Comet – exercices 5

Bring your answers to the course on oct. 19th

1 Up to parallel composition: a failed proof attempt

1. Consider the function f, defined by $f(\mathcal{R}) = \{(P|T,\ Q|T),\ P\mathcal{R}Q \text{ and } T \text{ is any process}\}$,

   Write down the proof that f is a compatible function, and explain where it fails. In other words, draw inspiration from the soundness proof seen in the course, show which parts of the proof go through, and which parts fail. You don't need to write a detailed proof, just provide meaningful hints.

2. Same question with $g(\mathcal{R}) = \mathcal{R} \cup \{(P|T,\ Q|T),\ P\mathcal{R}Q \text{ and } T \text{ is any process}\}$. Your answer can be shorter than for the previous question.

2 A distributivity property in the π-calculus

Here is a theorem in the π-calculus:

> For any π-calculus processes P, Q, R, we have
>
> $$(\nu a)\big(\,!a(x).R \mid P \mid Q\,\big) \qquad \sim \qquad (\nu a)\big(\,!a(x).R \mid P\,\big) \;\big|\; (\nu a)\big(\,!a(x).R \mid Q\,\big)$$
>
> if the name a is *used only in output subject* in P, Q and R.
>
> "Used only in output subject" here means that we can have outputs on a, but we cannot have inputs on a, nor send name a on some channel. *The terminology is as follows: a is used in output subject in $\overline{a}b$, in output object in $\overline{c}a$, in input subject in $a(x)$, and in input object in $d(a)$.*

The intution is that the process $!a(x).R$ is used as a resource, which in one case is shared by P and Q (process on the left), while in the other case each of P and Q have their own copy of the resource.

1. Give an example that shows that if a is used once in input subject in P or in Q, then the equivalence above fails. Other potential usages of a in P, Q, R are in output subject.
2. Same question if a is used once in output object in P or Q.
3. Give an example to show that if P and Q use a in output subject, but not R, the equivalence above fails.

In the questions above, you do not need to be overly precise: give your example, and provide a convincing reason as to why the equivalence fails.