gfun[borel] - compute the Borel transform of a generating function

Calling Sequence

borel(**expr**, **a(n)**, **t**)

Parameters

`expr` - linear recurrence with polynomial coefficients

`a,n` - name and index of the recurrence

`t` - (optional) 'diffeq'

Description

- If **(a(n),n=0..infinity)** is the sequence of numbers defined by the recurrence **expr**, the procedure computes the recurrence for the numbers **a(n)/n!**.
- If an optional argument 'diffeq' is given, **expr** is considered as a linear differential equation with polynomial coefficients for the function **a(n)**. In this case the procedure outputs a linear differential equation verified by the Borel transform of **a(n)**.

Examples

```
> with(gfun):
  rec:={a(n)=n*a(n-1)+a(n-2), a(0)=1, a(1)=1}:
  b:= borel(rec, a(n));
```

$$b := \{a(0)=1, a(1)=1, -a(n)+(-n^2-3n-2)\,a(n+1)+(n^2+3n+2)\,a(n+2)\} \quad \textbf{(2.1)}$$

invborel is the inverse command:

```
> invborel(b,a(n));
```

$$\{a(0)=1, a(1)=1, -a(n)+(-n-2)\,a(n+1)+a(n+2)\} \quad \textbf{(2.2)}$$

We can also perform Borel transforms on the corresponding differential equations:

```
> deq:=rectodiffeq(rec, a(n), f(x)):
  newdeq:= borel(deq, f(x), `diffeq`);
```

$$newdeq := \left\{-f(x)-2\left(\frac{d}{dx}f(x)\right)+(1-x)\left(\frac{d^2}{dx^2}f(x)\right), f(0)=1, D(f)(0)=1\right\} \quad \textbf{(2.3)}$$

```
> diffeqtorec(newdeq, f(x), a(n));
```

$$\{a(0)=1, a(1)=1, -a(n)+(-n^2-3n-2)\,a(n+1)+(n^2+3n+2)\,a(n+2)\} \quad \textbf{(2.4)}$$

See Also

gfun, gfun[invborel]