Master d'Informatique Fondamentale – 2020
Semantics and Verification (M1)

ENS DE LYON

TD de Sémantique et Vérification

VI– LTL

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Christophe Lucas
christophe.lucas@ens-lyon.fr

Recall that:

- $[\![\Diamond\varphi]\!]_\rho = \{\sigma \in (2^{\mathrm{AP}})^\omega \mid \exists i, \sigma\upharpoonright i \in [\![\varphi]\!]_\rho\}$
- $[\![\Box\varphi]\!]_\rho = \{\sigma \in (2^{\mathrm{AP}})^\omega \mid \forall i, \sigma\upharpoonright i \in [\![\varphi]\!]_\rho\}$
- $[\![\varphi \mathsf{U} \psi]\!]_\rho = \{\sigma \in (2^{\mathrm{AP}})^\omega \mid \exists i, \sigma\upharpoonright i \in [\![\psi]\!]_\rho, \forall j < i, \sigma\upharpoonright j \in [\![\varphi]\!]_\rho\}$
- $\varphi \mathsf{W} \psi := \neg(\neg\psi \mathsf{U} \neg(\varphi \vee \psi))$

Moreover, a state s of a transitition system satisfies a LTL formula φ with parameter ρ if and only if $\mathrm{Traces}(s) \subseteq [\![\varphi]\!]_\rho$.

Fixpoints

Exercise 1.
Let L be a complete lattice and let $f : L \to L$ be a monotone function. Show that $\mu(f) = \bigwedge\{a \in L \mid f(a) \leq a\}$ (resp. $\nu(f) = \bigvee\{a \in L \mid a \leq f(a)\}$) is the least fixpoint (resp. greatest fixpoint) of f.

Exercise 2.
Given formulae φ, ψ with parameters ρ, show that $[\![\varphi \mathsf{U} \psi]\!]_\rho$ is the least fixpoint of $[\![\psi\vee(\varphi\wedge\bigcirc X)]\!]_\rho(X)$.

Linear Time Logic

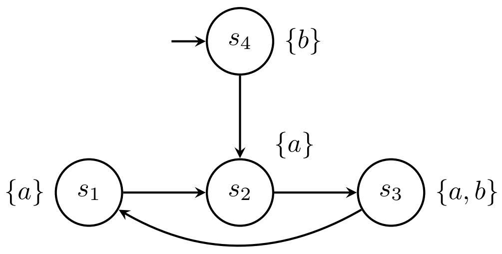
Exercise 3.
Show that:

1. $\neg(\varphi \mathsf{W} \psi) \equiv \neg\psi \mathsf{U} (\neg\varphi \wedge \neg\psi)$
2. $\neg(\varphi \mathsf{U} \psi) \equiv \neg\psi \mathsf{W} (\neg\varphi \wedge \neg\psi)$
3. $\bigcirc(\varphi \mathsf{U} \psi) \equiv \bigcirc\varphi \mathsf{U} \bigcirc\psi$
4. $\varphi \mathsf{W} \psi \equiv (\varphi \mathsf{U} \psi) \vee \Box\varphi$

Exercise 4.
Show that:

1. $[\![\top \mathsf{U} \varphi]\!]_\rho = \{\sigma \in (2^{\mathrm{AP}})^\omega \mid \exists i, \sigma\upharpoonright i \in [\![\varphi]\!]_\rho\}$
2. $[\![\varphi \mathsf{W} \bot]\!]_\rho = \{\sigma \in (2^{\mathrm{AP}})^\omega \mid \forall i, \sigma\upharpoonright i \in [\![\varphi]\!]_\rho\}$

Exercise 5.
Consider the following transition system over the set of atomic propositions $\{a, b\}$:



Indicate for each of the following LTL formulae the set of states for which these formulae are fulfilled:

A. $\bigcirc a$

B. $\bigcirc \bigcirc \bigcirc b$

C. $\Box b$

D. $\Box \Diamond a$

E. $\Box (b \, \mathcal{U} \, a)$

F. $\Diamond (a \, \mathcal{U} \, b)$