Progress in Algebraic Programming and Hypergraphs

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Humble and Hero Programming

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A central challenge: the **looming programmer productivity crisis**

- increasingly many hardware targets,
- increasingly heterogeneous hardware.

Compilers, libraries, and applications

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Solution: **re-define the classical boundaries** between

- compiler, library, *and* application.

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- perform **optimisations based on algebraic information**

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```
grb::Vector< double > x( n ), y( m ), z( n );
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grb::operators::min< double > minOp;
grb::operators::add< double, double, double > addOp;
```

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grb::Semiring<
   grb::operators::add< double >, grb::operators::mul< double >
   grb::identities::zero, grb::identities::one
> mySemiring;
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3) primitives: eWiseApply, reduction into scalar (fold), dot, mxv, ...

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grb::set( x, 1.0 );                     // x_i = 1, ∀i
grb::setElement( x, 3.0, n/2 );         // x_{n/2} = 3
grb::eWiseApply( y, x, x, addOp ); // y_i = x_i + x_i, ∀i
grb::eWiseApply( z, x, x, minOp ); // z_i = min{x_i, x_i}, ∀i
grb::mxv( y, A, x, mySemiring );    // y += Ax
```

Algebraic Type Traits

Compile-time inspection of algebraic properties **algebraic type traits**:

- grb :: is_associative< Operator >::value, `true` iff $(a \odot b) \odot c = a \odot (b \odot c)$;
- grb :: is_idempotent< Operator >::value, `true` iff $a \odot a = a$;
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2) decide which optimisations are applicable,

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3) reject expressions without recipe for **auto-parallelisation** exist.

$\alpha = x_0 \odot \ldots \odot x_{n-1}$, i.e., `grb::foldl( alpha, x, op )` with non-associative op? ✗

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Current backends:

- sequential auto-vectorising, shared-memory parallel (OpenMP), distributed-memory parallel (LPF+MPI/ibverbs), and hybrid.

Ref.: A C++ GraphBLAS: specification, implementation, parallelisation, and evaluation by A. N. Yzelman, D. Di Nardo, J. M. Nash, and W. J. Suijlen, pre-print (2020).

Performance

Upcoming **nonblocking** CG solve speedup:

- two-socket Intel Cascade Lake, 44 cores, no hyperthreads

	gyro_m	G2_circuit	bundle_adj	ecology2	Queen_4147
GSL	0.84	0.95	0.89	0.91	0.92
blocking ALP	2.30	4.53	**12.7**	6.91	17.5
SuiteSparse:GraphBLAS	1.57	1.11	5.82	3.52	11.6
Eigen	5.21	2.57	1.61	1.94	9.20
auto non-blocking ALP	**5.57**	**9.75**	2.87	**13.7**	**18.6**

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- Challenge: how to expose deeper pipelines?

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Graph algorithm performance using the **hybrid auto-parallelisation**

- Clueweb12 link matrix, approx. 978M vertices and 42.5**B edges**

	Ivy Bridge nodes						
	4	5	6	7	8	9	10
Input	1524	1271	1067	943	691	662	537
4-hop BFS	48.8	110	54.8	99.6	83.0	74.2	23.3
20-hop BFS	404	280	231	323	221	230	160
PageRank	13.3	10.3	9.68	8.00	21.0	22.9	21.6

The k-hop BFS and PageRank (PR) on Clueweb12, performance in seconds. Infiniband EDR interconnect.

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Interoperability with Spark, without re-writing software (2019):

				Spark				Spark with ALP/GraphBLAS			
	GB	Gnz	n_ϵ	$n = 1$	$n = 10$	$n = n_\epsilon$	$s/$it.	$n = 1$	$n = 10$	$n = n_\epsilon$	$s/$it.
uk-2002	4.7	0.3	73	168.6	1373.8	>4 hrs	133.9	8.7	13.9	48.7	0.56
clueweb12	786	42.5	45	-	-	-	-	658.8	963.2	1875.0	27.7

Pagerank performance in seconds using ten Ivy nodes with Infiniband EDR, Spark 2.3.1, and Hadoop 2.7.7.

- I/O: 19× faster, computation: 239× faster for `uk-2002`.
- Spark Clueweb PR: out of memory. Literature: Blogel, 128 nodes!

Ref.: Lightweight Parallel Foundations: a model-compliant communication layer by W. J. Suijlen and Y., `https://arxiv.org/abs/1906.03196` (2019)

Algebraic Programming

How far can we take this type of programming?

