gfun[rectodiffeq] - convert a linear recurrence into a differential equation

Calling Sequence

rectodiffeq (**eqns**, **u**,**n**, **f**,**z**)

Parameters

`eqns` - a single equation or a set of equations

`u,n` - the name and index of the recurrence

`f,z` - the name and variable of the function

Description

- Let **f** be the generating function associated to the sequence (**u(n)**): **f(z)=sum(u(n)*z^n,n=0..infinity)**. The procedure outputs a linear differential equation with polynomial coefficients verified by **f**.
- The input syntax is the same as for rsolve: the first argument should be a single recurrence relation or a set containing one recurrence relation and boundary conditions. The recurrence relation should be linear in the variable **u**, with polynomial coefficients in **n**. The terms of the sequence appearing in the relation should be of the form **u(n+k)**, with **k** an integer.
- The output is either a single differential equation, or a set containing a differential equation and initial conditions.

Examples

```
> with(gfun):
  deq:=rectodiffeq({(5*n+10)*u(n)+a*u(n+1)-u(n+2),u(0)=0,u(1)=0},u(n),f(t));
```

$$deq := \left(10\,t^2 + a\,t - 1\right) f(t) + 5\,t^3 \left(\frac{d}{dt} f(t)\right) \quad (2.1)$$

```
> diffeqtorec(deq,f(t),u(n));
```

$$\{(5\,n + 10)\,u(n) + a\,u(n+1) - u(n+2), u(0) = 0, u(1) = 0\} \quad (2.2)$$

```
> deq:=rectodiffeq((n-10)*u(n+1)-u(n),u(n),y(z));
```

$$deq := \{D(y)(0) = 0, D^{(6)}(y)(0) = 0, D^{(3)}(y)(0) = 0, y(0) = 0, D^{(2)}(y)(0) = 0, \quad (2.3)$$

$$D^{(4)}(y)(0) = 0, D^{(5)}(y)(0) = 0, D^{(7)}(y)(0) = 0, D^{(8)}(y)(0) = 0, D^{(9)}(y)(0) = 0,$$

$$D^{(10)}(y)(0) = 0, (-z - 11)\,y(z) + z\left(\frac{d}{dz} y(z)\right), D^{(11)}(y)(0) = \_C_0\}$$

```
> dsolve(deq,y(z));
```

(2.4)

$$y(z) = \frac{1}{39916800}\ \_C_0\ e^{z}\ z^{11} \qquad (2.4)$$

See Also

gfun, gfun[parameters], gfun[diffeqtorec], rsolve