

TD de Sémantique et Vérification
VIII– LTL and Büchi Automata
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- We denote the words characterised by an LTL formula φ by $\llbracket \varphi \rrbracket = \{\sigma \mid \sigma \models \varphi\}$.
- For a set of finite words W , we denote by \vec{W} the set of words having an infinite number of prefixes in W : $\vec{W} = \{\sigma \in \Sigma^\omega \mid \exists^\infty \hat{\sigma} \in W, \hat{\sigma} \subseteq \sigma\}$
- A regular safety property P is a safety property such that P_{bad} is regular (i.e., recognizable by a DFA)

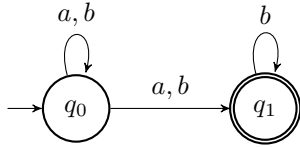
Let A be a set. Recall that

- For $u \in A^*$, $\text{ext}(u) = \{\sigma \in A^\omega \mid u \subseteq \sigma\}$
- We can equip A^ω with the topology $\mathcal{U} = \{\text{ext}(U) \mid U \subseteq A^*\}$

Decomposition of ω -regular Linear Time Properties

Exercise 1.

Let $L = (a + b)b^\omega$ and the following NBA \mathcal{A} recognizing L :



Show that there is a word $\sigma \notin \mathcal{L}_\omega(\mathcal{A})$ such that σ has an infinite number of prefixes recognized by \mathcal{A} as a DFA (i.e., $\exists^\infty \hat{\sigma} \subseteq_{\text{finite}} \sigma, \hat{\sigma} \in \mathcal{L}(\mathcal{A})$).

Exercise 2.

We want to show the decomposition theorem for ω -regular properties.

Let $U \subseteq \Sigma^*$.

1. Show that if $A \subseteq \Sigma^\omega$ is ω -regular, then $\text{pref}(A) \subseteq \Sigma^*$ is regular. (You may use that $\text{pref}(U)$ is regular if $U \subseteq \Sigma^*$ is regular.)
2. Show that $\text{cl}(U)$ is a safety property induced by $P_{bad} = U^c$.
3. Deduce that if $P \subseteq \mathcal{P}(\text{AP})^\omega$ is an ω -regular safety property, then P is a *regular* safety property.
4. Let P_{bad} be a set of bad prefix of $\text{cl}(U)$ and \mathcal{A} be a complete deterministic automaton recognizing P_{bad} such that for all final state q_F of \mathcal{A} and all $a \in \Sigma, \delta(q_F, a) = q_F$. Show that $\mathcal{L}_\omega(\mathcal{A}) = (2^{\text{AP}})^\omega \setminus \text{cl}(U)$.
5. Show that if U is regular then $\text{cl}(U)$ is ω -regular (we recall that if $\text{cl}(U)$ is a regular safety property, we can always find P_{bad} and \mathcal{A} satisfying the properties of the previous question. Try to adapt \mathcal{A} to recognize $\text{cl}(U)$).
6. Show that for every ω -regular linear time property P , there is a ω -regular safety property P_{safe} and a ω -regular liveness property P_{live} such that $P = P_{safe} \cap P_{live}$.

From LTL to NBAs

Exercise 3.

Let φ be a the LTL formula over $AP = \{a, b, c\}$ given by $\varphi = \Box a \wedge (b \mathcal{U} \neg c)$. Construct an (G)NBA \mathcal{A} , such that, $\mathcal{L}_\omega(\mathcal{A}) = \llbracket \varphi \rrbracket$.

Characterization of DBAs

Exercise 4.

1. Let $AP = \{a, b\}$. Show that $\{\sigma \mid \exists^\infty t, a \in \sigma(t)\} \subseteq (2^{AP})^\omega$ is not closed.
2. Show that there is a DBA \mathcal{A} such that $\mathcal{L}_\omega(\mathcal{A})$ is not a safety property.
3. Show that for every regular safety property P , there is a DBA \mathcal{A} such that $P = \mathcal{L}_\omega(\mathcal{A})$.