Disjunctive relational abstract interpretation for interprocedural program analysis

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Standard program analysis

Goal: compute (an upper approximation of) the reachable states of a program $(S, S_0, \rightarrow)$

$$\mathscr{A} = S_0 \cup \{s' \mid \exists s \in \mathscr{A}, s \rightarrow s'\}$$

Trace partitioning:

Assume S is partitioned: $S = S_1 \oplus S_2 \oplus \ldots \oplus S_k$,
and let $\mathscr{A}_i = \mathscr{A} \cap S_i, i = 1..k$:

$$\mathscr{A}_i = (S_0 \cap S_i) \cup \left(\bigcup_{j=1..k} \{s' \in S_i \mid \exists s \in \mathscr{A}_j, s \rightarrow s'\} \right)$$

$\rightarrow$ a system of recursive semantic equations

Numerical program analysis

A numerical state: n variables $\Rightarrow$ a vector $X \in \mathscr{N}^n$ ($\mathscr{N} = \mathbb{Z}$ or $\mathbb{Q}$)

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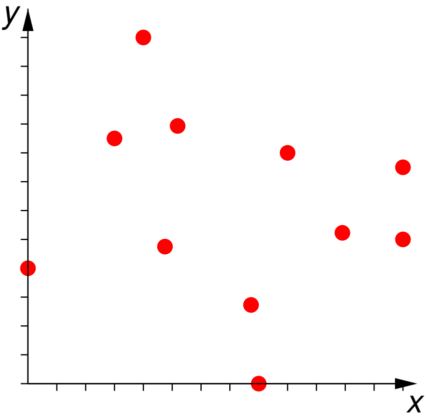
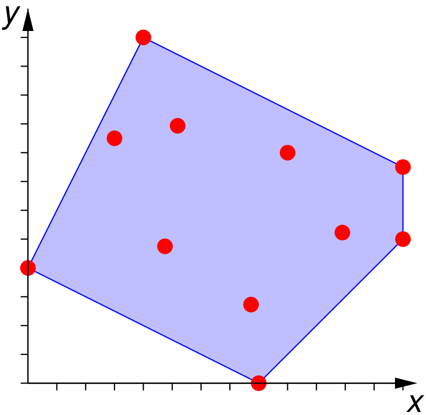
Example of semantic equations

	$\mathscr{A}_0 = \mathscr{N}^2$
x=0; y=0;	$\mathscr{A}_1 = \mathscr{A}_0[x \leftarrow 0][y \leftarrow 0]$
while	$\mathscr{A}_2 = \mathscr{A}_1 \cup \mathscr{A}_6$
x<=100 do	$\mathscr{A}_3 = \mathscr{A}_2 \cap (x \leq 100)$
if ?? then x=x+2	$\mathscr{A}_4 = \mathscr{A}_3[x \leftarrow x+2]$
else x=x+1; y=y+1	$\mathscr{A}_5 = \mathscr{A}_3[x \leftarrow x+1][y \leftarrow y+1]$
endif	$\mathscr{A}_6 = \mathscr{A}_4 \cup \mathscr{A}_5$
endwhile	$\mathscr{A}_7 = \mathscr{A}_2 \cap (x \geq 101)$

