Twincut graphs

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joint work with Édouard Bonnet, Romain Bourneuf,
Julien Duron, Stéphan Thomassé, and Nicolas Trotignon

ANR DIGRAPHS workshop
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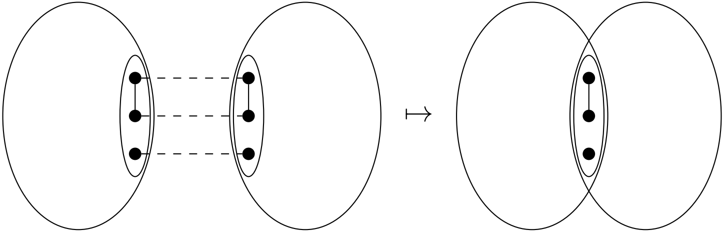
Glueing

Identify two graphs on some partial isomorphism:



Inverse of splitting on a separator.

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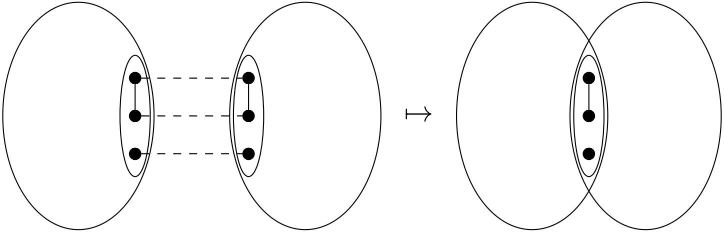

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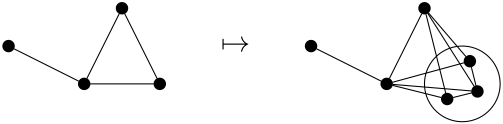
Theorem (Chudnovsky, Penev, Scott, Trotignon, '11)

If $\mathcal{C}$ is χ-bounded, then the closure of $\mathcal{C}$ under glueing on $\leq k$ vertices is also χ-bounded.

Substitution

Replace a vertex with a graph, making it a module:



Theorem (Chudnovsky, Penev, Scott, Trotignon, '11)

If $\mathcal{C}$ is (polynomially) χ-bounded, then the closure of $\mathcal{C}$ under substitution is also (polynomially) χ-bounded.

Glueing and Substitution

Question (Chudnovsky, Penev, Scott, Trotignon, '11)

If $\mathcal{C}$ is χ-bounded, is the closure of $\mathcal{C}$ under both substitution and glueing on $\leq k$ vertices also χ-bounded?

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Very special case:
Let $\mathcal{C}$ be the class obtained, starting with an edge, by

- making non-adjacent twins ($=$ substitution by a stable set of size 2),
- and glueing on 2 non-ajacent vertices,

$\mathcal{C}$ is triangle free. Is it χ-bounded?

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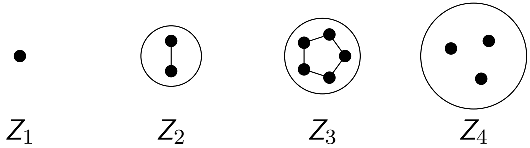
Remark

- $\mathcal{C}$ is closed under induced subgraphs
- $\mathcal{C}$ does not contain the cube

Zykov graphs

Z_k: add a vertex adjacent to each transversal of $Z_1, \ldots, Z_{k-1}$



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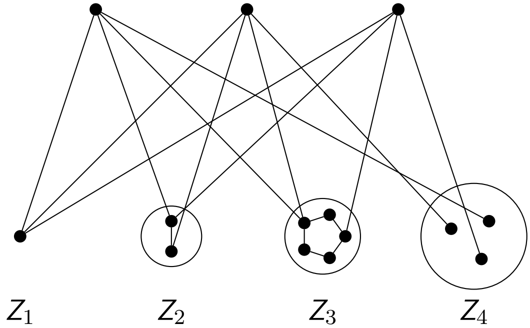

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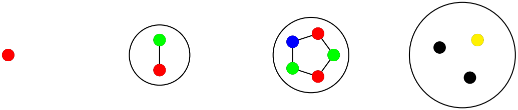

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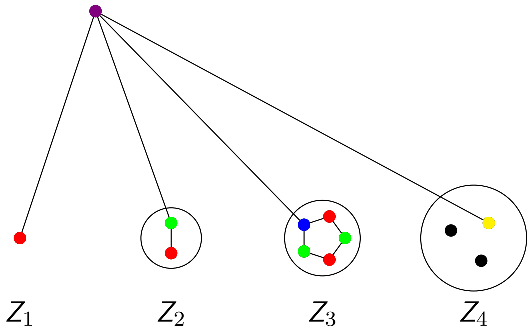

