Overview of the gfun[ContFrac] Package

Calling Sequence

gfun[ContFrac][**command**](**arguments**)
command(**arguments**)

Description

The ContFrac package provides tools to perform infinite corresponding continued fraction expansions of power series, given by *Riccati differential equations*.

Its main features include the ability to:

- guess continued fractions formulas, in C-fraction form.
- prove them

ContFrac is a subpackage of gfun.

List of ContFrac Package Commands

- Guessing and proving continued fractions formulas, given a Riccati equation. This equation can be heuristically guessed from an expression. It is then the users responsibility to check it is indeed satisfied by the original expression.

  guess_cfrac, riccati_to_cfrac, expr_to_cfrac

Informational Messages and Settings

- The verbosity level of ContFrac commands is determined by the value of infolevel[gfuncontfrac]. Levels 1 to 5 correspond to informational messages. Levels 6 and higher additionally turn on debugging information.

Examples

```
> with(gfun): with(ContFrac):
> riccati_to_cfrac( {diff(y(z),z) - 1 - y(z)^2, y(0)=0}, y(z),
  proc(n,z) series(tan(z),z,n) end );
```

$$\frac{z}{1+\frac{a_1 z^2}{1+\frac{\ldots}{1+\frac{a_n z^2}{1+\ldots}}}}, a_n = -\frac{1}{(2n-1)(2n+1)} \quad \textbf{(1.5.1)}$$

```
> expr_to_cfrac( exp(z), y, z );
```

(1.5.2)

$$1+\cfrac{z}{1+\cfrac{a_1 z}{1+\cfrac{\dots}{1+\cfrac{a_n z}{1+\dots}}}},\ a_n = \begin{cases} \frac{1}{2(n+1)} & n{::}even \\ -\frac{1}{2n} & n{::}odd \end{cases} \qquad (1.5.2)$$

The infolevel command enables to print details on the computation.
The 'demo' field concerns the main computation lines ; it can be set to 0 or 1.

```
> infolevel[demo]:=1;
```

$$infolevel_{demo} := 1 \qquad (1.5.3)$$

```
> expr_to_cfrac( tan(z), y, z, 20 );
computing series...
... done.
computing a Riccati equation (using a guessing approach)...
... done.
```

$$\left\{\frac{d}{dz}\,y(z) - 1 - y(z)^2,\ y(0) = 0\right\}$$

```
computing a C-fraction expansion...
... done. (in .461 seconds)
```

$$\cfrac{z}{1+\cfrac{a_1 z^2}{1+\cfrac{\dots}{1+\cfrac{a_n z^2}{1+\dots}}}},\ a_n = -\frac{1}{(2n-1)(2n+1)} \qquad (1.5.4)$$

```
> infolevel[demo]:=0;
```

$$infolevel_{demo} := 0 \qquad (1.5.5)$$

More information on how the continued fraction expansion is computed and proved can be printed using the 'gfuncontfrac' information field.

```
> infolevel[gfuncontfrac]:=1;
```

$$infolevel_{gfuncontfrac} := 1 \qquad (1.5.6)$$

```
> expr_to_cfrac( tan(z), y, z, 20 );
Guessing a formula
conjecture formula (on 8 coefficients)
```

$$\cfrac{z}{1+\cfrac{a_1 z^2}{1+\cfrac{\dots}{1+\cfrac{a_n z^2}{1+\dots}}}},\ a_n = -\frac{1}{(2n-1)(2n+1)}$$

```
defining a sequence H(n,z), which relates to convergence.
lemma: the conjecture holds iff there exists an unbounded i(n) s.t., H(i
(n),z) tends to 0.
  (i.e., their valuations tend to infinity)
proving Limit(val(H(n,z)),n = infinity) = infinity:
- computing a recurrence for H(n,z).
  (which does not conclude)
```

$$\dots z^8\,H(n) + \dots z^4\,H(n+1) + \left(\dots z^4 + \dots z^2\right) H(n+2) + \dots H(n+3) + \dots H(n+4) = 0$$

```
- reducing the recurrence order for H(n,z)...
```

```
- ... done.
QED.
```

$$\{-z^2 H(n) + (2n+3)^2 H(n+1), H(0) = -z^2\}$$

$$\frac{z}{1+\frac{a_1 z^2}{1+\frac{\cdots}{1+\frac{a_n z^2}{1+\ldots}}}}, a_n = -\frac{1}{(2n-1)(2n+1)} \quad \textbf{(1.5.7)}$$

Licence and Contact Information

- ContFrac is available under the GNU Lesser General Public Licence, version 2.1 or, at your option, any later version. See the file COPYING for details.
- The source code for ContFrac can be downloaded from th webpage.
- Please send your comments and bug reports to sebastien.maulat@ens-lyon.org.

See Also

gfun, UsingPackages, with