# Overview of the NewtonGF Package

## Calling Sequence

NewtonGF[**command**](**arguments**)
**command**(**arguments**)

## Description

- NewtonGF provides tools based on a combinatorial Newton iteration that operate over generating functions of combstruct grammars.
- The basis of this package is described in the article:
  *Algorithms for combinatorial structures: well-founded systems and Newton iterations*, by Carine Pivoteau, Bruno Salvy, Michèle Soria.
  Journal of Combinatorial Theory Series A, Vol. 119, pages 1711-1773, 2012.
  doi:10.1016/j.jcta.2012.05.007. A preliminary version is freely available as arXiv:abs/1109.2688.
- Each command in the **NewtonGF** package can be accessed by using either the long form or the short form of the command name in the command calling sequence.

  As the underlying implementation of the **NewtonGF** package is a module, it is also possible to use the form **NewtonGF:-command** to access a command from the package. For more information, see Module Members.

## List of NewtonGF Package Commands

- The following is a list of available commands.

  `SeriesNewtonIteration` `NumericalNewtonIteration` `Radius` `GFSeries` `BoltzmannExpectedSize` `BoltzmannParameter`

To display the help page for a particular **NewtonGF** command, see Getting Help with a Command in a Package.

## Examples

```
> with(NewtonGF);
```

$$[BoltzmannExpectedSize, BoltzmannParameter, GFSeries, NumericalNewtonIteration, Radius, SeriesNewtonIteration] \quad (3.1)$$

A grammar for functional graphs.

```
> FG := { G = Set(C), C = Cycle(T), T = Prod(Z, Set(T)) };
```

$$FG := \{C = Cycle(T), G = Set(C), T = Prod(Z, Set(T))\} \quad (3.2)$$

Here are the generating function equations :

$combstruct[gfeqns](FG, labeled, z)$;

$$\left[C(z) = \ln\left(\frac{1}{1 - T(z)}\right), G(z) = e^{C(z)}, T(z) = z\, e^{T(z)}, Z(z) = z\right]$$

```
> GFSeries(FG,labelled,z);
```

$$\left[C = z + \frac{3}{2}z^2 + \frac{17}{6}z^3 + \frac{71}{12}z^4 + \frac{523}{40}z^5 + O(z^6), G = 1 + z + 2z^2 + \frac{9}{2}z^3 + \frac{32}{3}z^4 + \frac{625}{24}z^5 + O(z^6), T = z + z^2 + \frac{3}{2}z^3 + \frac{8}{3}z^4 + \frac{125}{24}z^5 + O(z^6), Z\right. \quad (3.3)$$

$$= z + \mathrm{O}\left(z^{6}\right)\Big]$$

```
> egf:=SeriesNewtonIteration(FG, labelled, z);
```

$$egf := \mathbf{proc}(prec::posint)\ \ldots\ \mathbf{end\ proc} \tag{3.4}$$

```
> egf(20);
```

$$\Big[Z = z + \mathrm{O}\left(z^{20}\right), T = z + z^{2} + \frac{3}{2} z^{3} + \frac{8}{3} z^{4} + \frac{125}{24} z^{5} + \frac{54}{5} z^{6} + \frac{16807}{720} z^{7} \tag{3.5}$$
$$+ \frac{16384}{315} z^{8} + \frac{531441}{4480} z^{9} + \frac{156250}{567} z^{10} + \frac{2357947691}{3628800} z^{11} + \frac{2985984}{1925} z^{12}$$
$$+ \frac{1792160394037}{479001600} z^{13} + \frac{7909306972}{868725} z^{14} + \frac{320361328125}{14350336} z^{15}$$
$$+ \frac{35184372088832}{638512875} z^{16} + \frac{2862423051509815793}{20922789888000} z^{17} + \frac{5083731656658}{14889875} z^{18}$$
$$+ \frac{5480386857784802185939}{6402373705728000} z^{19} + \mathrm{O}\left(z^{20}\right), C = z + \frac{3}{2} z^{2} + \frac{17}{6} z^{3} + \frac{71}{12} z^{4}$$
$$+ \frac{523}{40} z^{5} + \frac{899}{30} z^{6} + \frac{355081}{5040} z^{7} + \frac{47259}{280} z^{8} + \frac{16541017}{40320} z^{9}$$
$$+ \frac{5719087}{5670} z^{10} + \frac{33306869867}{13305600} z^{11} + \frac{144619817}{23100} z^{12}$$
$$+ \frac{98139640241473}{6227020800} z^{13} + \frac{161741140763}{4054050} z^{14} + \frac{21844512889051}{215255040} z^{15}$$
$$+ \frac{2648261961071387}{10216206000} z^{16} + \frac{26265531568692365561}{39520825344000} z^{17}$$
$$+ \frac{458182173298217}{268017750} z^{18} + \frac{536484538620663729658993}{121645100408832000} z^{19} + \mathrm{O}\left(z^{20}\right), G = 1$$
$$+ z + 2 z^{2} + \frac{9}{2} z^{3} + \frac{32}{3} z^{4} + \frac{625}{24} z^{5} + \frac{324}{5} z^{6} + \frac{117649}{720} z^{7} + \frac{131072}{315} z^{8}$$
$$+ \frac{4782969}{4480} z^{9} + \frac{1562500}{567} z^{10} + \frac{25937424601}{3628800} z^{11} + \frac{35831808}{1925} z^{12}$$
$$+ \frac{23298085122481}{479001600} z^{13} + \frac{110730297608}{868725} z^{14} + \frac{4805419921875}{14350336} z^{15}$$
$$+ \frac{562949953421312}{638512875} z^{16} + \frac{48661191875666868481}{20922789888000} z^{17}$$
$$+ \frac{91507169819844}{14889875} z^{18} + \frac{1041273502979112415328 41}{6402373705728000} z^{19} + \mathrm{O}\left(z^{20}\right)\Big]$$

```
> ogf:=SeriesNewtonIteration(FG, unlabelled,z);
```

