gfun[rectodiffeq] - convert a linear recurrence into a differential equation

Calling Sequence

rectodiffeq (eqns, u,n, f,z)

Parameters

eqns  - a single equation or a set of equations
u,n  - the name and index of the recurrence
f,z  - the name and variable of the function

Description

- Let \( f \) be the generating function associated to the sequence \((u(n)) \): \( f(z)=\sum u(n)z^n, n=0..\infty \). The procedure outputs a linear differential equation with polynomial coefficients verified by \( f \).
- The input syntax is the same as for rsolve; the first argument should be a single recurrence relation or a set containing one recurrence relation and boundary conditions. The recurrence relation should be linear in the variable \( u \), with polynomial coefficients in \( n \). The terms of the sequence appearing in the relation should be of the form \( u(n+k) \), with \( k \) an integer.
- The output is either a single differential equation, or a set containing a differential equation and initial conditions.

Examples

```plaintext
> with(gfun):
deq:=rectodiffeq({(5*n+10)*u(n)+a*u(n+1)-u(n+2),u(0)=0,u(1)=0},u(n),f(t));

deq := \( 10t^2 + at - 1 \) f(t) + 5 t^3 \left( \frac{d}{dt} f(t) \right) \)  \hspace{1cm} (2.1)
```

```plaintext
> diffeqtorec(deq,f(t),u(n));

\{ (5 n + 10) u(n) + a u(n + 1) - u(n + 2), u(0) = 0, u(1) = 0 \} \hspace{1cm} (2.2)
```

```plaintext
> deq:=rectodiffeq((n-10)*u(n+1)-u(n),u(n),y(z));

\[
\begin{align*}
\text{deq} & := \begin{cases}
D(y)(0) = 0, D(6)(y)(0) = 0, D(3)(y)(0) = 0, y(0) = 0, D(2)(y)(0) = 0,
D(4)(y)(0) = 0, D(5)(y)(0) = 0, D(7)(y)(0) = 0, D(8)(y)(0) = 0, D(9)(y)(0) = 0,
D(10)(y)(0) = 0, (-z - 11) y(z) + z \left( \frac{d}{dz} y(z) \right), D(11)(y)(0) = -c_0
\end{cases}
\end{align*}
\] \hspace{1cm} (2.3)
```

```plaintext
> dsolve(deq,y(z));
```
```
\[ y(z) = \frac{1}{39916800} - e^z z^{11} \]

See Also

`gfun`, `gfun[parameters]`, `gfun[diffeqtorec]`, `rsolve`