25.1570
8,000	25.1120	100	25.1590
5,000	25.1110	800	25.1680
3,000	25.1100	8,000	25.1680
1,000	25.1100	5,000	25.1690
(237 more)		(119 more)	

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Volumes

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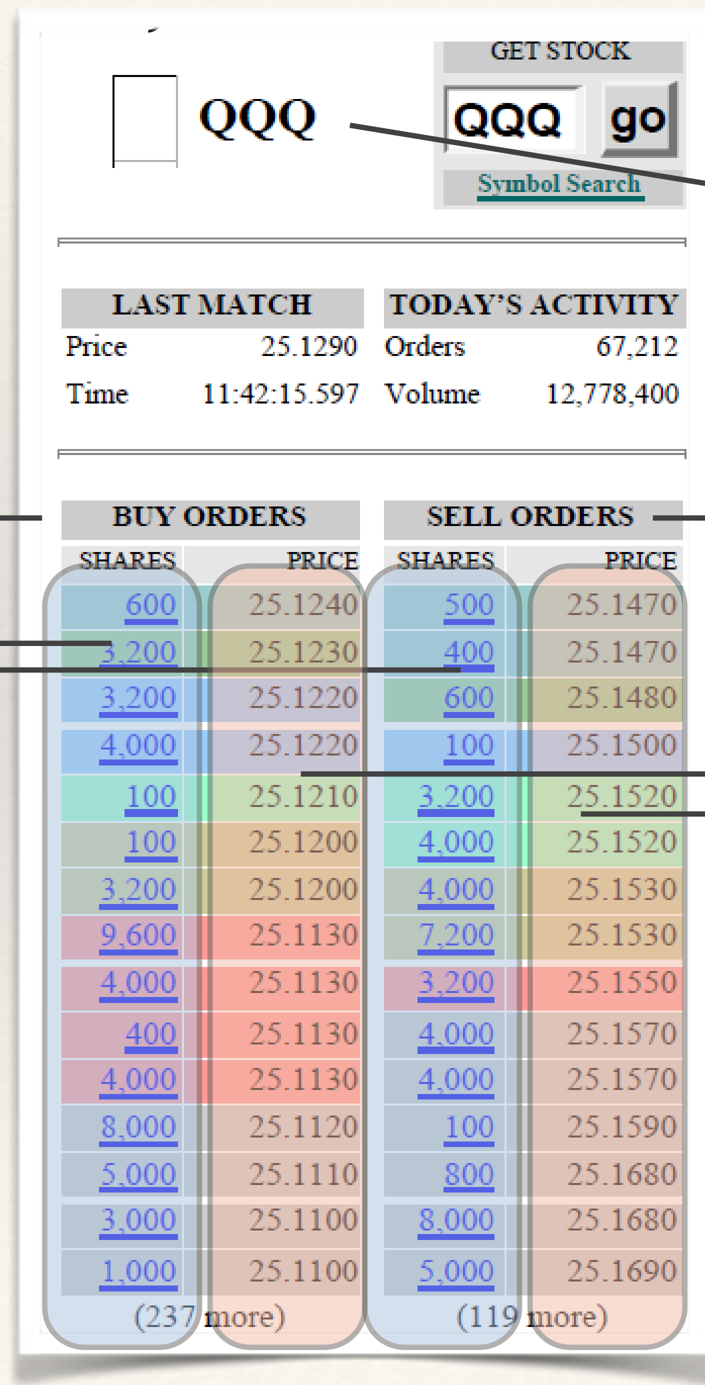

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Traded **contract**

Buy orders (***bid***)

Sell orders (***ask***)

Volumes

Prices:
highest bid < lowest ask
due to **bid-ask spread**

Order book (II)

How do you influence them?

Market orders:

