

NewtonGF[SeriesNewtonIteration] - returns a procedure to compute generating series by Newton iteration

Calling Sequence

SeriesNewtonIteration(**Sys**, **labelling**, **z**)

Parameters

Sys – set of equations: a grammar in the combstruct syntax

labelling: one of labelled, labeled, unlabelled, unlabeled, as in [combstruct](#)

z – name: name of the variable in the series

Description

- The **SeriesNewtonIteration** command returns a procedure that takes as input a nonnegative integer and returns the expansions of the generating series of Sys to that order.
- The procedure first computes the decomposition into strongly connected components and then generates a Newton iteration for each of them.
- This command is part of the **NewtonGF** package, so it can be used in the form **SeriesNewtonIteration(..)** only after executing the command **with(NewtonGF)**. However, it can always be accessed through the long form of the command by using **NewtonGF[SeriesNewtonIteration](..)**.

Examples

```
> with(NewtonGF);
[BoltzmannExpectedSize, BoltzmannParameter, GFSeries, NumericalNewtonIteration,
  Radius, SeriesNewtonIteration] (2.1)
```

A grammar for general plane trees.

```
> P := { T = Prod(Z, Sequence(T)) };
      P := { T = Prod(Z, Sequence(T)) } (2.2)
```

```
combstruct[gfeqns](P, labeled, z);
```

$$\left[T(z) = \frac{z}{1 - T(z)}, Z(z) = z \right]$$

```
> Pegf:=SeriesNewtonIteration(P, labelled, z);
      Pegf := proc(prec::posint) ... end proc (2.3)
```

```
> Pegf(10);
[Z = z + O(z10), T = z + z2 + 2 z3 + 5 z4 + 14 z5 + 42 z6 + 132 z7 + 429 z8
  + 1430 z9 + O(z10)] (2.4)
```

```
> Pegf(100);
[Z = z + O(z100), T = z + z2 + 2 z3 + 5 z4 + 14 z5 + 42 z6 + 132 z7 + 429 z8
  + 1430 z9 + 4862 z10 + 16796 z11 + 58786 z12 + 208012 z13 + 742900 z14
  + 2674440 z15 + 9694845 z16 + 35357670 z17 + 129644790 z18 + 477638700 z19
  + 1767263190 z20 + 6564120420 z21 + 24466267020 z22 + 91482563640 z23
  + 343059613650 z24 + 1289904147324 z25 + 4861946401452 z26] (2.5)
```

$$\begin{aligned} &+ 18367353072152 z^{27} + 69533550916004 z^{28} + 263747951750360 z^{29} \\ &+ 1002242216651368 z^{30} + 3814986502092304 z^{31} + 14544636039226909 z^{32} \\ &+ 55534064877048198 z^{33} + 212336130412243110 z^{34} \\ &+ 812944042149730764 z^{35} + 3116285494907301262 z^{36} \\ &+ 11959798385860453492 z^{37} + 45950804324621742364 z^{38} \\ &+ 176733862787006701400 z^{39} + 680425371729975800390 z^{40} \\ &+ 2622127042276492108820 z^{41} + 10113918591637898134020 z^{42} \\ &+ 39044429911904443959240 z^{43} + 150853479205085351660700 z^{44} \\ &+ 583300119592996693088040 z^{45} + 2257117854077248073253720 z^{46} \\ &+ 8740328711533173390046320 z^{47} + 33868773757191046886429490 z^{48} \\ &+ 131327898242169365477991900 z^{49} + 509552245179617138054608572 z^{50} \\ &+ 1978261657756160653623774456 z^{51} + 7684785670514316385230816156 z^{52} \\ &+ 29869166945772625950142417512 z^{53} \\ &+ 116157871455782434250553845880 z^{54} \\ &+ 451959718027953471447609509424 z^{55} \\ &+ 1759414616608818870992479875972 z^{56} \\ &+ 6852456927844873497549658464312 z^{57} \\ &+ 26700952856774851904245220912664 z^{58} \\ &+ 104088460289122304033498318812080 z^{59} \\ &+ 405944995127576985730643443367112 z^{60} \\ &+ 1583850964596120042686772779038896 z^{61} \\ &+ 6182127958584855650487080847216336 z^{62} \\ &+ 24139737743045626825711458546273312 z^{63} \\ &+ 94295850558771979787935384946380125 z^{64} \\ &+ 368479169875816659479009042713546950 z^{65} \\ &+ 1440418573150919668872489894243865350 z^{66} \\ &+ 5632681584560312734993915705849145100 z^{67} \\ &+ 22033725021956517463358552614056949950 z^{68} \\ &+ 86218923998960285726185640663701108500 z^{69} \\ &+ 337485502510215975556783793455058624700 z^{70} \\ &+ 1321422108420282270489942177190229544600 z^{71} \\ &+ 5175569924646105559418940193995065716350 z^{72} \\ &+ 20276890389709399862928998568254641025700 z^{73} \\ &+ 79463489365077377841208237632349268884500 z^{74} \\ &+ 311496878311103321137536291518809134027240 z^{75} \end{aligned}$$

