Bounded Multi Port Model: Motivations, Feasability & Application to Broadcast

Olivier Beaumont, **Lionel Eyraud-Dubois**, Shailesh Kumar Agrawal, Hejer Rejeb

Cepage team, LaBRI, Bordeaux, France

Scheduling in Aussois
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Outline

1. Introduction
2. TCP Bandwidth Sharing
3. Broadcast with Bounded Degree
4. Conclusions

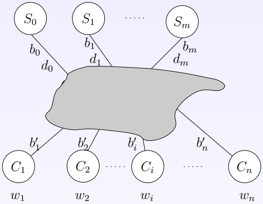
Introduction

Scheduling: what is a good model for communications ?

- Standard communication model: One-Port Model
  - a node is involved in at most one communication at the same time
  - corresponds well to old MPI implementations
- Problem: if the network is strongly heterogeneous, then the bandwidth of the server may be wasted
  - Imagine a server with 1GB/sec bandwidth sending 10MB to a client with 1MB/sec download bandwidth
  - It is not realistic to assume that the server will be busy for 10 secs
- In the context of large scale distributed and strongly heterogeneous platforms, one port model is not the right model

Explore the Bounded Multi Port model

- Simultaneous communications, with a per-node bandwidth bound (both upload and download)
- Internet-like: no contention inside the network



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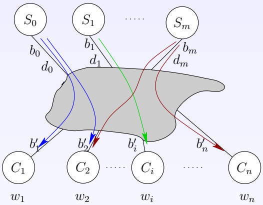


- In this talk, we will see:
  - A model for TCP bandwidth sharing
  - Its influence on several scheduling problems
  - A particular study of the broadcast operation

Outline

1 Introduction

2 TCP Bandwidth Sharing
- Model
- Influence on Scheduling Algorithms

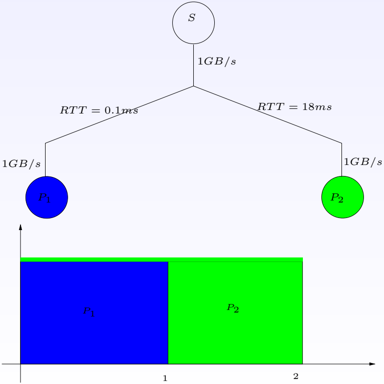
3 Broadcast with Bounded Degree

4 Conclusions

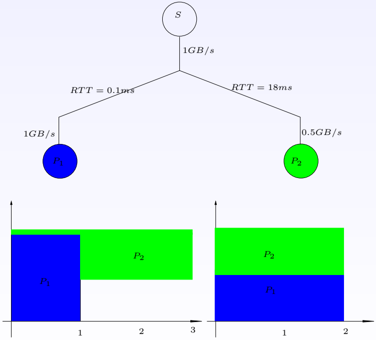
Why is it important ?

- Experiments using French Grid G5K
- The master has N threads, each sending data to N node

Experiment 1: master in Bordeaux, 1 client in Bordeaux (in a different cluster), 1 client in Nancy



Experiment 2: the same, except that the incoming bandwidth of the client in Nancy is twice smaller.



Modeling TCP Bandwidth Sharing

- Increase TCP window sizes until congestion
- TCP window increases quickly for nodes closer to the master
- $\Rightarrow$ closer nodes get higher bandwidth
- Max-Min Fairness algorithm

Model: Casanova and Marchal

Let b_i denote the achievable bandwidth between M and P_i (if alone)
Let λ_i denote the inverse of the RTT between M and P_i

- If $\sum b_i \leq B$, then $all(P_i) = b_i$
- Else
  - While $\exists i,\ b_i \leq \frac{\lambda_i B}{\sum \lambda_k}$: $all(P_i) = b_i$ and update $B \leftarrow B - b_i$
  - $\forall i$ s.t. $b_i > \frac{\lambda_i B}{\sum \lambda_k}$: set $all(P_i) = \frac{\lambda_i B}{\sum \lambda_k}$.

Note that $\sum all(P_i) = \min(B, \sum b_i)$.

An Upper Bound on Performance Degradation

- We consider a set of simultaneous communications between M and the P_is.
- Each communication has a release date r_i and starts immediately.