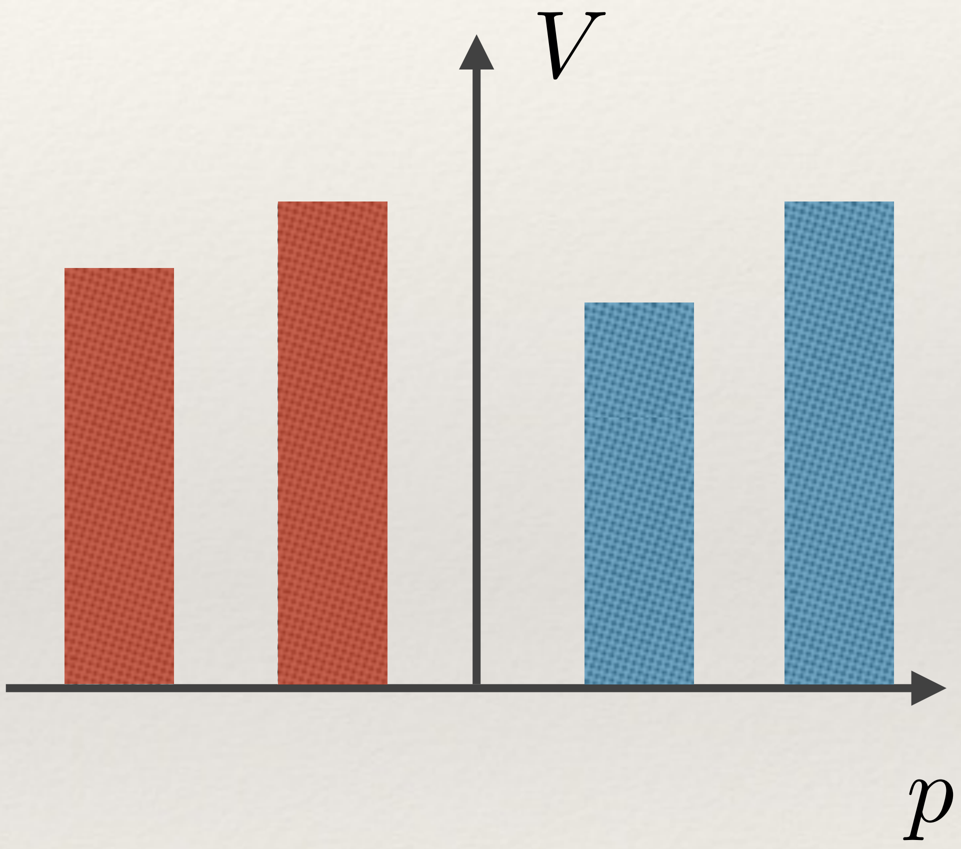
Unconditional orders to instantly buy/sell at best price a given volume (decreases liquidity)

Limit orders:

Add order to buy a given volume at specific price (increases liquidity)

Cancellations:

Removes previously added price



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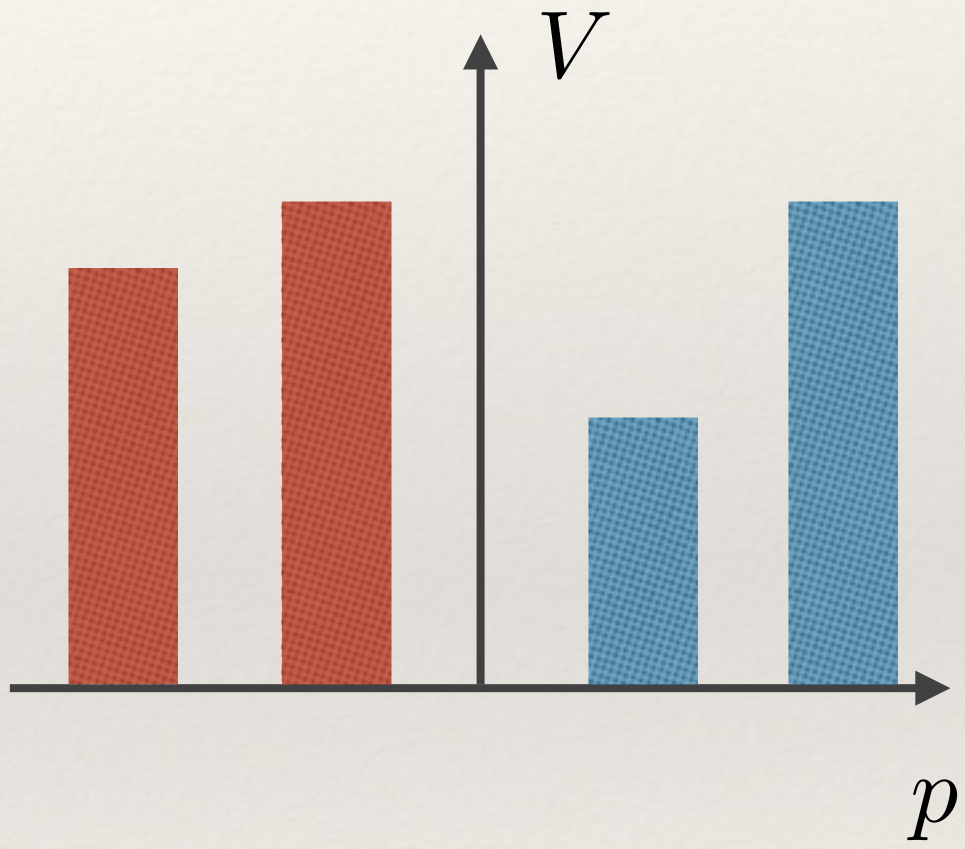

