ContFrac[expr_to_cfrac] - Compute a Corresponding Continued Fraction expansion for a power series given as an expression.

Calling Sequence

expr_to_cfrac(expr, y, z, [N, Nseries, names=Names, riccati=Riccati])

Parameters

expr – expression in z, assumed to satisfy a Riccati differential equation: { diff(y(z),z) = p y^2 + q y + r, y(0)=v},
with rational coefficients p,q,r and initial condition v at zero.

y – name; function name

z – name; variable of the function y

N – (optional) positive integer

Nseries – (optional) positive integer

Names – (optional) list of names used in the continued fraction and in the formal convergence proof:

n, a, alpha: index, coefficients and exponents names : the main coefficient is a(n) * x^(alpha(n)).

P, Q : name of the numerator and denominator sequences

H : remainder, s.t. H_n tends to zero (as a formal power series) iff. convergence is achieved.
it is polynomial in P,Q, their shifts and their derivatives.

Riccati – (optional) riccati equation satisfied by the expression.

Description

- This procedure computes and proves a Corresponding Fraction formula if it exists.
- It returns either:
  - a `cfrac` structure;
  - FAIL if none was found, in the limit of the number of initial terms used for "guessing". If a formula exists, using more and more terms ends up finding it.
- Internally, it guesses a Riccati equation for the series, and computes a continued fraction expansion for it. Information can be obtained using userinfo[demo] and userinfo[gfuncontfrac].

Examples

```
> restart;
> with(gfun): with(ContFrac):
> eq := {diff(y(z),z)-1-y(z)^2, y(0)=0};
```

$$eq := \left\{ \frac{d}{dz}\, y(z) - 1 - y(z)^2, y(0) = 0 \right\} \qquad (1.4.1)$$

```
> cf := expr_to_cfrac( arctan(z), y, z );
```

(1.4.2)

$$cf := \cfrac{z}{1+\cfrac{a_1 z^2}{1+\cfrac{\dots}{1+\cfrac{a_n z^2}{1+\dots}}}},\ a_n = \frac{n^2}{(2n-1)(2n+1)} \qquad (1.4.2)$$

use the information level 'gfuncontfrac' to get more information. Levels go from 1 to 5.

```
> infolevel[gfuncontfrac] := 2:
> cf := expr_to_cfrac( tan(z), y, z );
Guessing a formula
conjecture formula (on 8 coefficients)
```

$$\cfrac{z}{1+\cfrac{a_1 z^2}{1+\cfrac{\dots}{1+\cfrac{a_n z^2}{1+\dots}}}},\ a_n = -\frac{1}{(2n-1)(2n+1)}$$

```
defining a sequence H(n,z), which relates to convergence.
```

$$H(n,z) = \left(\frac{\partial}{\partial z} P(n,z)\right) Q(n,z) - Q(n,z)^2 - P(n,z)^2 - P(n,z)\left(\frac{\partial}{\partial z} Q(n,z)\right)$$

```
lemma: the conjecture holds iff there exists an unbounded i(n) s.t., H(i
(n),z) tends to 0.
  (i.e., their valuations tend to infinity)
proving Limit(val(H(n,z)),n = infinity) = infinity:
- computing a recurrence for H(n,z).
  (which does not conclude)
```

$$\dots z^8 H(n) + \dots z^4 H(n+1) + \left(\dots z^4 + \dots z^2\right) H(n+2) + \dots H(n+3) + \dots H(n+4) = 0$$

```
- reducing the recurrence order for H(n,z)...
- ... done.
```

$$\left\{-z^2 H(n) + (2n+3)^2 H(n+1), H(0) = -z^2\right\}$$

```
QED.
```

$$cf := \cfrac{z}{1+\cfrac{a_1 z^2}{1+\cfrac{\dots}{1+\cfrac{a_n z^2}{1+\dots}}}},\ a_n = -\frac{1}{(2n-1)(2n+1)} \qquad (1.4.3)$$

```
> infolevel[gfuncontfrac]:=0;
```

$$infolevel_{gfuncontfrac} := 0 \qquad (1.4.4)$$

```
> expr_to_cfrac( exp(z), y, z );
```

$$1+\cfrac{z}{1+\cfrac{a_1 z}{1+\cfrac{\dots}{1+\cfrac{a_n z}{1+\dots}}}},\ a_n = \begin{cases} \frac{1}{2(n+1)} & n::even \\ -\frac{1}{2n} & n::odd \end{cases} \qquad (1.4.5)$$

See Also

gfun, ContFrac, riccati_to_cfrac