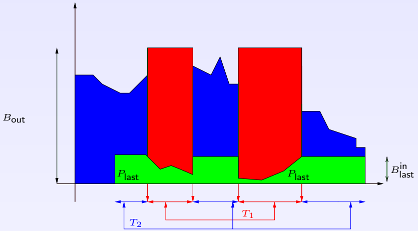
Lemma

If $\sum all(P_i) \leq B$, *then* $\forall i, \quad all(P_i) > 0 \Rightarrow all(P_i) = b_i$.

Theorem

The makespan obtained when relying on TCP Bandwidth Sharing mechanism can be at most twice the optimal makespan

An Upper Bound on Performance Degradation

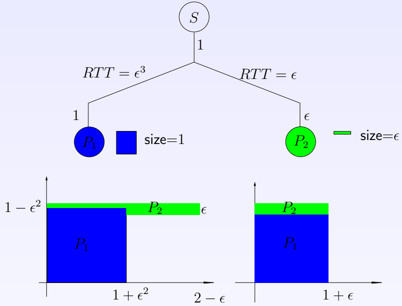


Proof.

- Same as Graham's proof... Consider P_{LAST}, whose last communication ends at T
- Partition $T - r_{\text{LAST}}$ into
  - T_1 instants when all bandwidth B is used, and T_2 the rest
- Then, if T_{OPT} denotes the optimal makespan
  - $T_1 \leq T_{\text{OPT}}$ (nothing is wasted during T_1)
  - $T_2 + r_{\text{LAST}} \leq T_{\text{OPT}}$ (P_{LAST} communicates at maximal rate during T_2)
- Therefore, $T = r_{\text{LAST}} + T_1 + T_2 \leq 2T_{\text{OPT}}$.

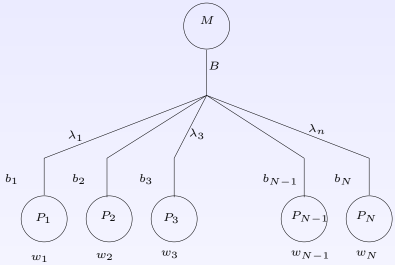
The above bound is tight

Let us consider the following platform



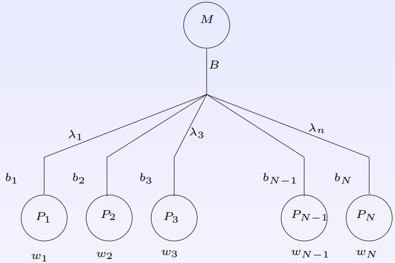
- If we rely on TCP bandwidth mechanism, then P_1 gets too much bandwidth: $1-\epsilon^2$ instead of $1-\epsilon$
- and it takes almost 1 time unit to finish the transfer with P_2
- If we enforce the bandwidth with P_1 to be at most $1-\epsilon$, both transfers end up in time $1+\epsilon$
- The ratio between both solutions is $2-3\epsilon$.

Steady State Scheduling of Independent Tasks



- b_i: number of tasks that can be sent to P_i in one time unit
- w_i: number of tasks that can be processed by P_i in one time unit
- Goal: Maximize the number of tasks that can be processed in steady state by the platform

Scheduling of Independent Tasks: Optimal Solution



- Let n_i denote the number of tasks processed by P_i
- Clearly, $n_i \leq b_i$, $n_i \leq w_i$ and $n_i \leq B$.
- Let us denote by $c_i = \min(b_i, w_i)$ and $C = \sum c_i$

Optimal Solution

- If $C \leq B$, then set $\forall i, \quad n_i = c_i$
- Else set $\forall i, \quad n_i = c_i \frac{B}{C}$.

Scheduling of Independent Tasks: Optimal Algorithm

Implementation 1

In order to avoid starvation, each slave node starts with two tasks in its local buffer. Each time P_i starts processing a new task, it asks for another task and the master node initiates the communication immediately with bandwidth rate n_i.

Proof.

- The bandwidth requested at master node is never larger than B since $\sum n_i \leq B$
- It takes $1/n_i$ time units to P_i to receive a task, and it takes at most $1/n_i$ time units to process it.
- Thus, the processing rate at P_i is exactly n_i.

□

Scheduling of Independent Tasks: Upper Bound

Implementation 1

Each time P_i starts processing a new task, it asks for another task and the master node initiates the communication immediately with bandwidth rate n_i.

Implementation 2

Each time P_i starts processing a new task, it asks for another task and the master node initiates the communication immediately.