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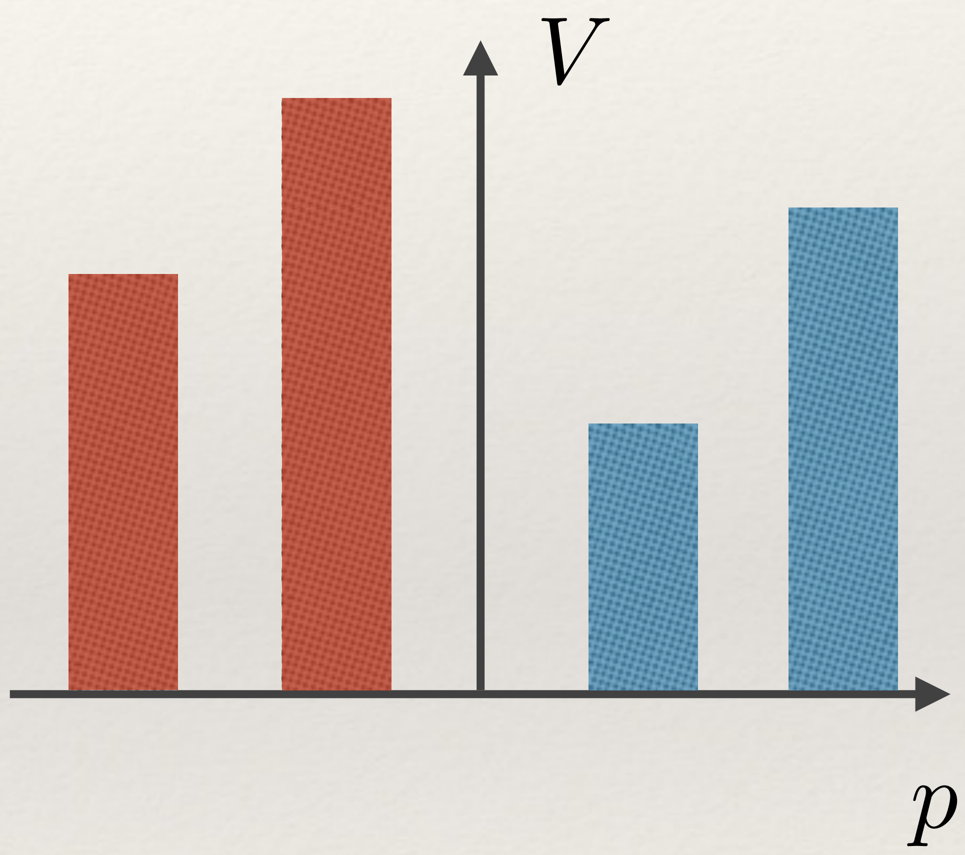

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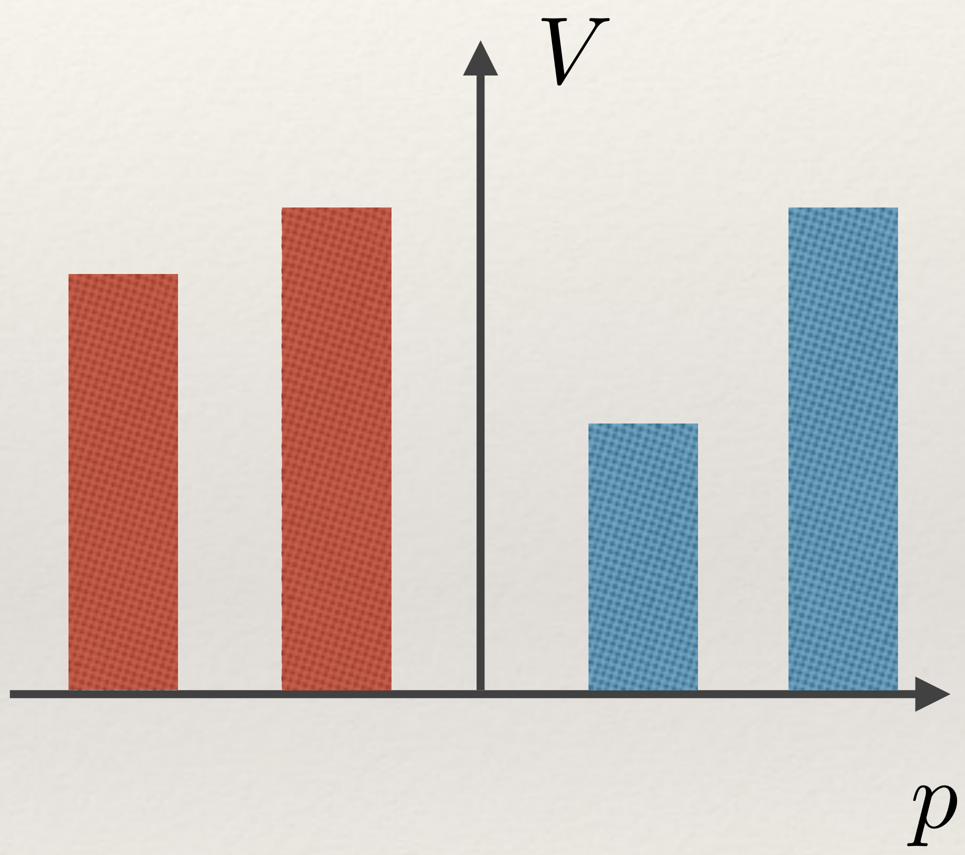
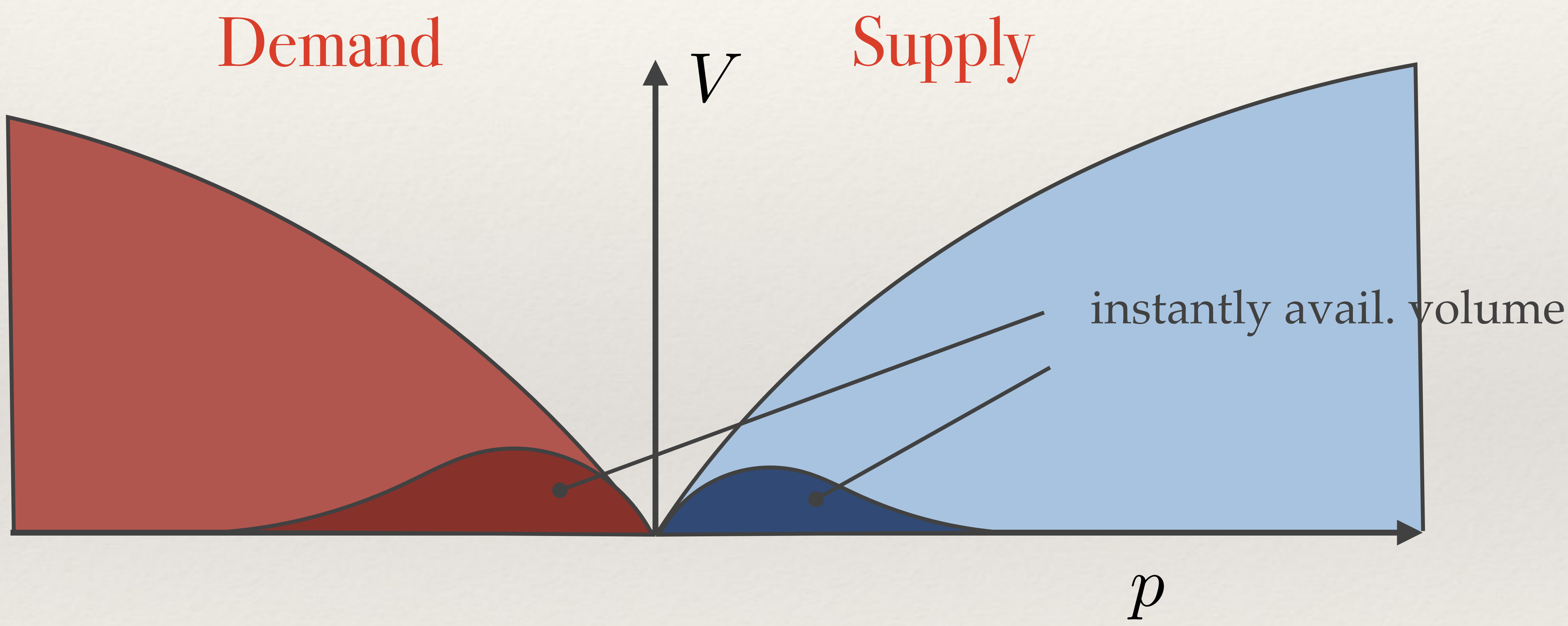