Numerical abstract domains



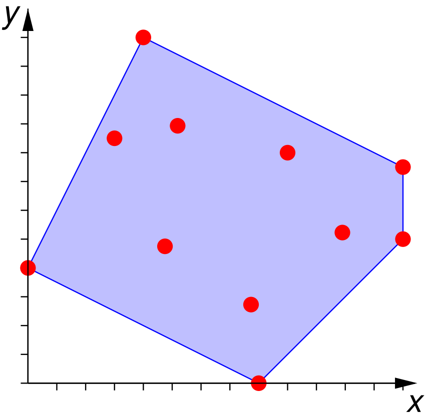
Numerical abstract domains



Polyhedra

$x + 2y \geq 8$
$x \leq y + 8$
$x \leq 13$
$y \leq 2x + 4$
$x + 2y \leq 28$

Numerical abstract domains



Polyhedra

relational

$x + 2y \geq 8$
$x \leq y + 8$
$x \leq 13$
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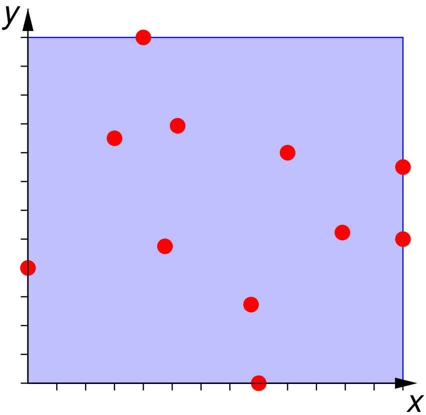
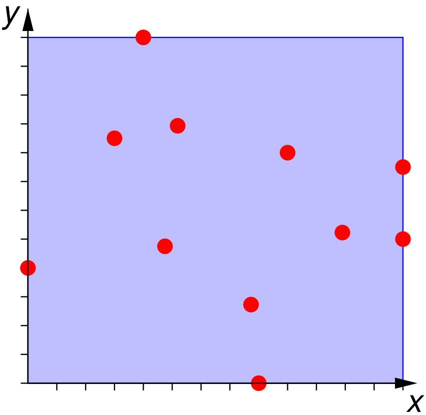
Numerical abstract domains



Intervals

$x \in [0, 13]$
$y \in [0, 12]$

Numerical abstract domains

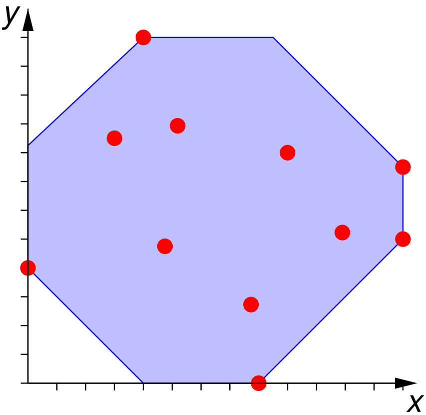


Intervals

not relational

$x \in [0, 13]$
$y \in [0, 12]$

Numerical abstract domains



Octagons

"weakly relational"

$0 \leq x \leq 13$
$0 \leq y \leq 12$
$4 \leq x + y \leq 20.5$
$-8 \leq x - y \leq 8$

An example with polyhedra

```
x := 0 ; y := 0;

while
    x ≤ 100 do
    if ?? then x:= x+2

    else x:= x+1; y:=y+1;

    endif

endwhile
```

An example with polyhedra

```
x := 0 ; y := 0;
      x=y=0
while
    x ≤ 100 do
    if ?? then x:= x+2


    else x:= x+1; y:=y+1;


    endif


endwhile
```

An example with polyhedra

```
x := 0 ; y := 0;
      x=y=0
while   x=y=0
    x ≤ 100 do
    if ?? then x:= x+2

    else x:= x+1; y:=y+1;

    endif

endwhile
∅
```

An example with polyhedra

```
x := 0 ; y := 0;
      x=y=0
while   x=y=0
    x ≤ 100 do
    if ?? then x:= x+2
      x=2, y=0
    else x:= x+1; y:=y+1;
      x=1, y=1
    endif

endwhile
∅
```

An example with polyhedra

x := 0 ; y := 0;

$x=y=0$

while $x=y=0$

$x \leq 100$ do

if ?? then x:= x+2

$x=2, y=0$

else x:= x+1; y:=y+1;

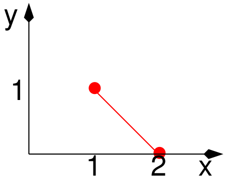
$x=1, y=1$

endif

$1 \leq x \leq 2, x+y=2$

endwhile

$\emptyset$



An example with polyhedra

x := 0 ; y := 0;

$x=y=0$

while

$x \leq 100$ do

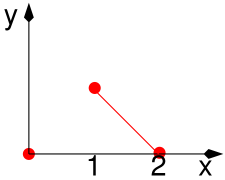
if ?? then x:= x+2

else x:= x+1; y:=y+1;

endif

$1 \leq x \leq 2,\ x+y=2$

endwhile



An example with polyhedra

x := 0 ; y := 0;