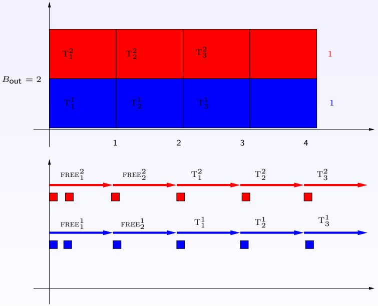
Theorem

The waste W experienced by Implementation 2 per unit time is bounded by $W \leq \frac{1}{4}B$, and hence its throughput verifies $T_2 \geq \frac{3}{4}T_1$

The bound is tight

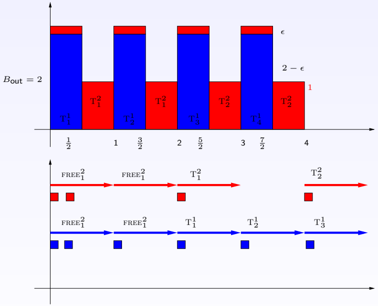
- 2 slave processors
  - P_1: $w_1 = 1,\ b_1 = 2,\ \mathrm{RTT} = \epsilon^2$
  - P_2: $w_2 = 1,\ b_2 = 1,\ \mathrm{RTT} = \epsilon$
- The ratio between both implementations is $\frac{4}{3}$

Using Implementation 1



2 tasks every 2 time units

Using Implementation 2



3 tasks every 4 time units

Summary of first part

- Multiport model is more realistic than 1-port model
- TCP Bandwidth sharing mechanism
  - is complicated
  - and strongly depends on RTT values.
- On the other hand,
  - we usually know what bandwidth should be allocated
  - and many mechanisms exist to limit the bandwidth of a connexion
- So, use them!

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    - Problem and Complexity
    - Algorithms
    - Evaluation
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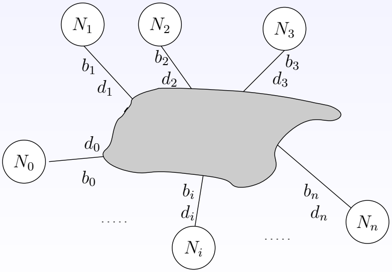
Introduction

From now on: broadcast/streaming operation

- One source node holds (or generates) a message
- All nodes must receive the complete message
- Steady-state: quantity of data per time unit
- Goal: optimize throughput
- Keep things reasonable: degree constraint



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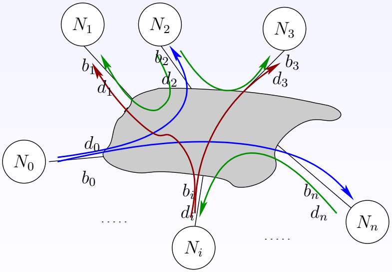
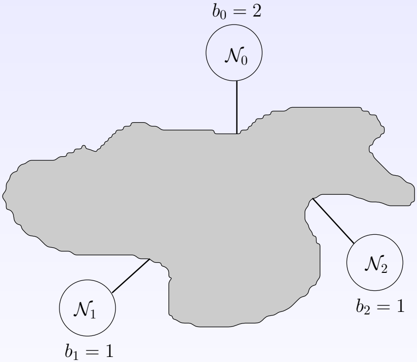


An example



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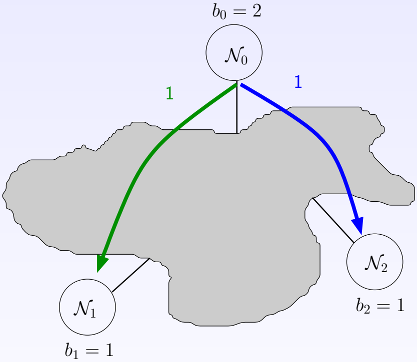
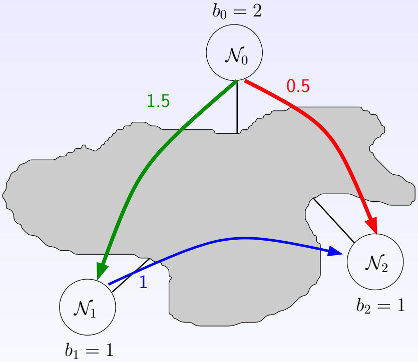


