Coq, an overview

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What is Coq?

- A “proof assistant”
- A “formal proof management system” (from Coq webpage)
- A programming language
- A specification language
- An interactive prover
- A project initiated by Thierry Coquand in 1984, and still under active development...

What is Coq useful for?

- Formally "certify" existing programs/libraries
- Build "certified" software
- Prove or certify mathematical theorems

What Coq is not?

- A fast/distributed/object-oriented programming language
- A Turing-complete programming language
- A model-checker
- A proof checker
- An automatic prover
- An oracle
- Something easy to work with

Principles: Poincaré

- Mathematical proofs can be arbitrarily complex (and thus difficult to find),
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Once we know what is a proof. . .

Principles - Curry-Howard correspondance

"proofs are programs"

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$$
\begin{array}{rccc}
p: & (A \to B) \wedge (B \to C) \wedge A & \to & C \\
 & (f, g, x) & \mapsto & g(f(x))
\end{array}
$$

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$$\begin{array}{llcc} p: & (A \to B) \wedge (B \to C) \wedge A & \to & C \\ & (f, g, x) & \mapsto & g(f(x)) \end{array}$$

property P	type T	(interface)
proof p	term t	(implementation)
proof-checking	type-checking	
$p \vdash P$	$\vdash t : T$	

Outline

Quick tour of syntax and basic principles

Three kinds of use

- Prove/certify mathematical theorems
- Certify existing software
- Build certified software

Conclusions

Syntax

Let's play with Coq.

What did we learn?

- There is a single language (gallina), for:
  - programs/functions,
  - specifications,
  - proofs.

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- There is another language (tactics: Ltac):
  - for building/searching proofs,
  - that can be used interactively.

  There are primitive tactics (`intros`, `apply`, `induction`), and rather complex ones (`tauto`, `ring`, `congruence`).

Principles - Gallina

- Checking a proof is easy: this is just type-checking...
  ... but we need to trust the type-checker.
- Gallina is a small language,
  - for which type-checking is (easily) decidable;
  - and still remains really expressive.
- It relies on a strong theoretical background:
  - the "Calculus of Inductive Constructions",
  - which comes from the λ-calculus.

Principles - Ltac

- Sequences of tactics do not constitute proofs:
  tactics produce gallina terms that can be checked by Coq.
- We don't need to trust tactics: any way to obtain a proof is valid since the proof will be checked.
- Proofs can actually be searched by other means than Ltac.

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Prove/certify mathematical theorems

- We just proved some elementary theorems,
  more complex ones can be proved too!
- Two major examples:
  - Georges Gonthier et al.'s proof of four-colours theorem;
  - Feit-Thompson's theorem (finite groups classification)

Certify existing software

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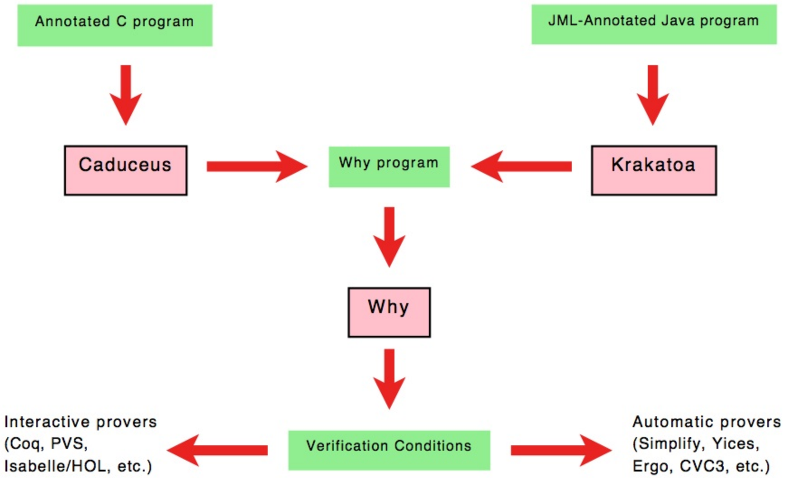
- Problem: sometimes, programs are not written in Coq...
- A solution: Why and Krakatoa/Caduceus tools.

  (see Jean-Christophe Filliâtre' gallery of certified programs: `http://why.lri.fr/examples/`)


Annotated C program
JML-Annotated Java program
Caduceus
Why program
Krakatoa
Why
Interactive provers
(Coq, PVS,
Isabelle/HOL, etc.)
Verification Conditions
Automatic provers
(Simplify, Yices,
Ergo, CVC3, etc.)

Build certified software

- If we have to write a new program, why not writing it and certifying it within Coq?
- Not so realistic, Coq is definitely too slow:
  - it's interpreted;
  - integers, floats... are not 'native'.
- However, Coq programs can be extracted to other languages: OCaml, Haskell and Scheme.
- This is how Xavier Leroy and Sandrine Blazy obtained their certified compiler for C:
  `http://compcert.inria.fr/`

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Sum-up

- Coq is a programming language:
  - purely functionnal;
  - interpreted (rather slow), but programs can be extracted to fast, compiled, languages;
- Coq is an expressive specification language:
  - any mathematical property can be stated.
- Coq certifies proofs by a simple type-checking algorithm.
- Coq is a proof assistant:
  - the interactive mode allows us to prove a theorem progressively, by using tactics;
  - tactics can be more or less elaborated, and can be defined by the user.

Related software

- Why, Krakatoa/Caduceus,
  - for the analysis of Java and C programs.
- Isabelle/HOL
  - Larry Paulson - U. of Cambridge
    & Tobias Nipkow - U. of München
- PVS
  - Sam Owre, Natarajan Shankar, John Rushby
- Twelf
  - Karl Crary & Robert Harper - Carnegie Mellon U., USA

History / people

- 1984: Thierry Coquand and Gérard Huet implement the Calculus of Constructions (INRIA-Rocquencourt)
- 1991: Christine Paulin extended it to the Calculus of Inductive Constructions
- 2005: Georges Gonthier et al. use Coq to prove the 4-colours theorem
- 2008: Xavier Leroy et al. build a certified compiler for C
- 2012: Feit-Thompson: any finite group of odd order is solvable
- Other developpers: Chet Murthy, Jean-Christophe Filliâtre, Bruno Barras, Hugo Herbelin, Assia Mahboubi and more than thirty people. . .
  TypiCal (formerly LogiCal), ProVal and Marelle projects