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> read "/Users/salvy/Google Drive/BinomialSums/BinomialSum_demo.mpl";
> libname:="/Users/salvy/lib/maple/FGb/FGb.lib",
"/Users/salvy/Documents/Pierre/PicardFuchs/maple/lib", libname:
> with(FGb) :
> with(RhamKoszul) :
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Dixon

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> F:=Sum(Sum(binomial(2*n,k)^3*(-1)^k*z^n,k=0..infinity),n=0..infinity);
```

$$F := \sum_{n=0}^{\infty} \left(\sum_{k=0}^{\infty} \text{binomial}(2n, k)^3 (-1)^k z^n \right) \quad (1)$$

```
> F=BinomialSum(F, t);
```

$$\sum_{n=0}^{\infty} \left(\sum_{k=0}^{\infty} \text{binomial}(2n, k)^3 (-1)^k z^n \right) = \left(\frac{1}{(2I\pi)^2} \right) \left(\left(\int \int \left(- \frac{t_1 t_2}{(t_2 + 1)^2 (t_1 + 1)^2 (t_1 t_2 - 1)^2 z - t_1^2 t_2^2} \right) dt_1 dt_2 \right) \right) \quad (2)$$

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> PicardFuchs(op([2,2,1$2],%), z, D, sheet=3);
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$$6 + (27z^2 + z) D^2 + (54z + 1) D \quad (3)$$

```
> DEtools[difop2de](%, [D, z], y(z))=0;
```

$$6y(z) + (54z + 1) \left(\frac{d}{dz} y(z) \right) + (27z^2 + z) \left(\frac{d^2}{dz^2} y(z) \right) = 0 \quad (4)$$

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> gfun:-diffeqtoec({%, y(0)=1}, y(z), u(n));
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$$\{(27n^2 + 27n + 6) u(n) + (n^2 + 2n + 1) u(n + 1), u(0) = 1\} \quad (5)$$

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> rsolve(%, u(n));
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$$\frac{1}{2} \frac{\Gamma\left(n + \frac{2}{3}\right) \Gamma\left(n + \frac{1}{3}\right) \sqrt{3} (-1)^n 27^n}{\pi \Gamma(n + 1)^2} \quad (6)$$

Essam-Guttman

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> F:=Sum(Sum(Sum(Sum((r[2]+1)*(r[2]+r[3]+2)*(r[3]+1)*binomial(n+2, r[1])*binomial(n+2, r[1]+r[2]+1)*binomial(n+2, r[1]+r[2]+r[3]+2)*z^n, r[3]=0..infinity), r[2]=0..infinity), r[1]=0..infinity), n=0..infinity);
```

$$F := \sum_{n=0}^{\infty} \left(\sum_{r_1=0}^{\infty} \left(\sum_{r_2=0}^{\infty} \left(\sum_{r_3=0}^{\infty} (r_2 + 1) (r_2 + r_3 + 2) (r_3 + 1) \text{binomial}(n + 2, r_1) \text{binomial}(n + 2, r_1 + r_2 + 1) \text{binomial}(n + 2, r_1 + r_2 + r_3 + 2) z^n \right) \right) \right) \quad (7)$$

```
> F=BinomialSum(F, t);
```

$$\sum_{n=0}^{\infty} \left(\sum_{r_1=0}^{\infty} \left(\sum_{r_2=0}^{\infty} \left(\sum_{r_3=0}^{\infty} (r_2+1)(r_2+r_3+2)(r_3+1) \text{binomial}(n+2, r_1) \text{binomial}(n \right. \right. \right. \quad (8)$$

$$\left. \left. \left. + 2, r_1 + r_2 + 1) \text{binomial}(n+2, r_1 + r_2 + r_3 + 2) z^n \right) \right) \right) = \left(\frac{1}{(2I\pi)} \right) \left(\left(\left(\right. \right. \right.$$

$$\left. \left. \left. - \frac{2t_1(t_1+1)^2}{8(t_1+1)^6 z^3 - 12t_1(t_1+1)^4 z^2 + 6t_1^2(t_1+1)^2 z - t_1^3} \right) dt_1 \right) \right)$$

> PicardFuchs(op([2,2,1],%),z,D,sheet=3);

$$(16z^2 - 10z + 1)D + 24z - 18 \quad (9)$$

> DEtools[diffof2de](%, [D,z], y(z))=0;

$$(24z - 18)y(z) + (16z^2 - 10z + 1) \left(\frac{d}{dz} y(z) \right) = 0 \quad (10)$$

> gfun:-diffeqtoec({%,y(0)=4},y(z),u(n));

$$\{(24 + 16n)u(n) + (-28 - 10n)u(n+1) + (n+2)u(n+2), u(0) = 4, u(1) = 72\} \quad (11)$$

> rsolve(%,u(n));

$$\frac{4 \cdot 8^n \Gamma\left(\frac{3}{2} + n\right) (n+2)}{\sqrt{\pi} \Gamma(n+1)} \quad (12)$$