

Mobile Ambients

Mobility

- name passing in the π -calculus models dynamically evolving network topology

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(*Higher-Order π -calculus, $HO\pi$*)
- **Mobile Ambients**: a model where term mobility is a primitive

From π -calculus to Mobile Ambients

$$\boxed{\pi} \quad P = \mathbf{0} \mid a(x).P \mid \bar{a}\langle x \rangle.P \mid (P_1 \mid P_2) \mid !P \mid (\nu a)P \mid P + Q$$

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+ *localities (ambients):*
spatial structure

$$\boxed{n[P]}$$

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constructs for *space*

+ localities (*ambients*):

$n[P]$

spatial structure

+ actions: *capabilities*

$M.P$

make the spatial structure evolve

Mobile Ambients – [Cardelli Gordon 1998]

$$P ::= \mathbf{0} \mid a[P] \mid P_1|P_2 \mid !P \mid (\nu x) P \mid M.P$$
$$M ::= \text{in } a \mid \text{out } a \mid \text{open } a$$

Reduction, movement

$$\overset{a}{\boxed{P|Q}} \mid \overset{b}{\boxed{R}} \longrightarrow \overset{b}{\boxed{\overset{a}{\boxed{P|Q}} \mid R}}$$

$$a[\text{in } b.P \mid Q] \mid b[R] \longrightarrow b[a[P|Q] \mid R]$$

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$$a[b[\text{out } a.P \mid Q] \mid R] \longrightarrow b[P|Q] \mid a[R]$$

Remark: movements are *subjective*

Reduction, opening

$$a \boxed{P} \mid Q \longrightarrow P \mid Q$$

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$$a[P] \mid \text{open } a.Q \longrightarrow P \mid Q$$

- ▷ opening is “objective”
- ▷ an ambient disappears, its content is thrown in its enclosing ambient (capabilities act on a different locality)

Where does non-determinism come from in Mobile Ambients?

$$\begin{array}{lll} a[\text{in } b.P \mid Q] \mid b[R] & \rightarrow & b[a[P|Q] \mid R] & \text{in} \\ a[b[\text{out } a.P \mid Q] \mid R] & \rightarrow & b[P|Q] \mid a[R] & \text{out} \\ a[P] \mid \text{open } a.Q & \rightarrow & P|Q & \text{open} \end{array}$$

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$$a[P] \mid b[\text{in } a.Q \mid Q'] \mid a[R]$$

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$$a[b[\text{in } b'.P \mid \text{out } a.P'] \mid b'[Q]]$$

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- structural congruence rules:

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- c is a secret that is carried along

Firewall

- $n[P]$ a firewall named n enclosing P
- capabilities $\text{in } n, \text{out } n$ are needed in order to cross the firewall
- the context = the network

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Example: an agent $Agent = k'[\text{open } k.k''[B]]$,
the firewall $Firewall = w[k[\text{out } w.\text{in } k'.\text{in } w.0]|\text{open } k'.\text{open } k''.A]$

$$(\nu w)(Agent \mid Firewall) \approx (\nu w)w[B \mid A]$$

$(\nu w)(w[k[\text{out } w.\text{in } k'.\text{in } w.\mathbf{0}]|\text{open } k'.\text{open } k''.A]) \mid k'[\text{open } k.k''[B]]$

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\downarrow \\
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& (\nu w)w[A \mid B]
\end{aligned}$$

Locks and sums

$acquire\ l.P \stackrel{\text{def}}{=} open\ l.P$ $release\ l.P \stackrel{\text{def}}{=} l[] \mid P$

handshake: $acquire\ n.release\ m.P \mid release\ n.acquire\ m.Q$

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$$\text{handshake: } \text{acquire } n.\text{release } m.P \mid \text{release } n.\text{acquire } m.Q$$

$$\llbracket n \Rightarrow P + m \Rightarrow Q \rrbracket \stackrel{\text{def}}{=} (\nu p, q, r) (\\ p[\text{in } n.\text{out } n.q[\text{out } p.\text{open } r.P]] \mid \\ p[\text{in } m.\text{out } m.q[\text{out } p.\text{open } r.Q]] \mid \\ \text{open } q \mid r[])$$

$$\llbracket n \Rightarrow P + m \Rightarrow Q \rrbracket \mid n[R] \rightarrow^* \approx P \mid n[R]$$

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- the model is “very distributed”
- ▷ movement every where, at any level (except under capabilities)
- ▷ autonomous movement: no synchronisation

Mobile Ambients are Turing complete

- ribbon cells: nested Ambients
“leftmost and rightmost cells”: Ambients to extend the ribbon
- every cell contains an Ambient that represents its state
- head of the machine: in **go right**, out **go left**

Communications

what we have been discussing are *Pure Mobile Ambients*
let us now put some π -calculus into the language

Messages: $\langle M \rangle$ Abstractions: $(x).P$

Reduction: $\langle M \rangle \mid (x).P \rightarrow P_{\{x:=M\}}$

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- ▶ *local communications* (no medium)
- ▶ communication of *capabilities* in a broad sense
(name, capability, path)

Well-formed terms

$$P \stackrel{\text{def}}{=} (\nu n) P \mid \mathbf{0} \mid P_1 | P_2 \mid !P \mid M[P] \mid M.P \mid (x).P \mid \langle M \rangle$$
$$M \stackrel{\text{def}}{=} x \mid n \mid \text{in } M \mid \text{out } M \mid \text{open } M \mid \epsilon \mid M_1.M_2$$

- variables (x) stand for names and capabilities
- badly formed terms: $n.P$, $(\text{in } n)[P]$
- a simple type system can be defined to rule out badly formed terms

π in MA

$$\begin{aligned} \llbracket (\nu n) P \rrbracket &\stackrel{\text{def}}{=} (\nu n) (n[!\text{open } io] \mid \llbracket P \rrbracket) \\ \llbracket n(x).P \rrbracket &\stackrel{\text{def}}{=} (\nu p) (io[\text{in } n.(x).p[\text{out } n.\llbracket P \rrbracket]] \mid \text{open } p) \\ \llbracket \bar{n}m \rrbracket &\stackrel{\text{def}}{=} io[\text{in } n.\langle m \rangle] \\ \llbracket P \mid Q \rrbracket &\stackrel{\text{def}}{=} \llbracket P \rrbracket \mid \llbracket Q \rrbracket \quad \llbracket !P \rrbracket \stackrel{\text{def}}{=} !\llbracket P \rrbracket \end{aligned}$$

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→ *remote inputs and outputs*

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Mobile Ambients in Pure Mobile Ambients?

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Mobile Ambients in Pure Mobile Ambients?
- similarly:
are recursive definitions and replication equivalent in MA?

A young calculus

- since the 1998 paper “*Mobile Ambients*”, there have been numerous criticisms/counterproposals

the calculus, as it is, is:

- ▶ very difficult to implement

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but then, what about a labelled transition system?