$x=y=0$

while $0 \leq y \leq x,\ x+y \leq 1$

$x \leq 100$ do

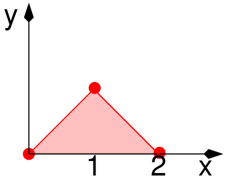
if ?? then x:= x+2

else x:= x+1; y:=y+1;

endif

$1 \leq x \leq 2,\ x+y=2$

endwhile



An example with polyhedra

x := 0 ; y := 0;

$x=y=0$

while ~~$0 \leq y \leq x,\ x+y \leq 1$~~ $0 \leq y \leq x$

$x \leq 100$ do

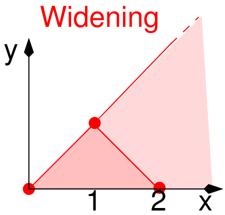
if ?? then x:= x+2

else x:= x+1; y:=y+1;

endif

$1 \leq x \leq 2,\ x+y=2$

endwhile



An example with polyhedra

x := 0 ; y := 0;

$x=y=0$

while

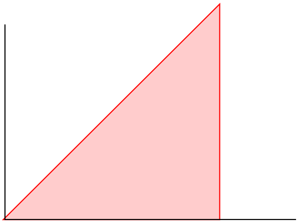
x $\leq$ 100 do $0 \leq y \leq x \leq 100$

if ?? then x:= x+2

else x:= x+1; y:=y+1;

endif

endwhile



An example with polyhedra

x := 0 ; y := 0;

$x=y=0$

while

x $\leq$ 100 do $0 \leq y \leq x \leq 100$

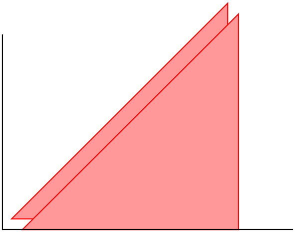
if ?? then x:= x+2

$0 \leq y \leq x-2 \leq 100$

else x:= x+1; y:=y+1;

$1 \leq y \leq x \leq 101$

endif



endwhile

An example with polyhedra

x := 0 ; y := 0;

$x=y=0$

while

x $\leq$ 100 do $0 \leq y \leq x \leq 100$

if ?? then x:= x+2

$0 \leq y \leq x-2 \leq 100$

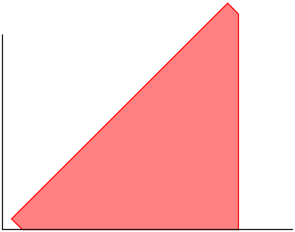
else x:= x+1; y:=y+1;

$1 \leq y \leq x \leq 101$

endif

$0 \leq y \leq x \leq 102, 2 \leq x+y \leq 202$

endwhile



An example with polyhedra

x := 0 ; y := 0;

$x=y=0$

while $0 \leq y \leq x \leq 102, x+y \leq 202$

x $\leq$ 100 do $0 \leq y \leq x \leq 100$

if ?? then x:= x+2

$0 \leq y \leq x-2 \leq 100$

else x:= x+1; y:=y+1;

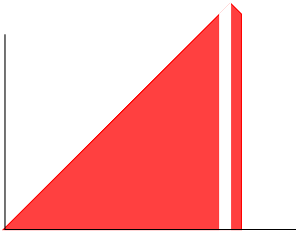
$1 \leq y \leq x \leq 101$

endif

$0 \leq y \leq x \leq 102, 2 \leq x+y \leq 202$

endwhile

$0 \leq y, 101 \leq x \leq 102, x+y \leq 202$



Motivation

- Relational domains (polyhedra, octagons, congruences, ...) are too expensive to scale up (on huge monolithic programs)

  $\rightarrow$ interprocedural analysis especially interesting

  (analyse medium size procedures once and for all)

- Bottom-up interprocedural analysis (build a procedure summary independently of the calling context) little used, because too imprecise with non-relational domains

  $\rightarrow$ take advantage of relational domains to build precise relational procedure summaries

Interprocedural program analysis

How to deal with procedures and calls ?

- Inlining
- Top-down: at each call, analyse the procedure in the context of its actual parameters.
- Bottom-up: analyse the procedure once and for all, synthesize a summary to be used at each call.
- Hybrid strategies: mix the two, e.g., compute a summary in the simplest aliasing context (no aliases), compute a new summary when a call with possible aliases is encountered.

