Exercice 1.

Two Poisson process

A database can have two types of request: writing and reading. They happen following two independent Poisson process of parameters $\lambda_R$ and $\lambda_W$.

1. What is the probability that the interval between two reading is bigger than $t$?
2. What is the probability that the next event is a reading?
3. What is the probability that at most $n$ writing happen is the interval $[0, t]$?
4. What is the probability that at least two events (reading or writing) happen in the interval $[0, t]$?

Exercice 2.

M/M/$\infty$ queue

The M/M/$\infty$ queue features a Poisson input traffic with intensity $\lambda$, an infinite buffer and an infinite number of independent servers, each one with exponential service time of parameter $\mu$. Let's consider $N_t$ the number of packets in the system at time $t$, it is a homogeneous continuous time Markov chain.

1. Describe its transition graph and its infinitesimal generator.
2. What is the stability condition(s), i.e. the condition(s) over the parameters such that the chain admits an invariant distribution $\pi$? In that case, what type of distribution is it?
3. In the stationary state, i.e. assuming that the current law of $N_t$ is $\pi$, what is the average number of packets in the system? What is the average waiting time in the system? Does the Little's law apply?

Exercice 3.

M/M/1 output traffic

Given an M($\lambda$)/M($\mu$)/1 queue, suppose that $\lambda < \mu$ and that the queue is in the stationary state. Show that the output traffic is a Poisson process with intensity $\lambda$.

Exercice 4.

Comparing three small queuing systems

We want to compare the three following systems:

1. Poisson process input traffic of parameter $\lambda$ in one queue with a server with exponential service time of parameter $2\mu$.
2. Poisson process input traffic of parameter $\lambda$ in one queue with two independent servers, each one with exponential service time of parameter $\mu$.
3. Poisson process input traffic of parameter $\lambda$ routed in one of two independent queues, with probability $1/2$ for each queue and an exponential service of parameter $\mu$ for each queue.

1. Intuitively, which system performs the best?
2. For each system, describe the infinitesimal generator.
3. For each system, give the stability condition, i.e. the condition over $\lambda$ and $\mu$ ensuring an invariant distribution.
4. For each system, compute the average number of clients in the system, assuming it is in the stationary state.
5. For each system, deduce the average waiting time in the system.
6. Where are the differences between the three models? Which one is the best?