You have at your disposal a function `Random`, each call of which returns a uniform random number in $[0, 1]$ independent of previous calls. We assume an infinite precision.

**Exercice 1.**

We are interested in the simulation of a discrete distribution on a finite set $S = \{1, \ldots, n\}$. Let $p_i$ be the probability of occurrence of the value $i$, with $\sum_{i=1}^{n} p_i = 1$. We allow a preprocessing time of $O(n)$.

1. Recall how to simulate this distribution in time $O(\log n)$ and with one call to `Random`.
2. Design a method to simulate this distribution in $O(1)$ time and calls to `Random`.

*Hint*: reshape the histogram of the distribution (where the bars indicate the frequency of each value).

**Exercice 2.**

*Simulation of a normal distribution*

1. Can we apply the inverse transformation method (*fr*: méthode de l'inverse) to easily simulate the normal distribution $\mathcal{N}(0, 1)$?
2. Using the inverse transformation method, simulate the law of density $f(x) = \frac{1}{\sqrt{1+x^2}}$ on $\mathbb{R}$ (Cauchy).
3. Deduce a way to simulate the law $\mathcal{N}(0, 1)$ by the rejection sampling (*fr*: méthode du rejet) of von Neumann.
Exercice 3.

Consider the following model of a communication channel (continuous time and discrete data):

- Traffic Input: packets of random length (according to uniform distribution on \{1, \ldots, M\}), where $T_n$ the arrival date of the $n$-th package follows a Poisson process of intensity $\lambda$, i.e., $T_0 = 0$ and will inter-arrivals $(T_n - T_{n-1})_{n \in \mathbb{N}}$ iid of law $Exp(\lambda)$.
- Server: FIFO of flow equal to 1 as long as there is work (transmission time of a packet = packet length).
- Queue: storage with $\infty$ memory.

Describe an simulation algorithm of this model, allowing in particular to plot the curve of the number of packets waiting in the queue as a function of time.

Exercice 4.

You know that this problem has been skipped every week from the beginning...

A long time ago, a number $p$ was chosen at random uniformly in $[0, 1]$, but this value was never revealed to mankind. Since this time, the sun rises every day with probability $p$ (still unknown). What happened during the preceding days is independent of what happens today. You know that the sun has risen every day from the beginning, that is $n$ times (and you know this number), what is the probability that it will rise tomorrow?