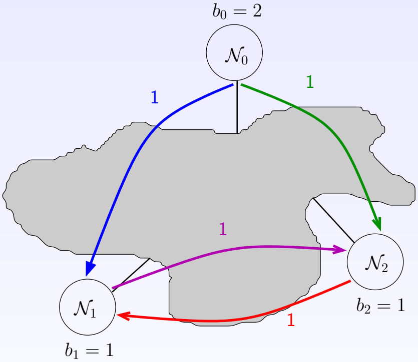
Best tree: $T = 1$

An example



Best DAG: $T = 1.5$

An example



Optimal: $T = 2$

Precise model

An instance

- n nodes, with output bandwidth b_i and maximal out-degree d_i
- node $\mathcal{N}_0$ is the master node that holds the data

A solution (Flows)

- Flow f_j^i from node $\mathcal{N}_j$ to $\mathcal{N}_i$
- $\forall j, \quad \left|\left\{i,\ f_j^i > 0\right\}\right| \leq d_j$ degree constraint at $\mathcal{N}_j$
- $\forall j, \quad \sum_i f_j^i \leq b_j$ capacity constraint at $\mathcal{N}_j$
- Maximize $T = \min_j \text{mincut}(\mathcal{N}_0, \mathcal{N}_j)$

Complexity

3-Partition

- $3p$ integers a_i such that $\sum_i a_i = pT$
- Partition into p sets S_l such that $\sum_{i \in S_l} a_i = T$

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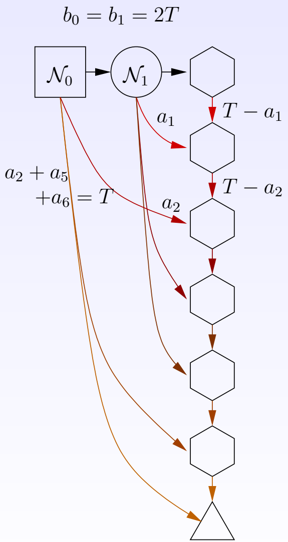
Reduction

- p "server" nodes, $b_j = 2T$ and $d_j = 4$
- $3p$ "client" nodes, $b_{j+p} = T - a_j$ and $d_{j+p} = 1$
- 1 "terminal" node, $b_{4p} = 0$, $d_{4p} = 0$



Upper bound

If $\mathcal{S}$ has throughput T

- Node $\mathcal{N}_i$ uses at most $X_i = \min(b_i, Td_i)$
- Total received rate: nT
- Thus $\sum_{i=0}^{n} \min(b_i, Td_i) \geq nT$
- Of course, $T \leq b_0$

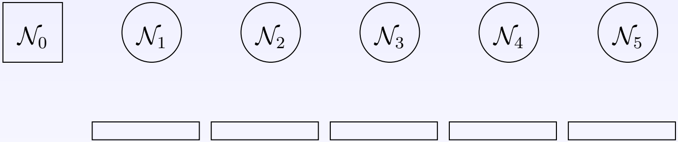
Our algorithms

- Inputs: an instance, and a goal throughput T
- Output: a solution with resource augmentation (additional connections allowed)

ACYCLIC algorithm

If $\sum_{i=0}^{n-1} \min(b_i, Td_i) \geq nT$

- Order nodes by capacity : $X_1 \geq X_2 \geq \cdots \geq X_n$
- Each node k sends throughput T to as many nodes as possible, in consecutive order



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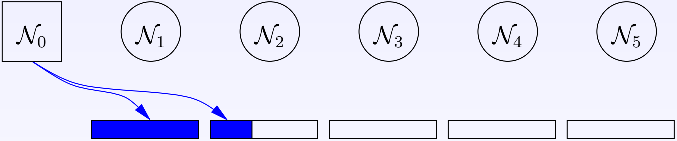

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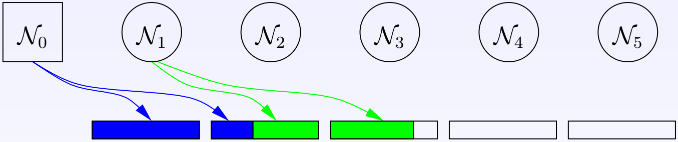

