Course on Mobility

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- focus on the π-calculus: a calculus of mobile processes based on *naming* (cf. R. Milner, Turing award lecture)

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- outline:
  π: definition - types - λ in π - behavioural equivalences

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R. Milner, *Communication and Concurrency*, Prentice Hall

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- notes for the course:
  not a tutorial, more to be used as a reference with the slides

Names and Processes

- nominal calculus:
  an infinite set of *names* (*channels, links, ports*)

$$a, b, \ldots, p, q, r, \ldots, x, y, \ldots$$

- we define *terms* (*processes*)

$$A, B, \ldots, P, Q, \ldots$$

Interaction, reduction, communication

$$P \quad = \quad \overline{a}\langle v\rangle.b(x).\mathbf{0} \quad | \quad a(y).\left(\overline{c}\langle y\rangle.\mathbf{0} \mid \overline{d}\langle y\rangle.\mathbf{0}\right)$$

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competition for a resource:

$$Q \quad = \quad a(x).Q_1 \mid a(x).Q_2 \mid \overline{a}\langle v\rangle.\mathbf{0}$$

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non confluence

A single entity: names

- prefixes:

$$a(b).\ \text{reception}, \quad \overline{a}\langle b\rangle.\ \text{emission} \qquad \begin{cases} a\text{: } \textit{subject} \\ b\text{: } \textit{object} \end{cases}$$

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  - ▷ substitution of a name with a name ($\neq \lambda$)

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- notation: $\overline{a}\langle b\rangle.\mathbf{0}$ is often written $\overline{a}\langle b\rangle$

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$$\overline{a}\langle c\rangle.\overline{c}\langle v\rangle.\mathbf{0}$$

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$$\overline{a}\langle c\rangle.\overline{c}\langle v\rangle.\mathbf{0} \mid a(x).x(t).\overline{r}\langle t\rangle.\mathbf{0}$$

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this is the way we reason on π-calculus terms

λ versus π

λ: functions that are applied to their arguments (β-reduction)
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π: a term *in a context*

λ: several kinds of reduction

- ▷ strategies (call-by-name, call-by-value,. . .)
- ▷ computing everywhere in the term (rule ξ)

π: reduction only "at top-level", *non deterministically*

Exercise: matching

- some π-calculi include a matching operator:
  $[n = m]\, P$ behaves like P if $n = m$, is stuck otherwise

examples:

▷ $a(x).b(y).[x = y]\, \overline{c}\langle x \rangle$ forwards a name if received twice

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- is matching encodable in a π-calculus without matching operator?

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Remarks:

- $\boldsymbol{\nu}$ is a binder: T is α-equivalent to
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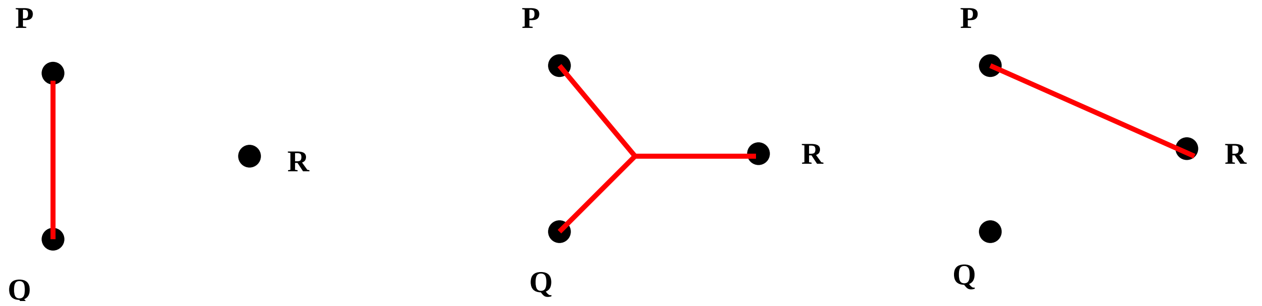
- $\boldsymbol{\nu}$ has greater priority than $|$

Name extrusion

the object of an output is a restricted name

$$(\boldsymbol{\nu} c)\,(P \mid \overline{a}\langle c\rangle.Q) \mid a(x).R \quad\to\quad (\boldsymbol{\nu} c)\,(P \mid Q \mid R_{\{x\leftarrow c\}}) \quad\equiv\quad (\boldsymbol{\nu} c)\,(P|R_{\{x\leftarrow c\}}) \mid Q$$

if $c \notin \mathsf{fn}(Q)$



→ ‘network topology’ is changing along computation

Exercise: localised π

- grammar so far: $P \;::=\; \mathbf{0} \mid P_1 \mid P_2 \mid a(b).P \mid \overline{a}\langle b\rangle.P \mid (\boldsymbol{\nu} n)\, P$

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- localised π: in $a(b).P$, b can only be used in *output*

$\hookrightarrow$ why the name *"localised π"*?

(consider a term of the form $(\boldsymbol{\nu} n)\, P$)

The polyadic π-calculus

- possibility of exchanging *name tuples*:

$$\overline{a}\langle u, v\rangle.P \mid a(x, y).Q \quad \longrightarrow \quad P \mid Q_{\{x,y \leftarrow u,v\}}$$

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- notation:

$a().P$ (resp. $\overline{a}\langle\rangle.P$) is written $a.P$ (resp. $\overline{a}.P$): cf. $\boxed{\text{CCS}}$

Booleans in the polyadic π-calculus

- an *abstraction:* true $\overset{\text{def}}{=} (t, f).\bar{t}$

cf. Milner's tutorial on π, abstractions and concretions

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better $\mapsto$ $\overset{\text{def}}{=} (\boldsymbol{\nu} t)(\boldsymbol{\nu} f)\,\bar{b}\langle t, f\rangle.(t.P \mid f.Q)$

Exercises

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- how can we 'program' the diadic π-calculus in the monadic π-calculus?

$$\overline{a}\langle u, v\rangle.P \mid a(x, y).Q \quad \longrightarrow \quad P \mid Q_{\{x,y \leftarrow u,v\}}$$

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▷ $\overline{a}\langle v\rangle.P \mid !a(x).Q \quad \longrightarrow \quad P \mid Q_{\{x \leftarrow v\}} \mid !a(x).Q$

▷ let $T \overset{\text{def}}{=} !\overline{c}\langle x\rangle \mid !c(y)\,, \qquad T \longrightarrow T$

→ *the replication operator brings* persistence

Replication and persistence

- persistent data

$$\mathsf{true}_b \overset{\mathsf{def}}{=} \,!b(t, f).\bar{t}$$

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- a resource: server for boolean $\vee$

$$!l(b_1,b_2,r).(\boldsymbol{\nu} b)\left(!b(t,f).(\boldsymbol{\nu} f')\left(\overline{b_1}\langle t,f'\rangle \mid f'.\overline{b_2}\langle t,f\rangle\right) \;\mid\; \overline{r}\langle b\rangle \right)$$

The language so far

$$P \quad ::= \quad \mathbf{0} \mid P_1 \,|\, P_2 \mid\, !P \mid a(b).P \mid \overline{a}\langle b\rangle.P \mid (\boldsymbol{\nu} a)\, P$$

this π-calculus is:

- monadic
- synchronous
- with replication

but there exist several other variations/extensions