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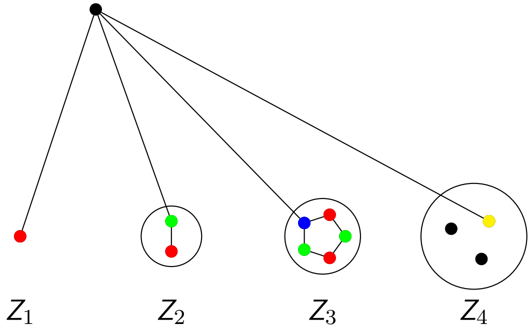
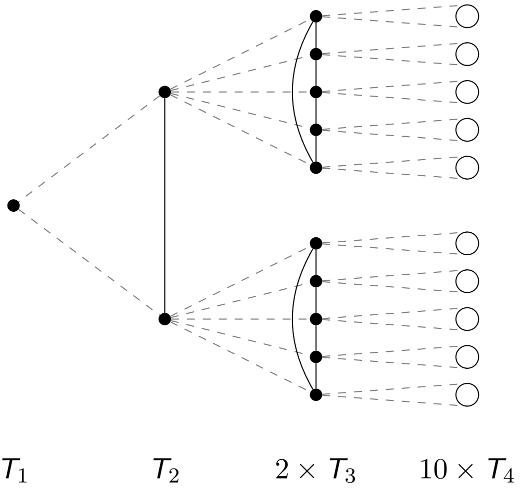


Zykov graphs induce all bipartite graphs.

Twincut graphs



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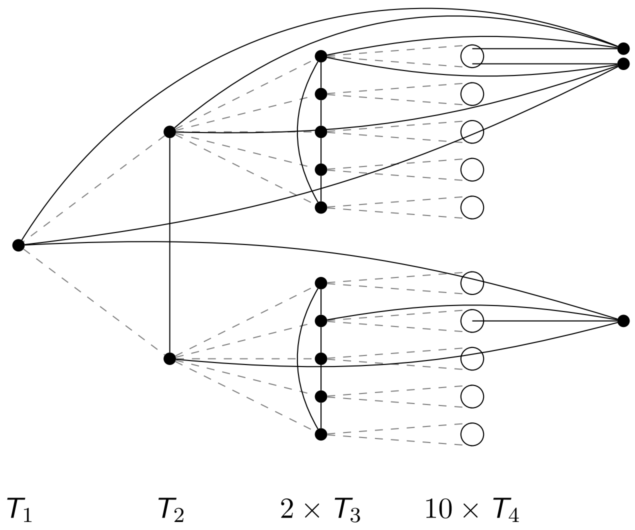

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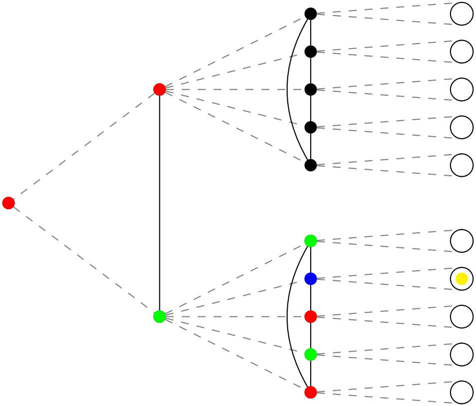
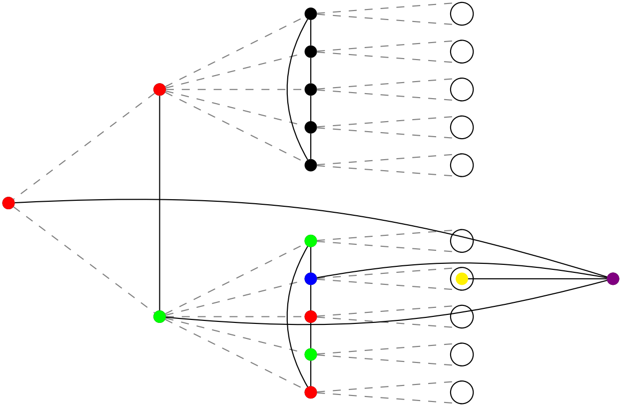