$$ogf := \mathbf{proc}(prec::posint)\ \ldots\ \mathbf{end\ proc} \tag{3.6}$$

```
> ogf(20);
```

$$\Big[Z = z + \mathrm{O}\left(z^{20}\right), T = z + z^{2} + 2 z^{3} + 4 z^{4} + 9 z^{5} + 20 z^{6} + 48 z^{7} + 115 z^{8} + 286 z^{9} \tag{3.7}$$
$$+ 719 z^{10} + 1842 z^{11} + 4766 z^{12} + 12486 z^{13} + 32973 z^{14} + 87811 z^{15}$$
$$+ 235381 z^{16} + 634847 z^{17} + 1721159 z^{18} + 4688676 z^{19} + \mathrm{O}\left(z^{20}\right), C = z + 2 z^{2}$$
$$+ 4 z^{3} + 9 z^{4} + 20 z^{5} + 51 z^{6} + 125 z^{7} + 329 z^{8} + 862 z^{9} + 2311 z^{10} + 6217 z^{11}$$

$$+ 16949\,z^{12} + 46350\,z^{13} + 127714\,z^{14} + 353272\,z^{15} + 981753\,z^{16}$$
$$+ 2737539\,z^{17} + 7659789\,z^{18} + 21492286\,z^{19} + O\left(z^{20}\right),\ G = 1 + z + 3\,z^2 + 7\,z^3$$
$$+ 19\,z^4 + 47\,z^5 + 130\,z^6 + 343\,z^7 + 951\,z^8 + 2615\,z^9 + 7318\,z^{10} + 20491\,z^{11}$$
$$+ 57903\,z^{12} + 163898\,z^{13} + 466199\,z^{14} + 1328993\,z^{15} + 3799624\,z^{16}$$
$$+ 10884049\,z^{17} + 31241170\,z^{18} + 89814958\,z^{19} + O\left(z^{20}\right)\ ]$$

```
> oracle:=NumericalNewtonIteration(FG, labelled);
```

$$oracle := \mathbf{proc}(x, \{ending\_block::integer := 3\})\ \ldots\ \mathbf{end\ proc} \quad (3.8)$$

```
> Digits:=10;
```

$$Digits := 10 \quad (3.9)$$

```
> Radius(FG, labelled);
```

$$0.367879441171442 \quad (3.10)$$

Newton iteration does not converge outside the radius of convergence of the generating function.

```
> oracle(0.4);
Error, (in recursivenewton) One coordinate at least has
decreased, Array(1..1, {(1) = 1.175601483009}), Array(1..1, {
(1) = .7687766174284})
> oracle(0.3);
```

$$[C = 0.6721731361,\ G = 1.958488762,\ T = 0.4894022272\ ] \quad (3.11)$$

```
> Digits:=20;
```

$$Digits := 20 \quad (3.12)$$

```
> oracle(0.3);
```

$$[C = 0.67217313607266260700,\ G = 1.9584887620591893800,\ T = 0.48940222718021496904\ ] \quad (3.13)$$

## See Also

combstruct, NewtonGF[SeriesNewtonIteration], NewtonGF[NumericalNewtonIteration], NewtonGF[Radius], NewtonGF[BoltzmannExpectedSize], NewtonGF[BoltzmannParameter], UsingPackages, with