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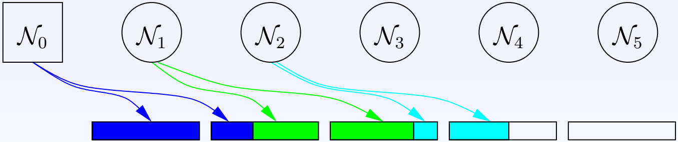

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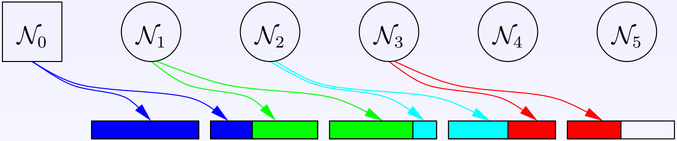

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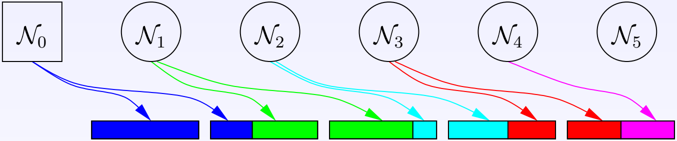
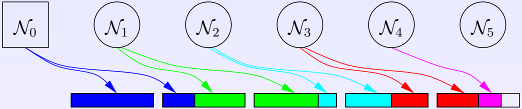


Provides a valid solution

- $b_0 \geq T$
- Sort by $X_i \implies \forall k, \sum_{i=0}^{k} X_i \geq (k+1)T$
- Since $X_k \leq Td_k$, the outdegree of $\mathcal{N}_k$ is at most $d_k + 1$

General case: $\sum_{i=0}^{n} X_i \geq nT$



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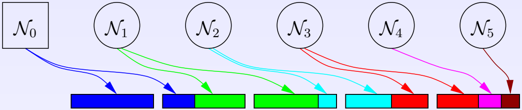

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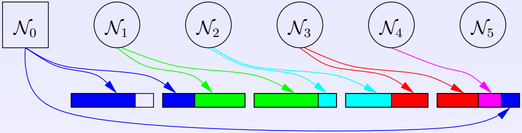

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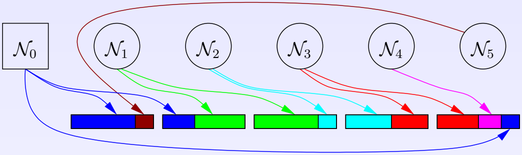

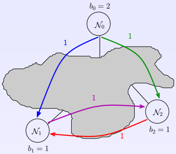

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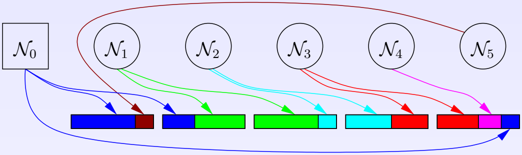

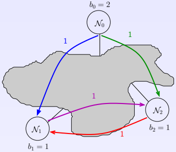

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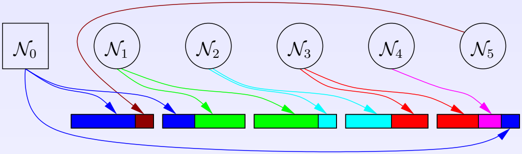
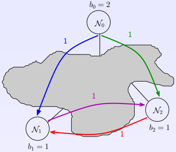
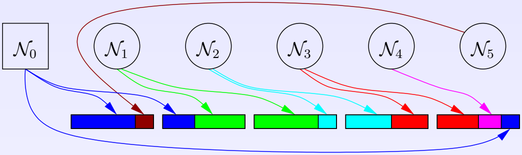
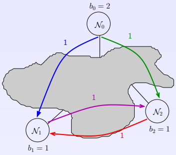




- Start with $\textsc{Acyclic}$, until k_0 such that $\sum_{i=0}^{k_0} X_i < (k_0+1)T$
- Succesively build partial solutions in which
  - All nodes up to $\mathcal{N}_k$ are served
  - Only node $\mathcal{N}_k$ has remaining bandwidth
- Use the source and $\mathcal{N}_{k_0-1}$ to serve $\mathcal{N}_{k_0}$ and $\mathcal{N}_{k_0+1}$
- Then for all k, $\mathcal{N}_{k+1}$ is served by $\mathcal{N}_k$ and $\mathcal{N}_{k-1}$

General case: $\sum_{i=0}^{n} X_i \geq nT$





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Final outdegree of $\mathcal{N}_i$: $o_i \leq \max(d_i + 2, 4)$

- Acyclic solution: $o_i \leq d_i + 1$
- Degree of the source and of $\mathcal{N}_{k_0-1}$ is increased by 1
- $\mathcal{N}_k$ has edges to $\mathcal{N}_{k-2}$, $\mathcal{N}_{k-1}$, $\mathcal{N}_{k+1}$ and $\mathcal{N}_{k+2}$.

