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Constant Aspect Ratio Tiling

Parametric Tiling is (sometimes) Polyhedral

Introduction

Tiling is a well-known effective transformation:
- ☛ Locality improvement, new level of granularity with parallelism opportunities.
- ☛ If the tile sizes are constant, polyhedral ($i = 4.\alpha + ii$)

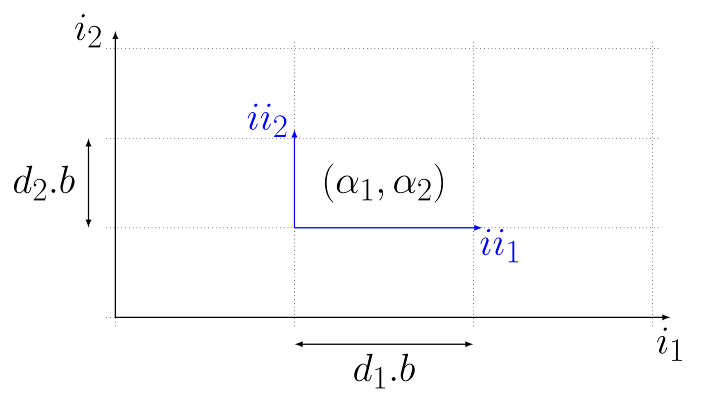
Parametric tiling: tiling were the tile size is a parameter
- ☛ Tile size can be picked at runtime (ex: autotuning)
- ☛ Non polyhedral ($i = b.\alpha + ii$)

Parametric tiling is usually embedded in the code generator:
- Fourier-Motzkin symbolic elimination
- Tile the bounding box of the iteration domain
- *D-tiling* [Kim, LCPC10] (computing the outset, inset, parallel)
- *PrimeTile* [Hartono, ICS09] (sequential), *DynTile* [Hartono, IDPDS10] and *PTile* [Baskaran, CGO10] (parallel)

☛ The transformations applied after parametric tiling must be "hard-coded"
(ex: wavefront/rectangular parallelism [Athanasios, Kelly, LCPC13])

Constant Aspect Ratio Tiling

Parametric tiling using only one tile size parameter and a fixed aspect ratio for every dimensions.



$i_k = d_k.b.\alpha_k + ii_k$ where $0 \leq ii_k < d_k.b$
α_k: blocked indexes
ii_k: local indexes
d_k: ratios
Parameters: $p_k = b.\lambda_k + pp_k$ where $0 \leq pp_k < b$

☛ **Main benefit:** Under these constraints, we are polyhedral !
☛ **Mathematical foundation:** How do we manage polyhedron and affine function?

