Large-dimensional and multi-scale effects in stocks volatility modeling

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work done at:

Chaire de finance quantitative, École Centrale Paris

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Journées MAS
Toulouse, 27–29 août 2014

Outline

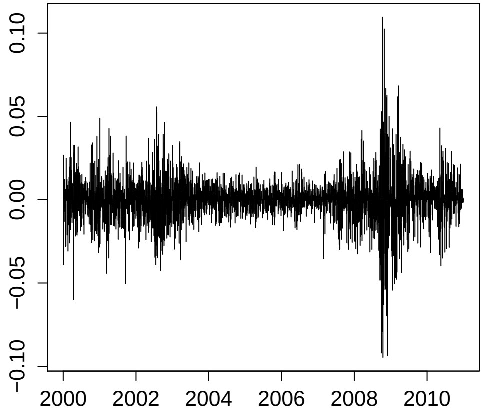
Stock prices log-returns: $x_t = \ln P_t - \ln P_{t+1}$

Stock's volatility: a measure of "typical amplitude" or "fluctuations"

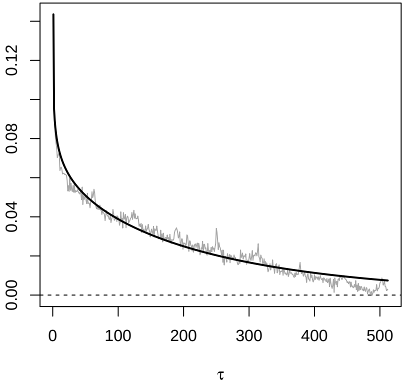
Can be understood globally (distributional sense, like empirical standard deviation) or dynamically (time-varying).

In this talk: time-varying volatility (several possible estimators: $|x_t|$, x_t^2, rolling std-dev, Rogers-Satchell, etc.)

Clustering



Long memory: auto-correlation



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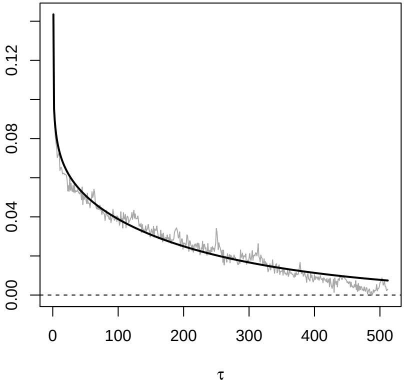

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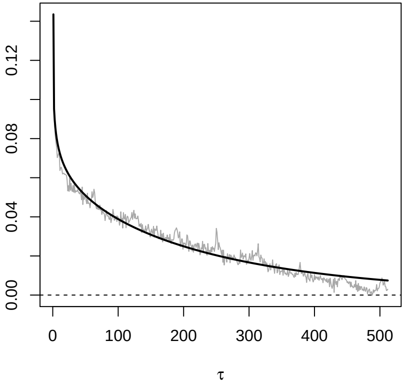

- $\langle (x_t^2 - \langle \sigma^2 \rangle) x_{t-\tau}^2 \rangle$
- power-law fit $\sim \tau^{-\beta}$

$$\begin{cases} x_t &= \sigma_t \, \xi_t \\ \xi_t &\sim F_\xi & \text{stochastic signed residuals (e.g. Student)} \\ \sigma_t^2 &= \mathcal{F}(\{x_{t-\tau}\}) & \text{positive fluctuating 'volatility'} \end{cases}$$

Not stochastic vol models, rather conditionally deterministic vol.
See discussion on 'Time-reversal asymmetry' later.

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QARCH: $\boxed{\mathcal{F}(\{x_{t-\tau}\}) = s^2 + \sum_{\tau>0} L(\tau) x_{t-\tau} + \sum_{\tau,\tau'>0} K(\tau,\tau') x_{t-\tau} x_{t-\tau'}}$

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$$\begin{aligned} \langle\sigma_t^2\rangle &= s^2 + \sum_{\tau=1}^{q} K(\tau)\langle x_{t-\tau}^2\rangle \\ &= s^2 + \sum_{\tau=1}^{q} K(\tau)\langle\sigma_{t-\tau}^2\rangle\langle\xi_{t-\tau}^2\rangle \\ \langle\sigma_t^4\rangle &= \dots \end{aligned}$$

need to be finite.

In particular, $\mathrm{Tr}\,K < 1/\langle\xi^2\rangle = 1$

Outline

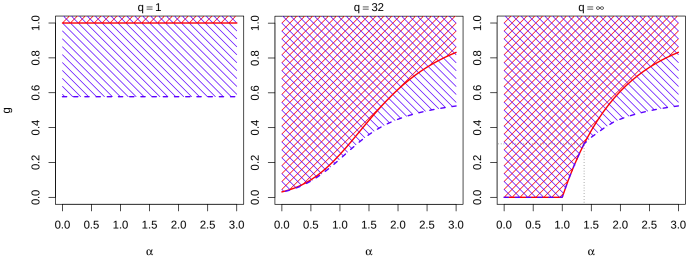

Figure: Allowed region in the α, g space for $K(\tau,\tau) = g\,\tau^{-\alpha}\mathbb{1}_{\{\tau\leq q\}}$ and $L(\tau) = 0$, according to the finiteness of $\langle\sigma^2\rangle$ and $\langle\sigma^4\rangle$. Divergence of $\langle\sigma^2\rangle$ is depicted by $45°$ (red) hatching, while divergence of $\langle\sigma^4\rangle$ is depicted by $-45°$ (blue) hatching. In the wedge between the dashed blue and solid red lines, $\langle\sigma^2\rangle < \infty$ while $\langle\sigma^4\rangle$ diverges.