Comparison of different solutions

Unconstrained solution

Best achievable throughput without degree constraints: $\frac{\sum_i b_i}{n}$

Best Tree

In a tree of throughput T, flow through all edges must be T. Counting the edges yield $\sum_i \min(d_i, \left\lfloor \frac{b_i}{T} \right\rfloor) \geq n$.

Best Acyclic

Computed by the $\textsc{Acyclic}$ algorithm

Cyclic

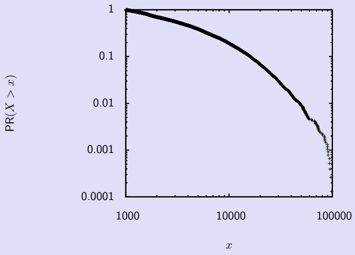
Throughput when adding cycles

Experimental setting

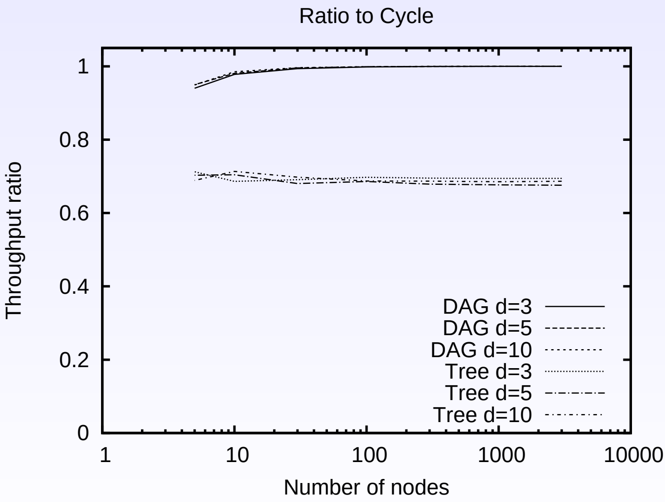
Random instance generation

- Outgoing bandwidths generated from the data of XtremLab project
- Nodes degrees are homogeneous

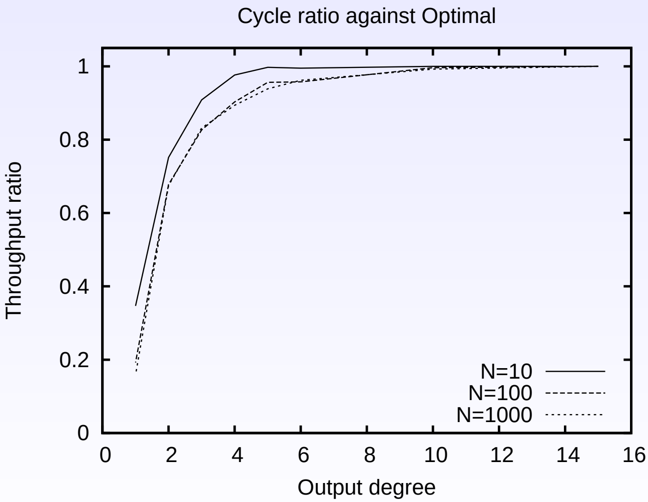
Complementary CDF of the data used



Results: comparisons to Cyclic



Results: Cyclic vs Unconstrained



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- TCP Bandwidth sharing mechanism
  - is complicated
  - and strongly depends on RTT values.
- On the other hand,
  - we usually know what bandwidth should be allocated
  - and many mechanisms exist to limit the bandwidth of a connexion
- So, use them!

Summary of second part

- Theoretical study of the broadcast problem:
  - optimal resource augmentation algorithm
- In practice:
  - a low degree is enough to reach a high throughput
  - an acyclic solution is very reasonable
  - once the overlay is computed, there exist distributed algorithms to perform the broadcast

Going further

- Worst-case approximation ratio of ACYCLIC ?
- Study the robustness of our algorithms
- Design on-line and/or distributed versions