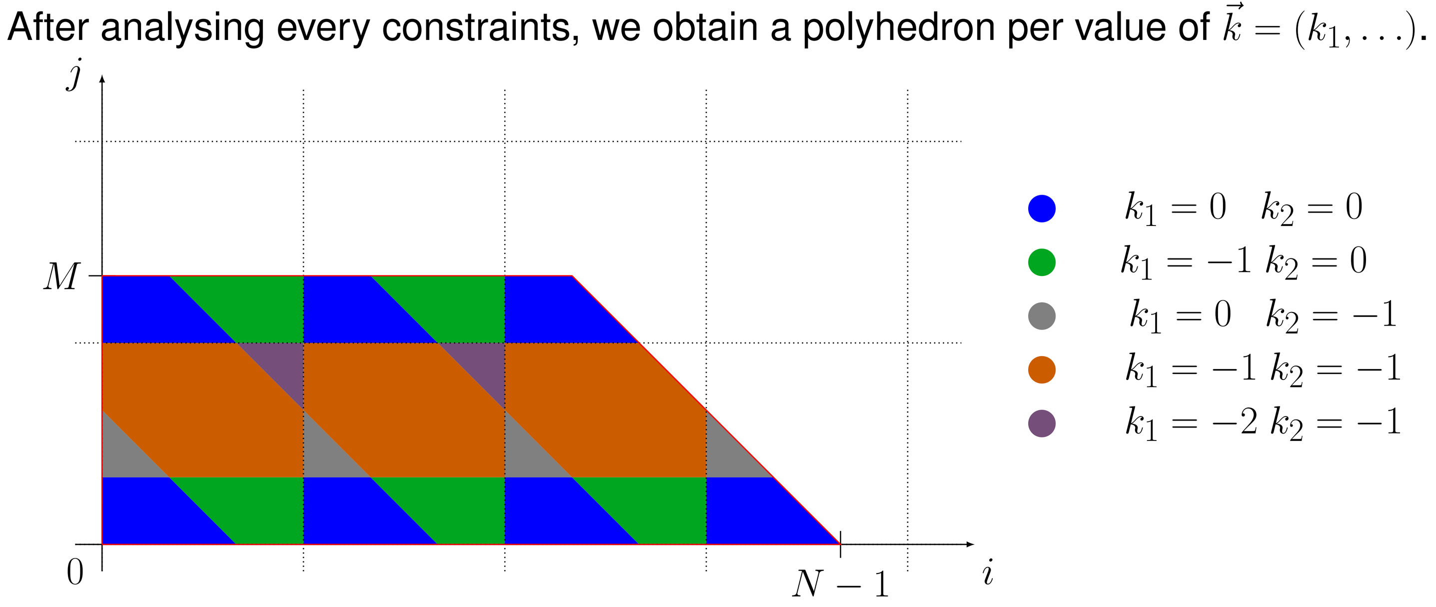
CART on polyhedron

Example: $\mathcal{D} = \{i, j \mid i + j \leq N - 1 \ \wedge \ j \leq M \ \wedge \ 0 \leq i, j\}$ with tiles of size $b \times b$.
- ☛ How to obtain its CART version: $\Delta = \{\alpha, \beta, ii, jj \mid \ldots\}$?

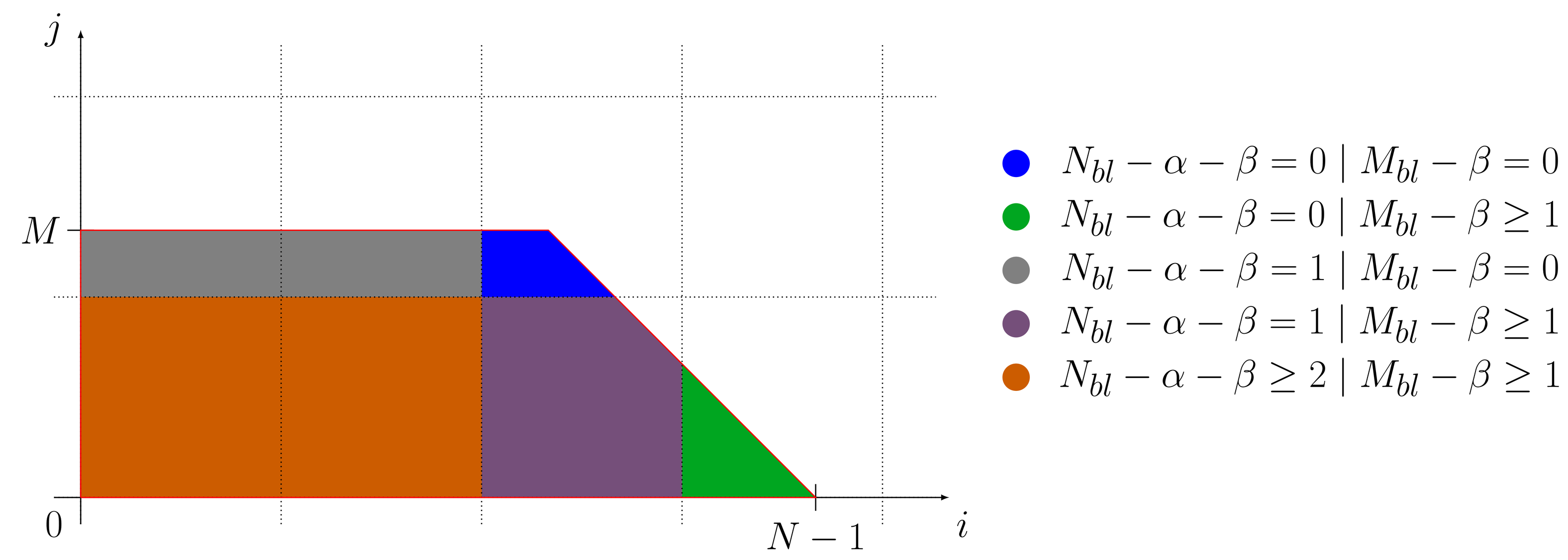
Let us focus on the first constraint:

$$N - i - j - 1 \geq 0$$
$$\Updownarrow (substitution)\ (N, i, j) = (N_{bl}, \alpha, \beta).b + (N_{loc}, ii, jj)$$
$$\boxed{(N_{bl} - \alpha - \beta).b} + (N_{loc} - ii - jj - 1) \geq 0$$
$$\Updownarrow b > 0$$
$$N_{bl} - \alpha - \beta + \boxed{\frac{N_{loc} - ii - jj - 1}{b}} \geq 0$$
$$\Updownarrow a \geq 0 \Leftrightarrow \lfloor a \rfloor \geq 0$$
$$N_{bl} - \alpha - \beta + \left\lfloor \frac{N_{loc} - ii - jj - 1}{b} \right\rfloor \geq 0 \quad \text{and } k_1 = \left\lfloor \frac{N_{loc} - ii - jj - 1}{b} \right\rfloor \in [| - 2; 0|]$$

After analysing every constraints, we obtain a polyhedron per value of $\vec{k} = (k_1, \ldots)$.