From state analysis to relational analysis

State analysis: Starting from a set S_0 of initial states, compute at each control point i, the set

$$\mathscr{A}_i = \{s \in S_i \mid \exists s_0 \in S_0, s.t., s_0 \rightarrow^* s\}$$

Relational analysis: Starting from a set S_0 of initial states, compute at each control point i, the *relation*

$$\mathscr{R}_i = \{(s_0, s) \in S_0 \times S_i \mid s_0 \in S_0, s_0 \rightarrow^* s\}$$

From state analysis to relational analysis

State analysis: Starting from a set S_0 of initial states, compute at each control point i, the set

$$\mathscr{A}_i = \{s \in S_i \mid \exists s_0 \in S_0, s.t., s_0 \to^* s\}$$

Relational analysis: Starting from a set S_0 of initial states, compute at each control point i, the *relation*

$$\mathscr{R}_i = \{(s_0, s) \in S_0 \times S_i \mid s_0 \in S_0, s_0 \to^* s\}$$

Important fact: $\exists s.\mathscr{R}_i$ is a necessary condition on s_0 for point i to be reachable from s_0

Relational analysis of a procedure

Procedures:

- All parameters are supposed to be passed by reference (but we ignore pointer manipulation, and we entrust existing analyses to detect aliasing problems).
- Global variables considered as additional parameters.

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- Global variables considered as additional parameters.

Relational analysis:

- Duplicate each parameter x with an auxiliary variable x^0, to record its initial value
- Perform a standard analysis of the body, starting from $x = x^0$ for each parameter x
- Project away local variables from the result at return point to obtain a relational summary $\mathscr{R}(X^0, X)$

Example: *div*

```
proc div (a, b, q, r: int){
assume a ≥0 && b ≥ 1;
   a0=a ; b0=b ; q0=q ; r0=r;
   q=0 ; r=a;
   while(r ≥ b){
      r = r-b ; q = q+1 ;
   }
}
```

Example: *div*

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}
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Standard Analysis

$$r \geq 0 \wedge q \geq 0 \wedge b \geq r+1$$
$$\wedge\, a = a0 \wedge b = b0$$

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Standard Analysis

$r \geq 0 \wedge q \geq 0 \wedge b \geq r+1$
$\quad \wedge a = a0 \wedge b = b0$

precondition $a0 \geq 0$ lost!

Widening limited by precondition

A precondition is obviously an invariant (a procedure may not modify the initial values of its parameters!)
$\Rightarrow$ one can limit (i.e., intersect) the widening by the precondition.

Example (cont.): Intersect the result of the widening with

$$a0 \geq 0 \wedge b0 \geq 1$$

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```

Standard Analysis

$r \geq 0 \wedge q \geq 0 \wedge b \geq r+1$
$\wedge\, a = a0 \wedge b = b0$

☹ precondition $a0 \geq 0$ lost!

With limited widening

$r \geq 0 \wedge q \geq 0 \wedge b \geq r+1$
$\wedge\, \underline{a \geq q+r} \wedge a = a0 \wedge b = b0$

😐 Better than expected, but still imprecise, as a summary

Disjunctive summaries

- Convex summaries are not expressive enough
- General disjunctions of polyhedra are difficult to manage (ex.: inclusion)
- Idea: disjunctive refinement by partitioning preconditions

  precondition P partitioned into $P_1 \oplus P_2 \oplus \ldots \oplus P_m$
- Disjunctive summary

  $$\mathscr{R} = \mathscr{R}_1 \oplus \mathscr{R}_2 \oplus \ldots \mathscr{R}_m \quad \text{with} \quad P_i = \exists X.\mathscr{R}_i,\ i = 1..m$$

