ON NEWTON'S METHOD FOR AN INVERSE FIRST PASSAGE PROBLEM

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In this poster, we will introduce the inverse first passage problem (IFPT) for an Ornstein-Uhlenbeck process and some numerical problems in our work. The IFPT problem is targeted to reproduce the corresponding boundary for a given stochastic process such that the first passage density can fit a given distribution function. This problem attracts plenty of attention recently in various applications such as financial risk management, reliability analysis [1] etc..

Cheng etc. [2] have proved the IFPT problem is well-posed, but some numerical problems remain in the solving procedures of IFPT problems. Following the integral equation method proposed in [3] and [4], the IFPT problem can be translated to an integral equation:

$$f(x,t) = 0, \forall x \geq L(t), \tag{0.1}$$

where $L(t)$ is the desired true solution and $L(0)$ is given. This property proposes a constraint for the numerical scheme to verify whether the solution is the minimal solution to the equation.

In the solving procedure, we tried the simplest secant method because $f_x(x,t)$ cannot be expressed for $x < L(t)$. An interesting thing is that although $f_x(x,t)$ cannot be provided, the left-side derivative at true solution can be given from preliminary analysis:

$$\lim_{x \to L(t)-} f_x(x,t) = g(t) > 0, \tag{0.2}$$

which is independent of $L(t)$.

How to maximally use this information for accelerating the numerical scheme remains a problem for us. The ordinary Newton's method requires the derivative information at every iterative value, and therefore it is hard to be used directly in this case. We are interested to try various searching methods in this problem to increase the numerical efficiency.

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