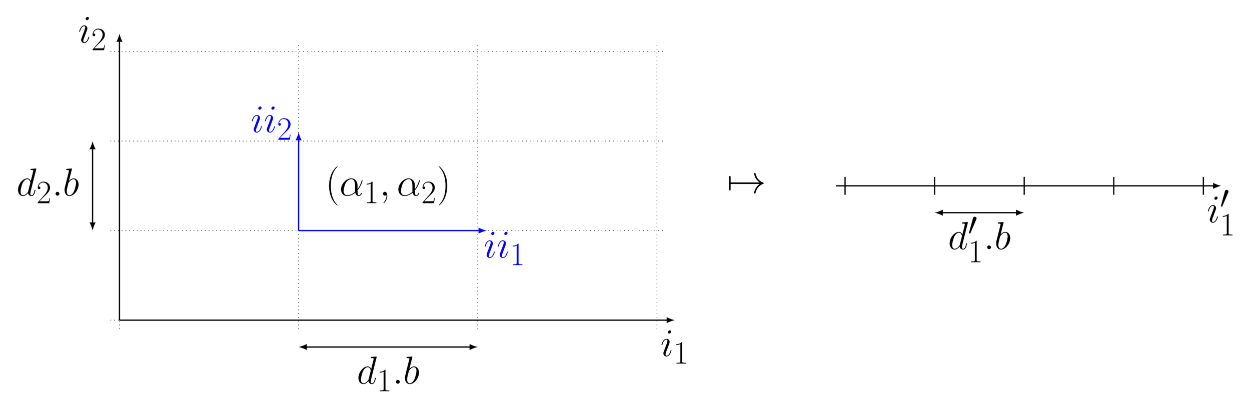


We can reorganize this union of polyhedron to have only one polyhedron per tile:



CART on affine function

We have two tilings: for the antecedent space and for the image space.



Example: $f : (i, j \mapsto i + j)$, with a tiling of the antecedent domain $b \times b$ and of the image domain b.
- ☛ How to obtain its CART version $\phi(\alpha, \beta, ii, jj) = (\alpha', ii')$?

Same method than for polyhedra (with equations) $\leadsto \phi$ is a piecewise affine function

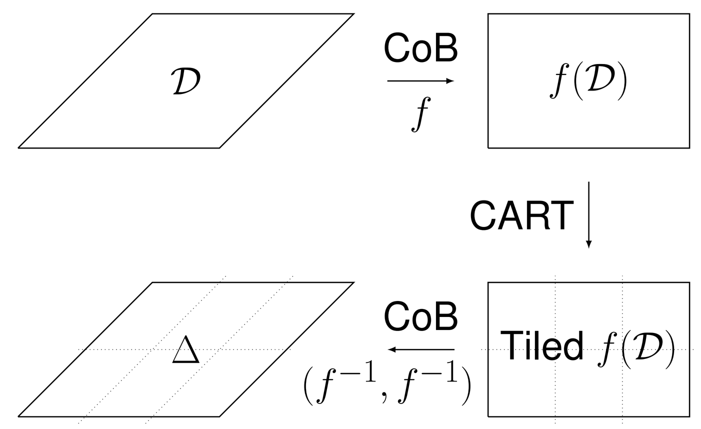
$$\phi(\alpha, \beta, ii, jj) = \begin{cases} (\alpha + \beta, ii + jj) & \text{if } ii + jj < b \\ (\alpha + \beta + 1, ii + jj - b) & \text{if } b \leq ii + jj \end{cases}$$

In general, the piecewise affine function might have modulo constraints
- ☛ **Example:** $f : (i \mapsto i)$ with a tiling of the antecedent domain of b and of the image domain of $2.b$

$$\phi(\alpha, ii) = \begin{cases} (\frac{\alpha}{2}, ii) & \text{if } \alpha \bmod 2 = 0 \\ (\frac{\alpha - 1}{2}, ii + b) & \text{if } \alpha \bmod 2 = 1 \end{cases}$$

Extensions

- **Tiling along non-canonical dimensions:**



- **Several tile size parameters:** two different tile size parameters must not interfere
  - ☛ Ex: matrix multiply with 3 tile size parameters

Conclusion and Future Work

- Standalone implementation (C++/Java): `http://compsys-tools.ens-lyon.fr/`
- Full CART transformation: currently being implemented in the AlphaZ compiler
- The code is still polyhedral after the CART transformation
  - ⇒ Allow polyhedral analysis and optimization after parametric tiling (for example, we can reapply another level of tiling for free)
- Used as the first step of the semantic tiling transformation

People Involved

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