Example of disjunctive summary

```
proc div (a, b, q, r: int){
assume a ≥0 && b ≥ 1;
   a0=a ; b0=b ; q0=q ; r0=r;
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   while(r ≥ b){
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Partitioning according to

$$a0 \geq b0 \; , \; b0 \geq a0+1$$

(entering the loop or not)

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```

Standard analysis:

$r \geq 0 \wedge q \geq 0 \wedge b \geq r+1$

With limited widening:

$r \geq 0 \wedge q \geq 0 \wedge b \geq r+1 \wedge a \geq q+r$

Partitioning according to

$$a0 \geq b0 \,,\; b0 \geq a0+1$$

(entering the loop or not)

Result:

either $b0-1 \geq a0 \geq 0$
 and $q = 0 \wedge r = a$

or $a0 \geq b0 \geq 1$
 and $r \geq 0 \wedge q \geq 0 \wedge$
 $q+r \geq 1 \wedge b \geq r+1$
 $\wedge\, a+1 \geq b+q$

Partitioning preconditions

Refinement according to local reachability: Let $\mathscr{R}_k(P)$ be the result at control point k of a relational analysis from precondition P. It relates the initial values X^0 and the current values X. Then,

$$R_k^0 = \exists X.\mathscr{R}_k(P)$$

is a necessary condition on X^0 for the point k to be reachable.

Partitioning preconditions

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$$R_k^0 = \exists X . \mathscr{R}_k(P)$$

is a necessary condition on X^0 for the point k to be reachable.

$R_k^0 \subseteq P$, thanks to widening limited by precondition, but if $R_k^0 \neq P$, any constraint c of R_k^0 not satisfied by P, may be used to split the precondition P into

$$P' = P \wedge c \quad \text{and} \quad P'' = P \wedge \neg c$$

and point k is not reachable from P''.

Partitioning preconditions: example

Example: From the the precondition $P = (a0 \geq 0 \wedge b0 \geq 1)$, the analysis of div provides

$$R = (a = a0 \wedge b = b0 \wedge r \geq b \geq 1 \wedge q \geq 0 \wedge a \geq q + r)$$

at the loop entry. Now,

$$R_0 = \exists(a, b, q, r).R = (a0 \geq b0 \geq 1)$$

Since $a0 \geq b0$ is not satisfied by the precondition P, it provides the (obvious) refinement

$$P' = (a0 \geq b0 \geq 1) \quad , \quad P'' = (b0 - 1 \geq a0 \geq 0)$$

that we used before.

Postponing loop feedback

The problem: In the div example, our partitioning separates the cases when the loop is entered at least once or not.
In the former case, our analysis provides

$$(a0 \geq b0 \geq 1) \Rightarrow$$
$$(r \geq 0 \wedge q \geq 0 \wedge q+r \geq 1 \wedge b \geq r+1 \wedge a+1 \geq b+q)$$

We should find $q \geq 1$ (the loop is executed at least once)

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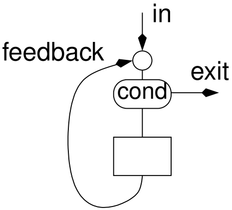
The explanation:
At loop exit, the semantic equation is



$$\mathscr{R}_{exit} = (\mathscr{R}_{in} \sqcup \mathscr{R}_{feedback}) \sqcap \neg cond$$

while $\mathscr{R}_{in} \sqcap \neg cond = \emptyset$

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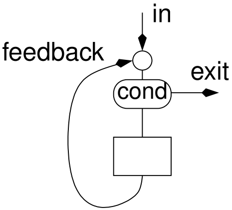

$$\mathscr{R}_{exit} = (\mathscr{R}_{in} \sqcup \mathscr{R}_{feedback}) \sqcap \neg cond$$

while $\mathscr{R}_{in} \sqcap \neg cond = \emptyset$

The solution: Compute instead

$$\mathscr{R}_{exit} = (\mathscr{R}_{in} \sqcap \neg cond) \sqcup (\mathscr{R}_{feedback} \sqcap \neg cond)$$

Postponing loop feedback

Previous result:

$$(r \geq 0 \wedge q \geq 0 \wedge q + r \geq 1 \wedge b \geq r + 1 \wedge a + 1 \geq b + q)$$

New result:

$$(r \geq 0 \wedge q \geq 1 \wedge b \geq r + 1 \wedge \underline{a + 1 \geq b + q + r})$$

Better than expected

Using procedure summaries

- Let p be a procedure, with a call $k : q(A)\ ;\ k' : \ldots$ where $A = (a1, a2, \ldots, a_n)$ are the actual parameters. Let $\mathscr{R}_k$ be the relation computed at point k.
- Let $F = (f_1, f_2, \ldots, f_n)$ be the formal parameters of q, and $(\mathscr{S}_1, \ldots, \mathscr{S}_m)$ be a summary of q.

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- Let p be a procedure, with a call $k : q(A)\ ;\ k' : \ldots$ where $A = (a1, a2, \ldots, a_n)$ are the actual parameters. Let $\mathscr{R}_k$ be the relation computed at point k.
- Let $F = (f_1, f_2, \ldots, f_n)$ be the formal parameters of q, and $(\mathscr{S}_1, \ldots, \mathscr{S}_m)$ be a summary of q.

Then, the relation at return point k' is

$$\mathscr{R}_{k'} = \bigsqcup_{\ell=1}^{m} \exists A'. \mathscr{R}_k[A/A'] \sqcap \mathscr{S}_\ell[F/A][F^0/A']$$

where

- $\mathscr{R}_k[A/A']$ is the result of renaming in $\mathscr{R}_k$ each variable a_i as a'_i
- $\mathscr{S}_\ell[F/A][F^0/A']$ is the result of renaming in $\mathscr{S}_\ell$ each variable f_i as a_i and each variable f_i^0 as a'_i

Partitioning preconditions (cont.)

Refinement according to called procedure summary

Since the relation at return point of a call is a $\bigsqcup$, its a good idea to make some of its terms empty.

Preconditions of the call: $P_\ell = (\exists F.\mathscr{S}_\ell), \ell = 1..m$

Making $\mathscr{C}_\ell = \exists A'.\mathscr{R}_k[A/A'] \sqcap P_\ell[F^0/A']$ empty: refine the precondition of the caller

$\exists X.\mathscr{C}_\ell$ is a necessary condition (on initial values in the caller) for $\mathscr{C}_\ell$ to be not empty. Use it to split the precondition, as before.

Example, a recursive procedure

McCarthy's 91 function