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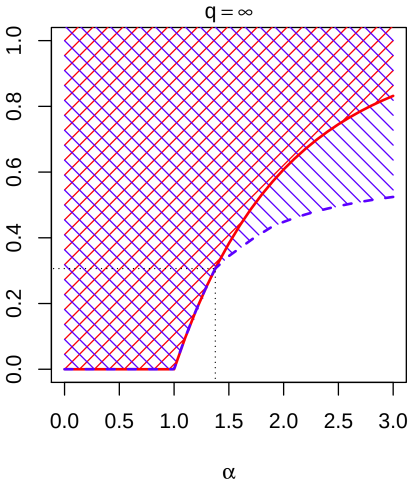
- a long-ranged power-law decaying correlation function $(0 < \beta < 1)$ can be obtained theoretically with a power-law volatility-feedback kernel with exponent $\alpha = (3 - \beta)/2 \in (1, 1.5)$.
- Empirically, the estimated (exponentially truncated) power-law kernel is found to have $g \approx 0.081$ and $\alpha \approx 1.11$.


q = ∞
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
α

Outline

$$\mathcal{F}(\{x_{t-\tau}\}) = s^2 + \sum_{\tau,\tau'>0} K(\tau,\tau') x_{t-\tau} x_{t-\tau'}$$

$$\sum_{\tau',\tau''=1}^{q} \left(\sum_n \lambda_n v_n(\tau') v_n(\tau'') \right) r_{t-\tau'}\, r_{t-\tau''} \equiv \sum_n \lambda_n \langle r | v_n \rangle_t^2$$

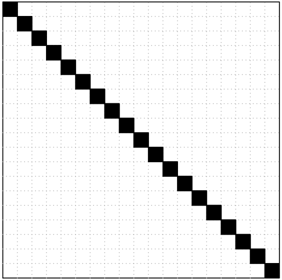
The square volatility σ_t^2 picks up contributions from various past returns eigenmodes. The modes associated to the largest eigenvalues λ are those which have the largest contribution to volatility spikes.

Examples of non-diagonal quadratic kernels (0)

ARCH(q): purely diagonal [engle1982autoregressive, bollerslev1986generalized, bollerslev1994arch]

$$\sigma_t^2 = s^2 + \sum_{\tau=1}^{q} K(\tau,\tau) x_{t-\tau}^2$$

$K(\tau, \tau')$

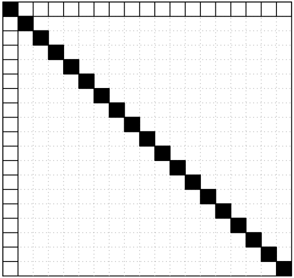


Examples of non-diagonal quadratic kernels (1)

Correlation between past 1-day returns and q-days weighted trends

$K(\tau, \tau')$

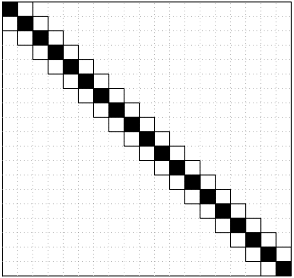
$$\sigma_t^2 = \text{ARCH} + x_{t-1} \sum_{\tau=1}^{q} k_{\text{LT}}(\tau) x_{t-\tau}$$



Examples of non-diagonal quadratic kernels (2)

Past squared 2-days returns over q lags

$K(\tau,\tau')$



$$\sigma_t^2 = \mathsf{ARCH} + \sum_{\tau=0}^{q-2} k_2(\tau)[R_{t-\tau}^{(2)}]^2$$

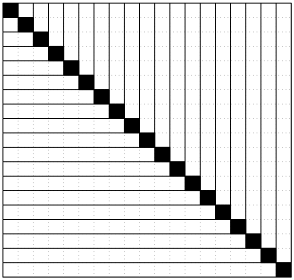
where $R_t^{(\ell)} \equiv \sum_{\tau=1}^{\ell} x_{t-\tau}$

Examples of non-diagonal quadratic kernels (3)

Squared last ℓ-days trends [borland2005multi]

$K(\tau, \tau')$

$$\sigma_t^2 = \text{ARCH} + \sum_{\ell=1}^{q} k_{\text{BB}}(\ell)[R_t^{(\ell)}]^2$$

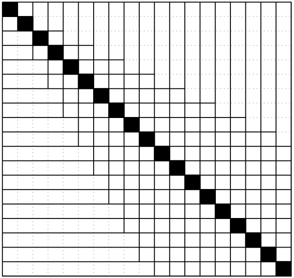


Examples of non-diagonal quadratic kernels (4)

Correlations between past ℓ-days trends [zumbach2010volatility]

$$\sigma_t^2 = \mathsf{ARCH} + \sum_{\ell=1}^{\lfloor q/2 \rfloor} k_{\mathsf{Z}}(\ell) R_t^{(\ell)} R_{t-\ell}^{(\ell)}$$

$K(\tau, \tau')$



Estimation methods

- Method of Moments
  - pros: no distributional hypothesis, computationally easy (inverting a linear system)
  - cons: very noisy, in particular with high moments

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Compromise: one-step ML with GMM prior.

