Lecture 5: Linear Loop Tiling

CR11 – Hardware Compilation and Simulation
ENS-Lyon – M2IF

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Loop Tiling

\[
\begin{align*}
&\text{for } i := 0 \text{ to } N \\
&\text{for } j := 0 \text{ to } N \\
&B: \quad a[i,j] := a[i-1,j] + a[i,j-1];
\end{align*}
\]

- **Partition** into atomic blocks (tiles)
- **Scope**: perfect loop nests with uniform dependences
for $i := 0$ to $N$
for $j := 0$ to $N$

$B: a[i,j] := a[i-1,j] + a[i,j-1]$;
Parallelepipedic Loop Tiling

\[ \vec{\tau}_1, \vec{\tau}_2, b_1, b_2, T_1, T_2 \]

Tiled domain: \((T_1, T_2, i, j) \in \hat{D}_B \)
Slice with hyperplanes $\vec{\tau}_i$ by $b_i$
Parallelepipedic Loop Tiling

- **Slice** with hyperplanes $\vec{\tau}_i$ by $b_i$
- **Tiled domain:** $(T_1, T_2, i, j) \in \hat{D}_B$
Draw the tile execution order with $\theta(T_1, T_2, i, j) = (T_2, T_1, i, j)$
Find a relation between $T_k$, $\vec{\tau}_k \cdot \vec{x}$ and $b_k$
Generate the loop tiling for \( \theta(T_1, T_2, i, j) = (T_2, T_1, i, j) \)
The tile dependence graph is **schedulable**.
for $i := 1$ to $N$
for $j := 1$ to $N$
$B: \ a[i] := a[i-1] + a[i] + a[i+1]$;

Is the tiling valid?
\[ \text{for } i := 1 \text{ to } N \]
\[ \text{for } j := 1 \text{ to } N \]
\[ B: \quad a[i] := a[i-1] + a[i] + a[i+1]; \]

\( \vec{\tau} \) is valid if \( \vec{\tau} \cdot \vec{d} \geq 0 \) \( \forall \vec{d} \in \Delta D \)
for $i := 0$ to $N$
  for $j := 0$ to $N$
    $B: a[i] := a[i-1] + a[i] + a[i+1]$;
for $i := 0$ to $N$
for $j := 0$ to $N$

B: $a[i] := a[i] + 1$;

Tend to 0 communication, minimize $\vec{\tau} \cdot \vec{d}$ \thinspace \forall \vec{d} \in \Delta D$
for $i := 0$ to $N$
for $j := 0$ to $N$
B: $a[i] := a[i] + 1$;

for $i := 1$ to $\dim D_B$
Solve the ILP:

$$\min M$$
\begin{align*}
\text{s.t.} & \quad \vec{r}_i \in C_T \setminus \{\vec{0}\} \\
& \quad (\vec{r}_1, \ldots, \vec{r}_i) \text{ linearly independent} \\
& \quad \vec{r}_i \cdot \vec{d} \leq M \quad \forall \vec{d} \in \Delta D
\end{align*}$$
for $i := 0$ to $N$
    for $j := 0$ to $N$
        $B$: $a[i] := a[i] + 1$;

Apply the algorithm
for $t := 1$ to $N$
    for $i := 1$ to $N$
        $B: \quad a[t,i] := a[t-1,i-1] + a[t-1,i] + a[t-1,i+1];$

1. Find a communication-minimal tiling
2. Generate the code with \texttt{iscc}