T_1 T_2 $2 \times T_3$ $10 \times T_4$

Twincut graphs



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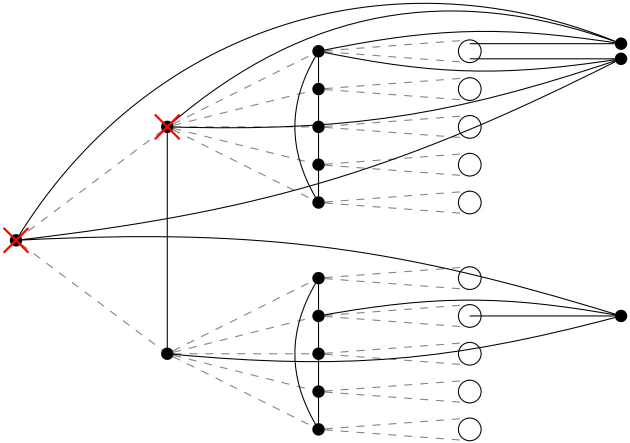

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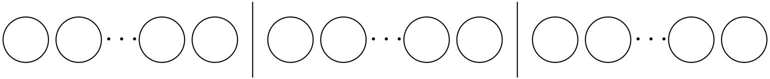
Bad orientation

$k+1$ vertices per transversal



$k(k+1) \times \vec{Z}_k$

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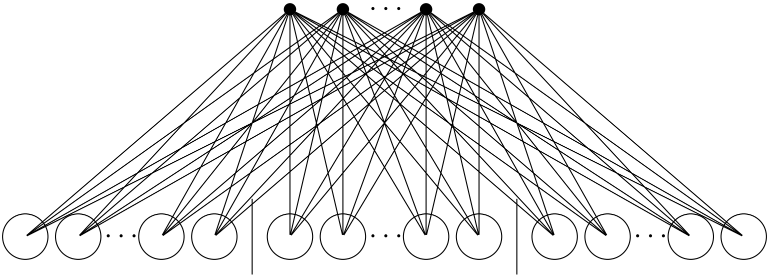

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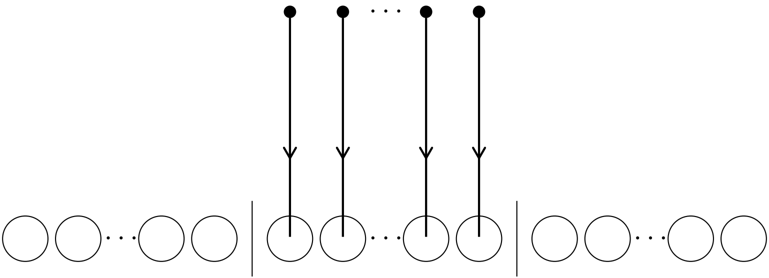

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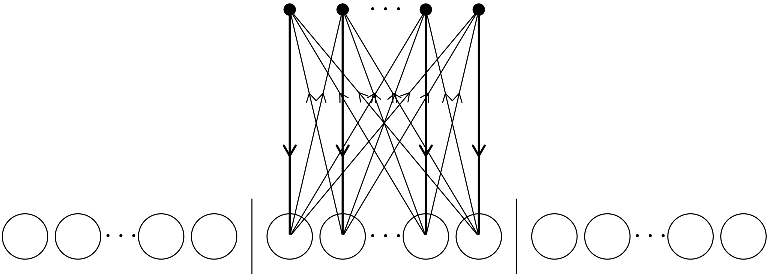

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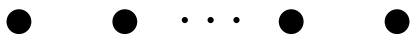

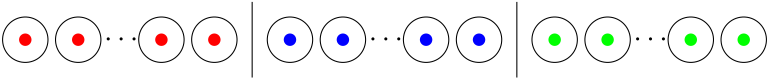

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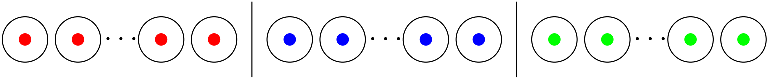

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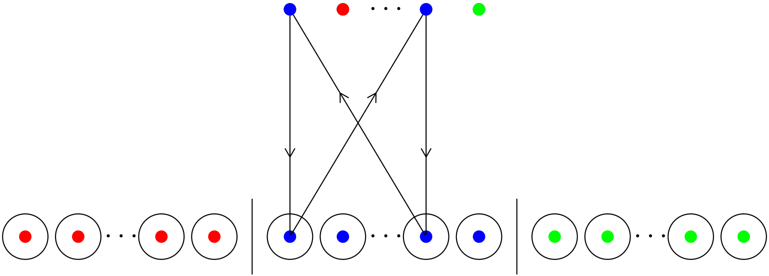

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