$+ 1221395654430378811828760722007962130791020 z^{76}$
 $+ 4790408930363303911328386208394864461024520 z^{77}$
 $+ 18793142726809884575211361279087545193250040 z^{78}$
 $+ 73745243611532458459690151854647329239335600 z^{79}$
 $+ 289450081175264899454283846029490767264392230 z^{80}$
 $+ 1136359577947336271931632877004667456667613940 z^{81}$
 $+ 4462290049988320482463241297506133183499654740 z^{82}$
 $+ 17526585015616776834735140517915655636396234280 z^{83}$
 $+ 68854441132780194707888052034668647142985206100 z^{84}$
 $+ 270557451039395118028642463289168566420671280440 z^{85}$
 $+ 1063353702922273835973036658043476458723103404520 z^{86}$
 $+ 4180080073556524734514695828170907458428751314320 z^{87}$
 $+ 16435314834665426797069144960762886143367590394940 z^{88}$
 $+ 64633260585762914370496637486146181462681535261000 z^{89}$
 $+ 254224158304000796523953440778841647086547372026600 z^{90}$
 $+ 1000134600800354781929399250536541864362461089950800 z^{91}$
 $+ 3935312233584004685417853572763349509774031680023800 z^{92}$
 $+ 15487357822491889407128326963778343232013931127835600 z^{93}$
 $+ 60960876535340415751462563580829648891969728907438000 z^{94}$
 $+ 239993345518077005168915776623476723006280827488229600 z^{95}$
 $+ 944973797977428207852605870454939596837230758234904050 z^{96}$
 $+ 3721443204405954385563870541379246659709506697378694300 z^{97}$
 $+ 14657929356129575437016877846657032761712954950899755100 z^{98}$
 $+ 57743358069601357782187700608042856334020731624756611000 z^{99} +$
 $O(z^{100})]$

> Pogf:=SeriesNewtonIteration(P, unlabelled, z);
Pogf := proc(prec::posint) ... end proc (2.6)

> Pogf(10);
 $[Z = z + O(z^{10}), T = z + z^2 + 2z^3 + 5z^4 + 14z^5 + 42z^6 + 132z^7 + 429z^8$
 $+ 1430z^9 + O(z^{10})]$ (2.7)

A grammar for general non-plane trees.

> NP := { T = Prod(Z, Set(T)) };
 $NP := \{ T = Prod(Z, Set(T)) \}$ (2.8)

combstruct[geqns](NP, labeled, z);
 $[T(z) = z e^{T(z)}, Z(z) = z]$

> NPegf:=SeriesNewtonIteration(NP, labelled, z);

```
NPogf := proc( prec::posint) ... end proc (2.9)
```

```
> NPogf(10);
```

$$\left[Z = z + O(z^{10}), T = z + z^2 + \frac{3}{2} z^3 + \frac{8}{3} z^4 + \frac{125}{24} z^5 + \frac{54}{5} z^6 + \frac{16807}{720} z^7 + \frac{16384}{315} z^8 + \frac{531441}{4480} z^9 + O(z^{10}) \right] \quad (2.10)$$

```
> NPogf := SeriesNewtonIteration(NP, unlabelled, z);
```

```
NPogf := proc( prec::posint) ... end proc (2.11)
```

```
> NPogf(10);
```

$$\left[Z = z + O(z^{10}), T = z + z^2 + 2 z^3 + 4 z^4 + 9 z^5 + 20 z^6 + 48 z^7 + 115 z^8 + 286 z^9 + O(z^{10}) \right] \quad (2.12)$$

See Also

[combstruct\[gfseries\]](#), [NewtonGF\[NumericalNewtonIteration\]](#), [NewtonGF\[Radius\]](#), [NewtonGF](#)