\textbf{gfun}[\texttt{algeqtodiffeq}] - compute a differential equation satisfied by an algebraic function

\textbf{Calling Sequence}

\texttt{algeqtodiffeq(p, y(z), ini, options)}

\textbf{Parameters}

\begin{itemize}
  \item \texttt{p} \quad \text{polynomial in } y \text{ and } z \text{ (or a polynomial equation)}
  \item \texttt{y} \quad \text{name of the holonomic function}
  \item \texttt{z} \quad \text{name of the generic variable associated with } y
  \item \texttt{ini} \quad \text{(optional) initial conditions to specify a solution of eq}
  \item \texttt{options} \quad \text{(optional) equation(s) of the form homogeneous=true and/or ini\textunderscore cond=false}
\end{itemize}

\textbf{Description}

The polynomial \texttt{p} defines an algebraic function, \texttt{RootOf(p,y)} in Maple terms. This procedure computes a linear differential equation with polynomial coefficients verified by the function \texttt{y(z)}. This equation is of order at most \texttt{degree(p,y)-1}.

The output contains initial conditions in zero (\texttt{y(0)}, \texttt{D(y)(0)}, and so on), and can thus be given directly to \texttt{dsolve}. In general, \texttt{y(0)} is a \texttt{RootOf} a polynomial, \texttt{D(y)(0)} a rational expression in \texttt{y(0)}, \texttt{(D@@2)(y)(0)} a rational expression in \texttt{y(0),D(y)(0)}, and so on.

If the optional argument "homogeneous=true" is given, the differential equation will be forced to be homogeneous.

If the optional argument "ini\textunderscore cond=false" is given, no attempt at computing initial conditions at 0 will be made and the equation will be returned without initial conditions.

\textbf{Examples}

\begin{verbatim}
> with(gfun):
algeqtodiffeq(y=1+z*y^2,y(z));
\end{verbatim}

\begin{equation}
1 + (-1 + 2 z) y(z) + (-z + 4 z^2) \left( \frac{d}{dz} y(z) \right) \quad (2.1)
\end{equation}

\begin{verbatim}
> algeqtodiffeq(56*a^3+7*a^3*y^3-14*y*z,y(z),{y(0)=-2});
\end{verbatim}

\begin{equation}
\begin{cases}
-y(z) + 3 \left( \frac{d}{dz} y(z) \right) z^2 + (-108 a^9 + 2 z^3) \left( \frac{d^2}{dz^2} y(z) \right), y(0) = -2, D(y)(0) = \frac{1}{3 a^3} \\
\end{cases} \quad (2.2)
\end{equation}

We can use \texttt{algeqtodiffeq} with \texttt{diffeqtorec} to determine fast Taylor expansions.

\begin{verbatim}
> p:=y=1+z*y+z*y^5;
p := y = 1 + z y + z y^5 \quad (2.3)
\end{verbatim}

\begin{verbatim}
> deq:=algeqtodiffeq(p,y(z));
\end{verbatim}
rec:=diffeqtorec(deq,y(z),u(n)):
 p_generator:=rectoproc(rec,u(n),list):
p_generator(30);  

See Also
  gfun, gfun[parameters], dsolve, gfun[diffeqtorec]