ALP/Dense

Classical **dense** linear algebra algorithms:

- submatrix selection, row permutations, ... random sampling?
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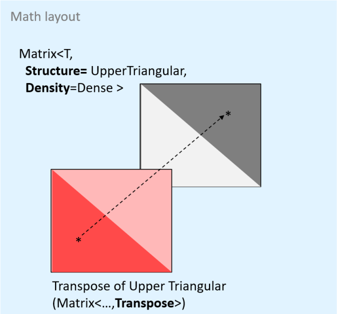

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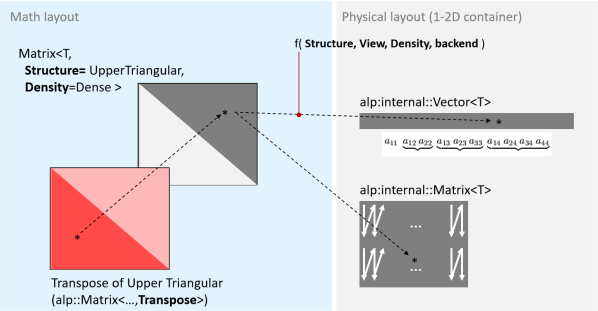
- ALP is free to choose the underlying data layout:



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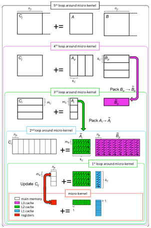

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    - use offline auto-tuning, **once** per new architecture: ✓
    - by J. Ye, G. Rossini, S. Varoumas, a.o. at Huawei Cambridge

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Amounts to **delayed compilation** of 'universal binaries', with

- high-level MLIR dialects as an **architecture-agnostic** language.
- Capture algebraic information and optimisations in compiler logic.

Ref.: MOM: Matrix Operations in MLIR by L. Chelini, H. Barthels, P. Bientinesi, M. Copic, T. Grosser, D. G. Spampinato, in IMPACT at HiPEAC Budapest, Hungary (2022).

ALP as a more fundamental programming model

ALP/Pregel: vertex-centric programming

- arguably, like MapReduce, a more humble API:

```
static void program(
        VertexIDType &current_max_ID,    // each vertex starts with its unique ID
  const VertexIDType &incoming_message,  // IDs will propagate from neighbours
        VertexIDType &outgoing_message,  // new max IDs will be broadcast
  grb::interfaces::PregelData &pregel
) {

  if( pregel.round > 0 ) {                           // messages arrive after round 1

    if( current_max_ID < incoming_message ) {  // a larger ID has arrived; join the
      current_max_ID = incoming_message;           // component 'led' by this ID

    } else {                                          // otherwise no change: if everyone
      pregel.voteToHalt = true;                        // has no change, stop execution
    }
  }
  outgoing_message = current_max_ID;              // as long as we're running, keep
}                                                      // broadcasting my component ID
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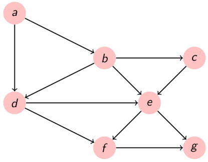

Ref.: Y., "Humble Heroes", Communications of Huawei Research, accepted. http://albert-jan.yzelman.net/publications.php#yzelman22

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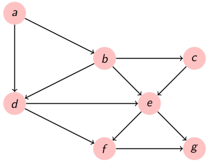

Compiles using the standard ALP software stack:

- `grbcxx -b hybrid myPregelAlgo pregelAlgo.cpp`
- in 'develop' branch on GitHub, slated for v0.7 release

Ref.: Y., "Humble Heroes", Communications of Huawei Research, accepted. http://albert-jan.yzelman.net/publications.php#yzelman22

HyperDAGs

As another novel backend example, the following two steps:

1) `grbcxx -b hyperdags myProgram example.cpp`
2) `grbrun -b hyperdags ./myProgram`

dumps a **HyperDAG** corresponding to the computation.

HyperDAGs generalise DAGs: each net corresponds to one output, possibly consumed by one or more subsequent computations.

Papp, Anegg, and Y. show, amongst others, that:

- HyperDAG partitioning remains NP-Complete,
- $\nexists$ algorithm in P with reasonable approximation factors[1], and
- hierarchical NUMA-aware partitioning can be very suboptimal.

On GitHub: HyperDAG backend (branch 274), HyperDAG database.

1: assuming that the exponential time hypothesis holds.

Ref. Pál András Papp, Georg Anegg, Y., "Partitioning Hypergraphs is Hard: Models, Inapproximability, and Applications" (submitted), ArXiV:2208.08257.

Conclusion

Solving the looming software portability and productivity crisis requires

blurring the lines between compiler, library, and application

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Our take with C++ ALP and MLIR is **open source**:

- `github.com/Algebraic-Programming`, Apache 2.0;
- `algebraic-programming.github.io`.

Acknowledgements

Thank you

The **GraphBLAS community**

J. Kepner and J. Gilbert. *Graph Algorithms in the Language of Linear Algebra*. SIAM, 2011.
Kepner, Gilbert, Buluç, Mattson, Moreira, and many others: https://www.graphblas.org

Algebraic programming inspired by

Generic Programming and C++ STL principles, **Alexander Stepanov**'s work in particular.

Contributors and inspirators to ALP/GraphBLAS:

Aristeidis Mastoras, Sotiris Anagnostidis, Anders Hansson;
Lorenzo Chelini, Daniele G. Spampinato, Valdimir Dimić;
Alberto Scolari, Denis Jelovina, Verner Vlačić, Auke Booij;
Aikaterini Karanasiou, Dan Iorga, Gabriel Gjini, Pouya Pourjafar;
Daniel Di Nardo, Jonathan M. Nash, Wijnand J. Suijlen;
Pál András Papp, Georg Anegg, Bill McColl;
and other colleagues – within Huawei, and our external research partners

Open source, Apache 2.0, welcome to try, use, and work together!

- https://github.com/Algebraic-Programming
- https://algebraic-programming.github.io

Backup slides

Backup slides

History

Algebraic Programming:

The Design and Analysis of Computer Algorithms,
Aho, Hopcroft, Ullman (1974)

Introduction to Algorithms (first edition only),
Cormen, Leiserson, Rivest (1990)

Elements of Programming,
Alexander Stepanov & Paul McJones (2009)

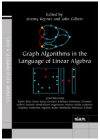
Graph Algorithms in the Language of Linear Algebra,
Jeremy Kepner & John Gilbert (2011)

From Mathematics to Generic Programming,
Alexander Stepanov & Daniel Rose (2015)

GraphBLAS.org, following work by Kepner & Gilbert,
Kepner, Gilbert, Buluç, Mattson, et alii (2016)

ALP (2022), and perhaps more?

ALP/GraphBLAS



`GraphBLAS.org`; Kepner, Gilbert, Buluç, Mattson, Moreira, ...

- for example, $y = A^k x$, parametrised in a semiring:

```
template< typename Semiring, typename NonzeroT, typename VectorT >
grb::RC mpv(
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        const   grb::Vector< VectorT >  &x,
                grb::Vector< VectorT >  &buffer,
        const   Semiring                &ring = Semiring()
) {
        // error checking and error propagation omitted
        grb::vxm( y, x, A, ring );
        for( size_t i = 1; i < k; ++i ) {
                std::swap( y, buffer );
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}
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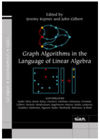
ALP/GraphBLAS



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Solves different problems for different semirings:

- plus-times: numerical linear algebra

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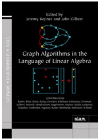

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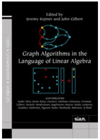


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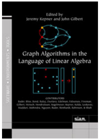

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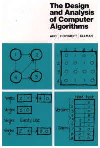

Solves different problems for different semirings:

- plus-times: numerical linear algebra
- Boolean: reachability / connectivity
- min-plus: shortest paths
- ...and more – see e.g. Aho, Hopcroft, and Ullman '74; Kepner & Gilbert '11

The ALP/GraphBLAS backends in more detail

- **reference**; sparse accumulators for vectors, Gustavson's matrix format, direction optimisation, and auto-vectorisation;
- **shared-memory parallel**: OpenMP and NUMA-aware;
- **distributed-memory parallel**: LPF (MPI or ibverbs)
  - (1D) block-cyclic vector and matrix distribution;
  - sparsity-aware allgather and reduce-scatter;
  - parallel I/O.
- **hybrid**: composes distributed- with shared-memory backend.

Non-blocking ALP/GraphBLAS

Nonblocking backend. Suppose $s = r + \alpha v$ over a given `semiring`:

1) `grb::set( s, r );`

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Blocking execution: the vector s is accessed *twice*

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Manual fusion (Y. et al., '20): performance ✓

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grb::eWiseLambda( [ &s, &r, &alpha, &v, &ring ] (const size_t i) {
    grb::apply( s[ i ], alpha, v[ i ], ring.getMultiplicativeOperator() );
    grb::foldl( s[ i ], r[ i ], ring.getAdditiveOperator() );
}, s, r, v );
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Ref.: A C++ GraphBLAS: specification, implementation, parallelisation, and evaluation by Y., D. Di Nardo, J. M. Nash, and W. J. Suijlen (2020).