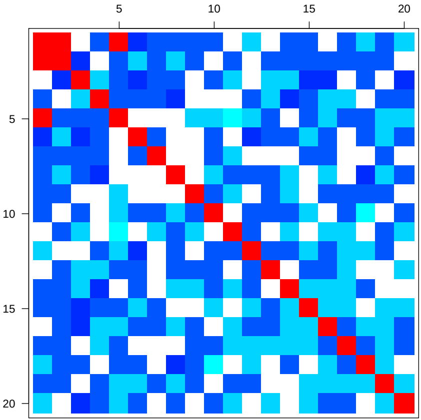
Dataset

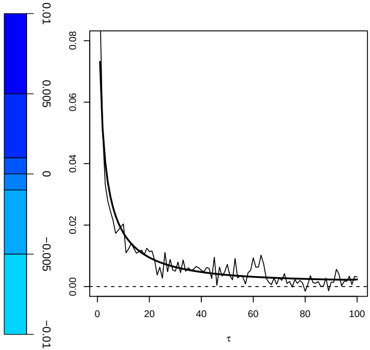
Daily stock prices for $N = 280$ names: universality hypothesis

Present in the SP500 index during 2000 – 2009 ($T = 2515$ days)

Removing market “low-frequency” fluctuations (separate calibration for volatility of the index)

Results


5
10
15
20
5
10
15
20


0.01
0.005
0
−0.005
−0.01
0.08
0.06
0.04
0.02
0.00
0
20
40
60
80
100
τ

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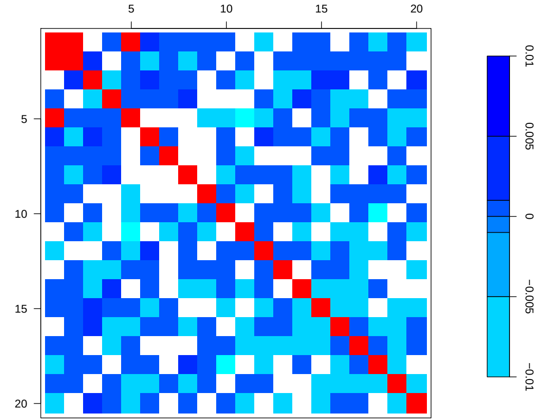

- no evident structure
- diagonal dominates
- still significant off-diag content

Outline

Explicitly backward looking construction: $\sigma_t^2 = \mathcal{F}(\{x_{t-\tau}\})$

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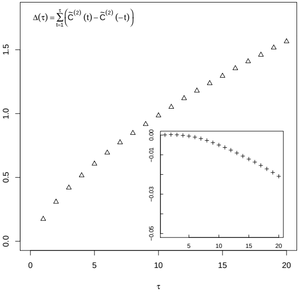
Generates a too large Time-reversal asymmetry:
$\widetilde{C}^{(2)}(\ell) = \langle(\sigma_t^2 - \langle\sigma^2\rangle)x_{t-\ell}^2\rangle$



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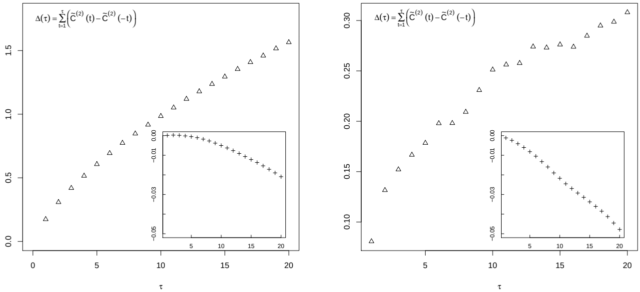

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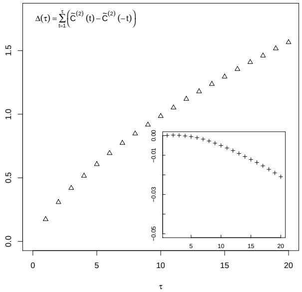

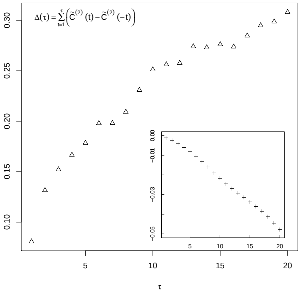

Put in more randomness: ARCH mechanism + TRI stochastic volatility !

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Extensions:

- specific day/night self- and cross-excitement effects
- similarities with Hawkes modelling

Pierre Blanc, Rémy Chicheportiche, and Jean-Philippe Bouchaud.
The fine structure of volatility feedback II: Overnight and intra-day effects.
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