Demand and supply

Can the order book be considered as a proxy for demand and supply curves?



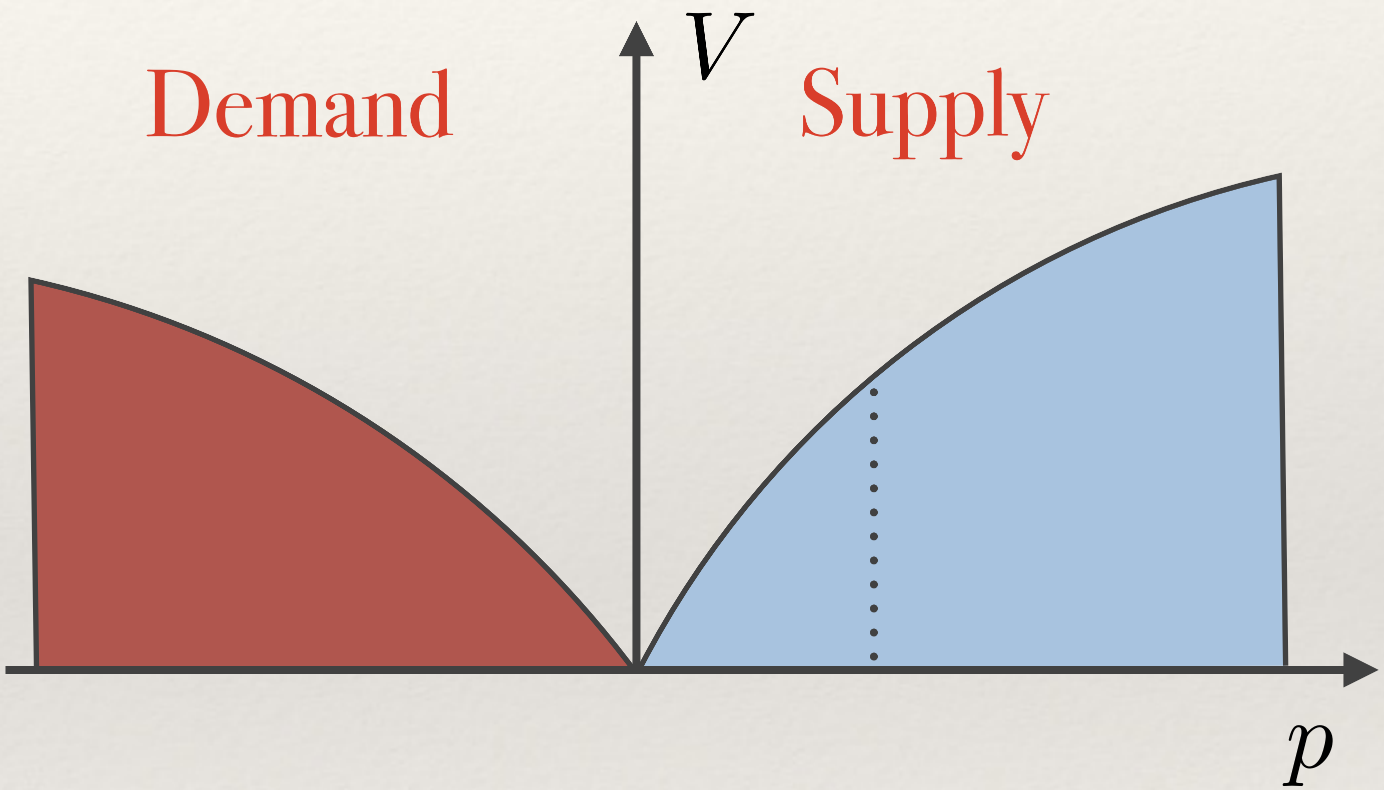
Not exactly: that is a small fraction of the latent demand and supply curve

