

Comet, exercises 6

Bring your answers to next course (Oct 26)

1 Universal Coalgebra

Let $F = 2 \times Id^A$ (i.e., $FX = 2 \times X^A$) be the functor for deterministic automata.

Question 1.1 *Give its action on morphisms (i.e., what is Ff for some $f : X \rightarrow Y$?). Prove that it is a functor.*

Recall that the final coalgebra for this functor is the coalgebra of formal languages on the alphabet A , with derivatives describing the dynamics: $\langle \mathcal{P}(A^*), \langle \epsilon, \delta \rangle \rangle$ with $\epsilon(L) = "\epsilon \in L"$ and $\delta_a(L) = a^{-1}L = \{w \mid aw \in L\}$.

Question 1.2 *Describe the final coalgebra for the functors $B \times Id^A$ and $B \times Id$, where B is an arbitrary set (justify your answers).*