```
proc f91 (x,y: int) {
  var z, t: int ;
1:if (x > 100)
2:    y = x -10 ;
3:else {
4:    z = x + 11 ;
5:    f91 (z, t) ;
6:    f91 (t, y) ;
7:}
}
```

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1:if (x > 100)
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4:    z = x + 11 ;
5:    f91 (z, t) ;
6:    f91 (t, y) ;
7:}
}
```

Semantic equations
(without parameters duplication)

$$\begin{aligned}
\mathscr{R} &= \mathscr{R}_2 \sqcup \mathscr{R}_7 \\
\mathscr{R}_2 &= (x \geq 101 \wedge y = x - 10) \\
\mathscr{R}_7 &= (x \leq 100) \sqcap \\
&\qquad (\exists t. \mathscr{R}(x+11, t) \sqcap \mathscr{R}(t, y))
\end{aligned}$$

22mm]

Example, a recursive procedure

McCarthy's 91 function

```
proc f91 (x,y: int) {
   var z, t: int ;
1:if (x > 100)
2:    y = x -10 ;
3:else {
4:    z = x + 11 ;
5:    f91 (z, t) ;
6:    f91 (t, y) ;
7:}
}
```

Semantic equations
(without parameters duplication)

$$
\begin{aligned}
\mathscr{R} &= \mathscr{R}_2 \sqcup \mathscr{R}_7 \\
\mathscr{R}_2 &= (x \geq 101 \wedge y = x - 10) \\
\mathscr{R}_7 &= (x \leq 100) \sqcap \\
&\qquad (\exists t. \mathscr{R}(x+11, t) \sqcap \mathscr{R}(t, y))
\end{aligned}
$$

22mm]Standard analysis:

$$
\begin{aligned}
\mathscr{R}_2 &= (x \geq 101 \wedge y = x - 10) \\
\mathscr{R}_7 &= (x \leq 100 \wedge y + 9 \geq x \wedge y \geq 91) \\
\mathscr{R} &= (x \leq y + 10 \wedge y \geq 91)
\end{aligned}
$$

Example, a recursive procedure (cont.)

