gfun[borel] - compute the Borel transform of a generating function

Calling Sequence
borel(expr, a(n), t)

Parameters
expr  - linear recurrence with polynomial coefficients
a, n  - name and index of the recurrence
t    - (optional) 'diffeq'

Description
• If (a(n),n=0..infinity) is the sequence of numbers defined by the recurrence expr, the procedure computes the recurrence for the numbers a(n)/n!.

• If an optional argument 'diffeq' is given, expr is considered as a linear differential equation with polynomial coefficients for the function a(n). In this case the procedure outputs a linear differential equation verified by the Borel transform of a(n).

Examples
> with(gfun):
rec:={a(n)=n*a(n-1)+a(n-2), a(0)=1, a(1)=1}:
b:= borel(rec, a(n));

\[
b := \{a(0) = 1, a(1) = 1, -a(n) + (-n^2 - 3 n - 2) a(n + 1) + (n^2 + 3 n + 2) a(n + 2)\}
\]  

(2.1)

invborel is the inverse command:

> invborel(b,a(n));

\[
{a(0) = 1, a(1) = 1, -a(n) + (-n - 2) a(n + 1) + a(n + 2)}
\]  

(2.2)

We can also perform Borel transforms on the corresponding differential equations:

> deq:=rectodiffeq(rec, a(n), f(x));
newdeq:= borel(deq, f(x), 'diffeq');

\[
newdeq := \left\{-f(x) - 2 \left(\frac{\partial}{\partial x} f(x)\right) + (1 - x) \left(\frac{\partial^2}{\partial x^2} f(x)\right), f(0) = 1, D(f)(0) = 1\right\}
\]  

(2.3)

> diffeqtorec(newdeq, f(x), a(n));

\[
{a(0) = 1, a(1) = 1, -a(n) + (-n^2 - 3 n - 2) a(n + 1) + (n^2 + 3 n + 2) a(n + 2)}
\]  

(2.4)

See Also
gfun, gfun[invborel]