($V_{avail} << V_{daily}$)

The idea

We formulate a **mechanical theory of market impact** based on universal principles

- Prices live on a one-dimensional line
- Demand and supply curves vanish **at** the traded price



…if curve is locally linear

$$Q = \int_0^{\Delta p} dp\, V(p) \propto \Delta p^2$$

This is a static picture… Does this hold when one has a proper dynamics (slow execution)?

Our model: ingredients

We consider a **one-dimensional reaction-diffusion** system:

$$A + B \to \emptyset$$

in order to model the **latent liquidity** process

Hopping: *Particles have probability D per unit time of **jumping** left/right*

Annihilation: *Particles of different type on the same site **annihilate** with probability λ per unit time (eventually, we want $\lambda \to \infty$)*

Insertion: *New particles are **inserted** at the boundaries at a rate J per unit time*

we are interested in studying the statistics of the interface among the **rightmost B** and the **leftmost A**

The mean-field equation

The master equation for the process is rather complicated to write. Indeed, one can extract the dynamics of the **mean density**

$$\frac{\partial \langle b(x,t)\rangle}{\partial t} = D\frac{\partial^2 \langle b(x,t)\rangle}{\partial x^2} - \lambda \langle a(x,t) b(x,t)\rangle$$
$$\frac{\partial \langle a(x,t)\rangle}{\partial t} = D\frac{\partial^2 \langle a(x,t)\rangle}{\partial x^2} - \lambda \langle a(x,t) b(x,t)\rangle$$

with boundaries

$$J = -D\frac{\partial \langle b(x,t)\rangle}{\partial x}\bigg|_{x=0} \qquad 0 = -D\frac{\partial \langle b(x,t)\rangle}{\partial x}\bigg|_{x=L}$$
$$0 = -D\frac{\partial \langle a(x,t)\rangle}{\partial x}\bigg|_{x=0} \qquad -J = -D\frac{\partial \langle a(x,t)\rangle}{\partial x}\bigg|_{x=L}$$

where we remark that

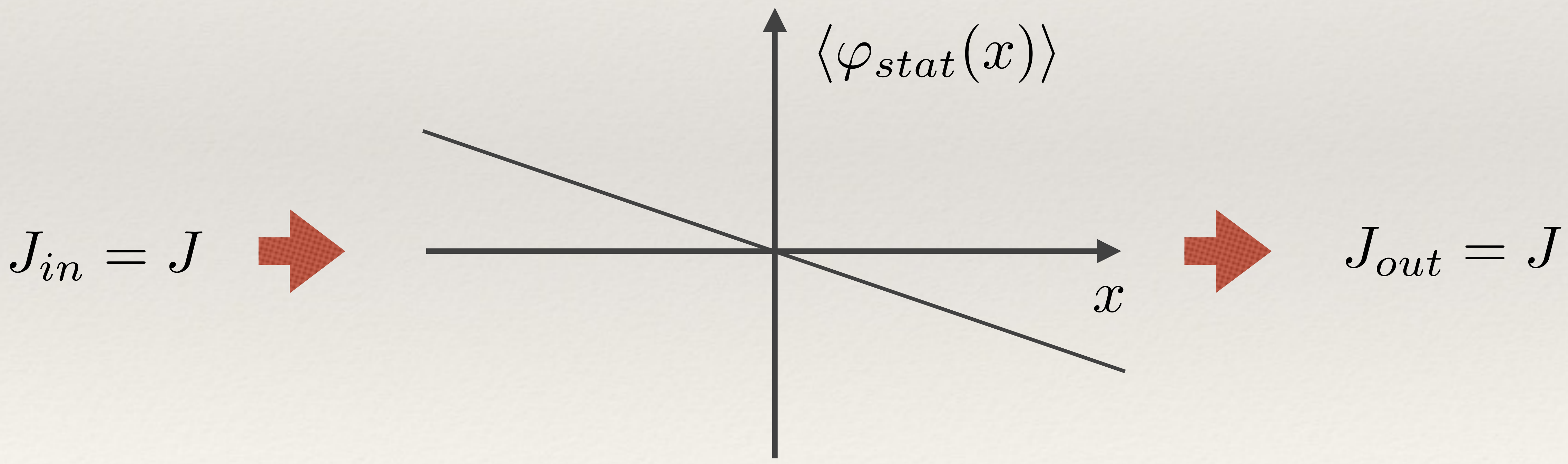
$$\langle a(x,t) b(x,t)\rangle \neq \langle a(x,t)\rangle \langle b(x,t)\rangle$$