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Automatic non-blocking execution (Mastoras et al., '22): humble ✓

- *lazily* evaluate ALP/GraphBLAS calls, build pipelines
- triggered when needed, **fusion without ALP program changes**

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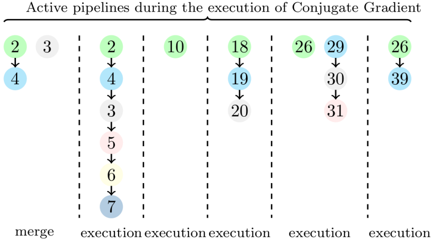
Non-blocking Conjugate Gradients in ALP/GraphBLAS

Dynamic **on-line** dependence analysis:




```
// six-stage pipeline, vectors(temp, r, x, b, u)
grb::set(temp, 0);
grb::set(r, 0);
grb::mxv(temp, A, x, ring);
grb::eWiseApply(r, b, temp, minus);
grb::set(u, r);
grb::dot(sigma, r, r, ring);

// single-stage pipeline, vector(b)
grb::dot(bnorm, b, b, ring);

tol *= sqrt(bnorm);

iter = 0;

do {
    // three-stage pipeline, vectors(temp, u)
    grb::set(temp, 0);
    grb::mxv(temp, A, u, ring);
    grb::dot(residual, temp, u, ring);

    grb::apply(alpha, sigma, residual, divide);

    // part of a two-stage pipeline, vectors (x, u, r)
    // the eWiseMulAdd at the bottom is the second stage
    grb::eWiseMulAdd(x, alpha, u, x, ring);

    // three-stage pipeline, vectors(temp, r)
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    grb::eWiseApply(r, r, temp, minus);
    grb::dot(residual, r, r, ring);

    if (sqrt(residual) < tol) break;

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    grb::eWiseMulAdd(u, alpha, u, r, ring);

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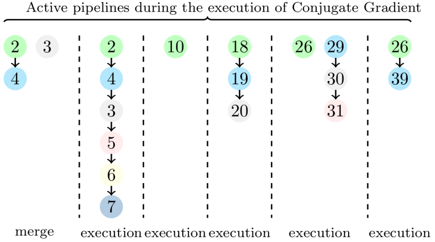

Fused execution can **cross control flow**:

- e.g., lines 26, 39 cross an if-statement;

```
1  // six-stage pipeline, vectors(temp, r, x, b, u)
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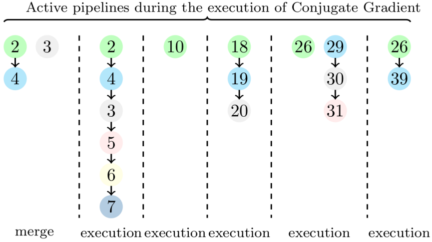

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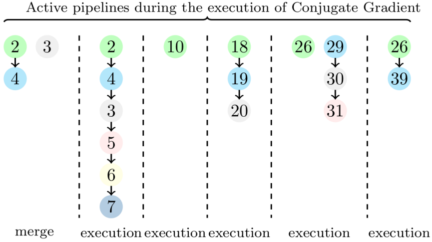

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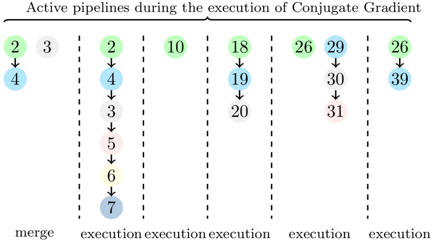
Non-blocking Conjugate Gradients in ALP/GraphBLAS

Dynamic **on-line** dependence analysis:




```
1  // six-stage pipeline, vectors(temp, r, x, b, u)
2  grb::set(temp, 0);
3  grb::set(r, 0);
4  grb::mxv(temp, A, x, ring);
5  grb::eWiseApply(r, b, temp, minus);
6  grb::set(u, r);
7  grb::dot(sigma, r, r, ring);
8
9  // single-stage pipeline, vector(b)
10 grb::dot(bnorm, b, b, ring);
11
12 tol *= sqrt(bnorm);
13
14 iter = 0;
15
16 do {
17     // three-stage pipeline, vectors(temp, u)
18     grb::set(temp, 0);
19     grb::mxv(temp, A, u, ring);
20     grb::dot(residual, temp, u, ring);
21
22     grb::apply(alpha, sigma, residual, divide);
23
24     // part of a two-stage pipeline, vectors (x, u, r)
25     // the eWiseMulAdd at the bottom is the second stage
26     grb::eWiseMulAdd(x, alpha, u, x, ring);
27
28     // three-stage pipeline, vectors(temp, r)
29     grb::eWiseMul(temp, alpha, temp, ring);
30     grb::eWiseApply(r, r, temp, minus);
31     grb::dot(residual, r, r, ring);
32
33     if (sqrt(residual) < tol) break;
34
35     grb::apply(alpha, residual, sigma, divide);
36
37     // part of a two-stage pipeline, vectors (x, u, r)
38     // the eWiseMulAdd aboce is the first stage
39     grb::eWiseMulAdd(u, alpha, u, r, ring);
40
41     sigma = residual;
42 } while (++iter < max_iterations);
```

Fused execution can **cross control flow**:

- e.g., lines 26, 39 cross an if-statement;
- elect chunk size s.t. all vectors cached;
- reduce #threads if vectors too small;
- **analytic model** automatically selects performance parameters: ✓.