Standard analysis:

$$
\begin{aligned}
\mathscr{R}_2 &= (x \geq 101 \wedge y = x - 10) \\
\mathscr{R}_7 &= (x \leq 100 \wedge y + 9 \geq x \wedge y \geq 91) \\
\mathscr{R} &= (x \leq y + 10 \wedge y \geq 91)
\end{aligned}
$$

Example, a recursive procedure (cont.)

Standard analysis:

$$\begin{array}{rcl} \mathscr{R}_2 & = & (x \geq 101 \wedge y = x - 10) \\ \mathscr{R}_7 & = & (x \leq 100 \wedge y + 9 \geq x \wedge y \geq 91) \\ \mathscr{R} & = & (x \leq y + 10 \wedge y \geq 91) \end{array}$$

Since $\exists y.\mathscr{R}_2 = (x \geq 101)$, split the initial precondition $\top$ into $(x \geq 101)$ and $(x \leq 100)$.

Example, a recursive procedure (cont.)

Standard analysis:

$$\begin{array}{lcl}
\mathscr{R}_2 & = & (x \geq 101 \wedge y = x - 10) \\
\mathscr{R}_7 & = & (x \leq 100 \wedge y + 9 \geq x \wedge y \geq 91) \\
\mathscr{R} & = & (x \leq y + 10 \wedge y \geq 91)
\end{array}$$

Since $\exists y . \mathscr{R}_2 = (x \geq 101)$, split the initial precondition $\top$ into $(x \geq 101)$ and $(x \leq 100)$.

Result:

$$\begin{array}{ll}
 & (x \geq 101 \wedge y = x - 10) \\
\text{or} & (x \leq 100 \wedge y \geq 91)
\end{array}$$

not much better

Example, a recursive procedure (cont.)

Now, refine according to the summary at the first recursive call, i.e., according to $(x + 11 \geq 101)$.

Example, a recursive procedure (cont.)

Now, refine according to the summary at the first recursive call, i.e., according to $(x+11 \geq 101)$.

Final result:

$$
\begin{array}{ll}
 & (x \geq 101 \wedge y = x - 10) \\
\text{or} & (90 \leq x \leq 100 \wedge y = 91) \\
\text{or} & (x \leq 89 \wedge y = 91)
\end{array}
$$

☺ most precise summary

Implementation

Method implemented by Rémy Boutonnet in his prototype analyzer MARS (Mars Abstract interpretation Research System)

- Based on CLANG, takes a significant subset of C
- Uses the Apron library of abstract domains
- Applies a standard data-flow analysis to identify pure “value” parameters and pure “result” parameters, which don’t need to be duplicated
- Recursive procedures not yet taken into account

Experiments

Goal: Compare the interprocedural analysis to a standard analysis of inlined programs, answer the following questions:

- The construction of summaries involves several analyses, does it induce a significant overhead?
- Does the bottom-up approach result in a significant loss of precision?

Problems:

- Find an interesting benchmark (numerical programs with procedures)
- How to compare the precision?

Experiments (cont.)

- Use of the Mälardalen benchmark (worst case execution time), 19 programs sometimes augmented with statement counters
- Precision: Compare the results at the end of the main program (after projection on the variables of the main)
  - Qualitative comparison: is the result of interprocedural analysis better ($\subset$), worse ($\supset$), equal ($=$), or incomparable ($<>$) w.r.t. the one of inlining analysis?
  - Quantitative comparison: compare the number of constraints

First experimental results

- Interprocedural analysis faster on 13 programs over 19
  (average speedup 2.9).
  Most procedures called once. The speedup increases rapidly with the number of calls.
- Some precision lost on 3 programs
  (worst: from 32 to 12 constraints)
- Some precision gained on 5 programs
  (best: from 1 to 6 constraints)

Future work

Extend to modular analysis of procedures with memories (objects, reactive software)

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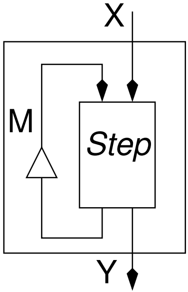
Extend to modular analysis of procedures with memories (objects, reactive software)

A reactive module

- X : input variables
- Y : output variables
- M : memory variables

M = M^0;
forever {
    read(X);
    (Y,M) = *Step*(X,M);
    write(Y);
}



Future work

Can we construct a summary of each module (including an invariant on the memory) and use these summaries to analyse the whole system?