Stationary model

The field

$$\varphi(x,t) = b(x,t) - a(x,t)$$

diffuses freely due to the **conservation law** for *B - A*



while the stationary value of the interface is at the center of the system

Perturbed model (I)

We model the presence of an extra buyer with a modified reaction law:

$$
\begin{aligned}
A + B &\to \emptyset \quad &\text{w. prob.} \quad & 1-p \\
A + B &\to B \quad &\text{w. prob.} \quad & p\,\frac{1+m}{2} \\
A + B &\to A \quad &\text{w. prob.} \quad & p\,\frac{1-m}{2}\,.
\end{aligned}
$$

for *p=0* we get the old model, while for *p ≠ 0* we get a bias governed by *m*

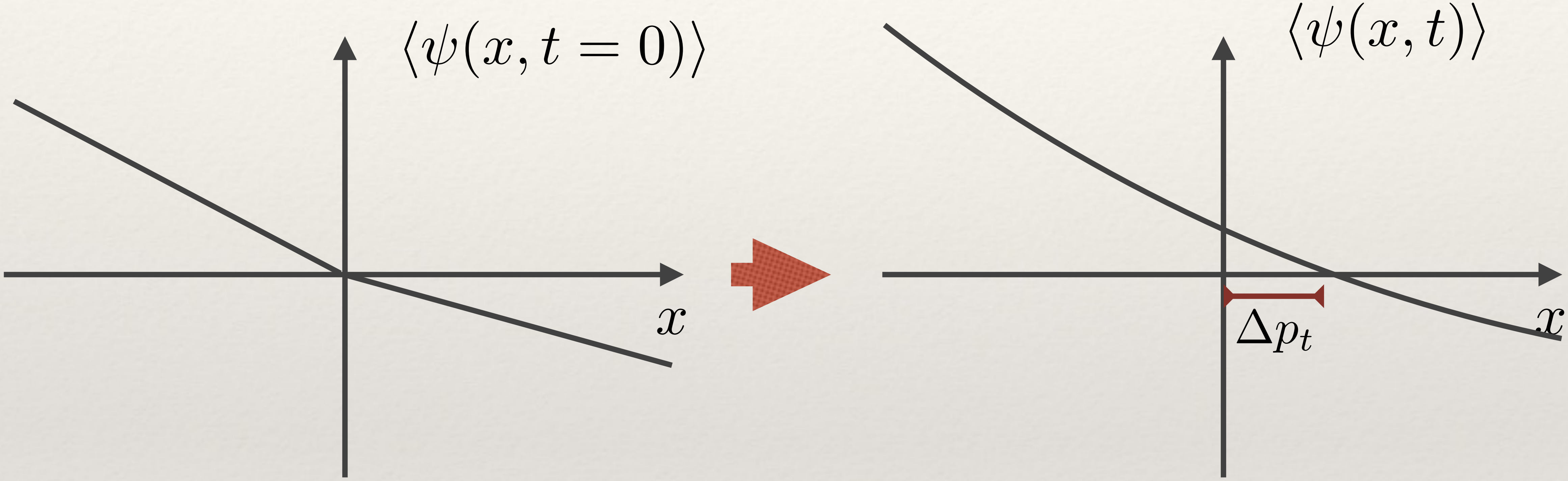
$$
\frac{\partial \langle b(x,t)\rangle}{\partial t} = D\frac{\partial^2 \langle b(x,t)\rangle}{\partial x^2} - \lambda u_A \langle a(x,t) b(x,t)\rangle \qquad u_A = 1 - p\left(\frac{1+m}{2}\right)
$$

$$
\frac{\partial \langle a(x,t)\rangle}{\partial t} = D\frac{\partial^2 \langle a(x,t)\rangle}{\partial x^2} - \lambda u_B \langle a(x,t) b(x,t)\rangle \qquad u_B = 1 - p\left(\frac{1-m}{2}\right)
$$

and the new conserved field is $\psi = u_B b - u_A a$

Perturbed model (II)

