Overview of the gfun[ContFrac] Package

Calling Sequence

\[ \text{gfun[ContFrac][command][arguments]} \]
\[ \text{command(arguments)} \]

Description

The ContFrac package provides tools to perform infinite corresponding continued fraction expansions of power series, given by *Riccati differential equations*.

Its main features include the ability to:

- guess continued fractions formulas, in C-fraction form.
- prove them

ContFrac is a subpackage of *gfun*.

List of ContFrac Package Commands

- Guessing and proving continued fractions formulas, given a Riccati equation.
  This equation can be heuristically guessed from an expression. It is then the users responsibility to check it is indeed satisfied by the original expression.

  \[ \text{guess_cfrac, riccati_to_cfrac, expr_to_cfrac} \]

Informational Messages and Settings

- The verbosity level of ContFrac commands is determined by the value of *infolevel[gfuncontfrac]*.
  Levels 1 to 5 correspond to informational messages. Levels 6 and higher additionally turn on debugging information.

Examples

\[ \text{with(gfun): with(ContFrac):} \]
\[ \text{riccati_to_cfrac(}\{\text{diff(y(z),z) - 1 - y(z)^2, y(0)=0}, y(z), \text{proc(n,z) series(tan(z),z,n) end }\});} \]
\[ \frac{z}{1 + \frac{a_1 z^2}{1 + \frac{\ldots}{1 + \frac{a_n z^2}{1 + \ldots}}}}, a_n = -\frac{1}{(2 n - 1) (2 n + 1)} \]  
\[ \text{(1.5.1)} \]
\[ \text{expr_to_cfrac( exp(z), y, z );} \]
\[ \text{(1.5.2)} \]
\[
1 + \frac{z}{1 + \frac{a_1 z}{1 + \ldots}} = a_n = \begin{cases} 
\frac{1}{2(n+1)} & \text{n:even} \\
-\frac{1}{2n} & \text{n:odd}
\end{cases}
\] (1.5.2)

The infolevel command enables to print details on the computation. The 'demo' field concerns the main computation lines; it can be set to 0 or 1.

\[
\text{infolevel}\text{[demo]}:=1;
\]

> \text{infolevel}\text{[demo]}:=1
\]

\[
\text{expr_to_cfrac} (\tan(z), y, z, 20);
\]

computing series...

... done.

computing a Riccati equation (using a guessing approach)...  
... done.

\[
\left\{ \frac{d}{dz} y(z) - 1 - y(z)^{2}, y(0) = 0 \right\}
\]

computing a C-fraction expansion...

... done. (in .461 seconds)

\[
1 + \frac{a_1 z^2}{1 + \frac{a_n z^2}{1 + \ldots}} = a_n = -\frac{1}{(2n-1)(2n+1)}
\] (1.5.4)

\[
\text{infolevel}\text{[demo]}:=0;
\]

> \text{infolevel}\text{[demo]}:=0
\]

More information on how the continued fraction expansion is computed and proved can be printed using the 'gfuncontfrac' information field.

\[
\text{infolevel}\text{[gfuncontfrac]}:=1;
\]

> \text{infolevel}\text{[gfuncontfrac]}:=1
\]

\[
\text{expr_to_cfrac} (\tan(z), y, z, 20);
\]

Guessing a formula

conjecture formula (on 8 coefficients)

\[
1 + \frac{a_1 z^2}{1 + \frac{a_n z^2}{1 + \ldots}} = a_n = -\frac{1}{(2n-1)(2n+1)}
\]

defining a sequence \(H(n,z)\), which relates to convergence.

lemma: the conjecture holds iff there exists an unbounded \(i(n)\) s.t., \(H(i(n),z)\) tends to 0.  
(i.e., their valuations tend to infinity)

proving \(\text{Limit}(\text{val}(H(n,z)), n = \text{infinity}) = \text{infinity}\):  
- computing a recurrence for \(H(n,z)\).  
(which does not conclude)

... \(z^8 H(n) + \ldots z^4 H(n+1) + (\ldots z^4 + \ldots z^2) H(n+2) + \ldots H(n+3) + \ldots H(n+4) = 0\)

- reducing the recurrence order for \(H(n,z)\)...
- ... done.

\[ \{ -z^2 H(n) + (2n + 3)^2 H(n + 1), H(0) = -z^2 \} \]

QED.

\[
\frac{z}{1 + \frac{a_1 z^2}{1 + \frac{a_2 z^2}{1 + \frac{...}{1 + \frac{a_n z^2}{1 + ...}}}}} = a_n = \frac{1}{(2n - 1)(2n + 1)}
\]

(1.5.7)

**Licence and Contact Information**

- ContFrac is available under the GNU Lesser General Public Licence, version 2.1 or, at your option, any later version. See the file COPYING for details.
- The source code for ContFrac can be downloaded from the webpage.
- Please send your comments and bug reports to sebastien.maulat@ens-lyon.org.

**See Also**

- gfun, UsingPackages, with