The system hasn't a stationary state anymore!



$$J_{in} = J u_B \neq J_{out} = J u_A$$

In fact, the interface drifts as

$$\Delta p_t = 2\alpha(u_B/u_A)\sqrt{Dt} \qquad \textit{with} \quad \alpha(z)\left(\frac{z+1}{z-1} - \mathrm{erf}[\alpha(z)]\right) - \frac{1}{\sqrt{\pi}} e^{-\alpha^2(z)}$$

Y-ratio: executed volumes

As one would like to determine the relation with respect to the volume, one can calculate:

Executed volume: $\langle Q \rangle = \beta(u_B/u_A)(JT)$

Market volume: $\langle V \rangle = \gamma(u_B/u_A)(JT)$

so that finally

$$\Delta p_t = 2\alpha(QD/\beta J)^{1/2}$$

While the value of $Y = 2\alpha / (D/\beta J)^{1/2}$ is fixed by the **participation ratio**

$$\phi(z) = \frac{(\text{trader volume})}{(\text{market volume})} = \frac{2\beta(z)}{\beta(z) + \gamma(z)}$$

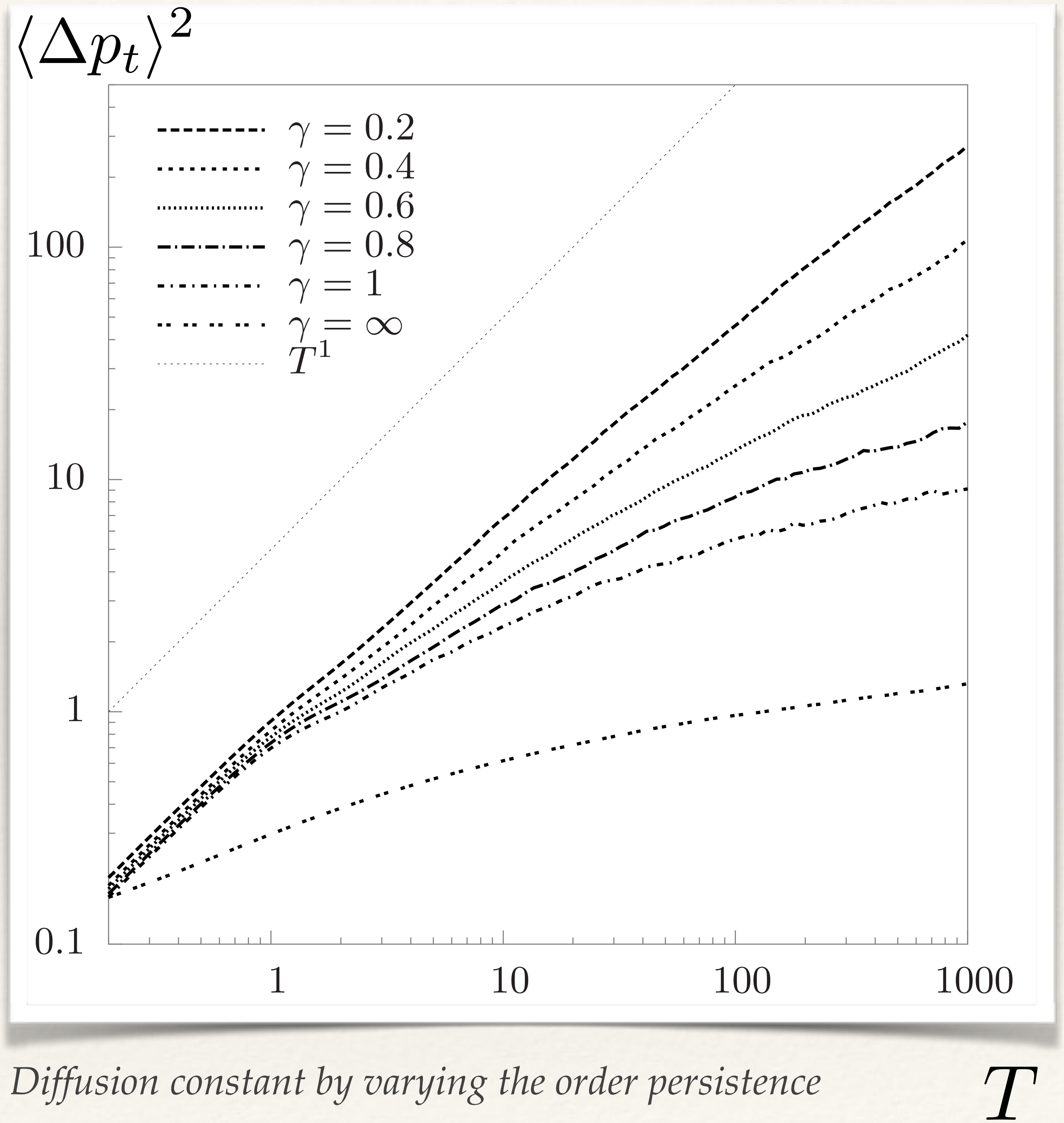
Generalizations

Any generalization preserving the asymmetric part of the dynamics **yields the same impact relation**.

The variance of the price p_t can be tuned

$$\langle m_t m_{t+\tau} \rangle \sim \tau^{-\gamma}$$

so to enforce consistency with empirical data



Diffusion constant by varying the order persistence

ε-intelligence model

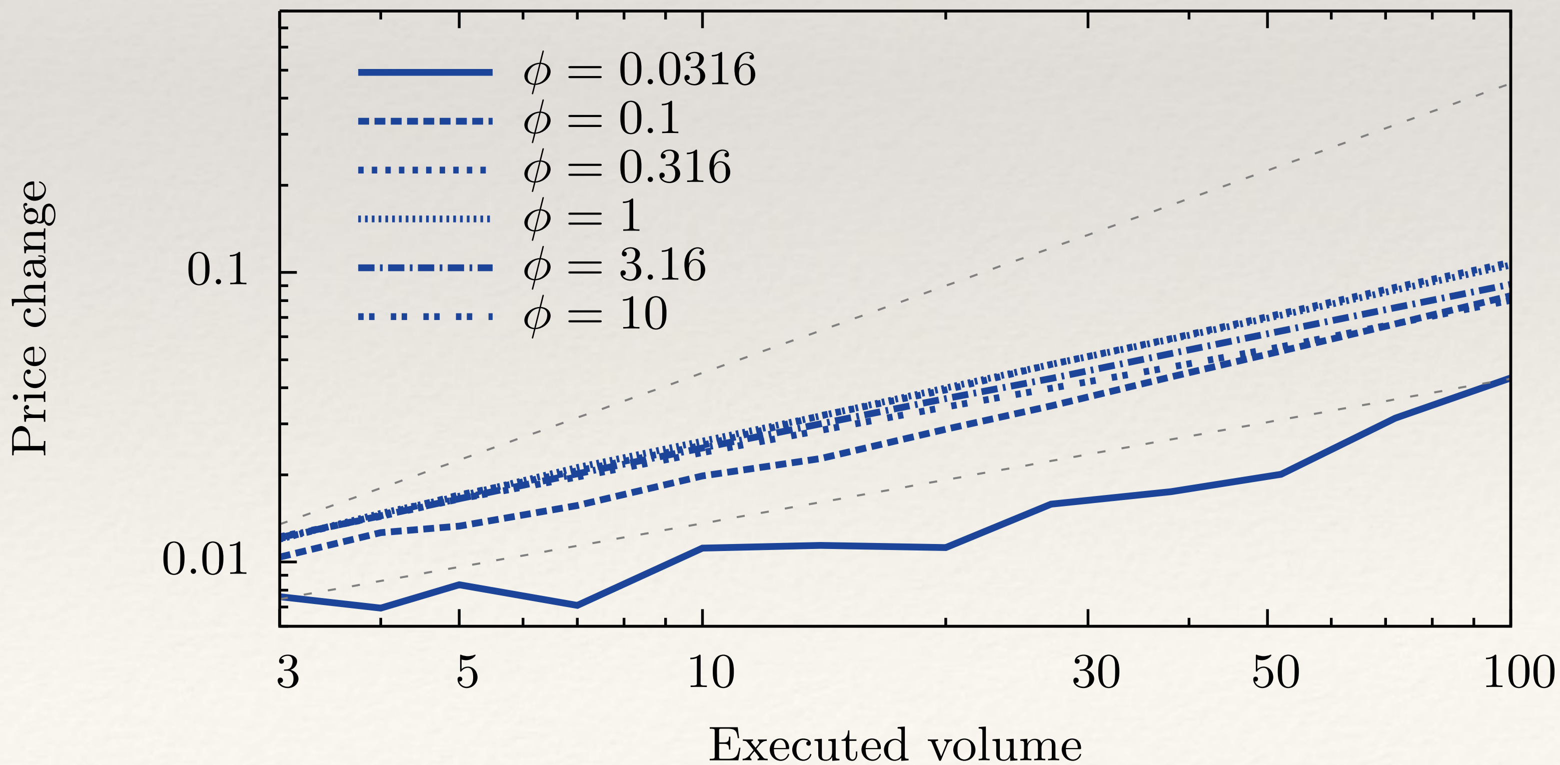
Different type of models sharing the same ingredients (dimensionality and vanishing liquidity at the mid-price) yield **qualitatively similar results**

[Mastromatteo, I., *et al.* (2014) Physical Review E, 89(4), 042805.
Tóth, B., *et al.* (2011). Physical Review X, 1(2), 021006]

Gain: Closer to empirical data (faithfully describes market, limits and cancellations)

Lose: Analytical tractability

This are the empirically grounded models which inspired the **stylized one** which has been illustrated.



Conclusions

- Anomalous market impact arises from the anomalous properties of a market as an information processing system
- Empirically, impact is universal and concave
- A simple model reproducing the minimal ingredients (dimensionality and locally linear book) is able to reproduce a square root impact
- Generalizations of these ideas still yield concave impact

Thank you

References

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