Nested Timed Automata

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Joint work with Xiaojuan Cai, Mizuhito Ogawa and Shoji Yuen.

Motivation

Hybrid automata extend timed automata with various rates of clocks;
We would like to extend timed automata with (time-sensitive) context switches.

- (Recursive) Procedure calls
- Multi-level interrupt handlings

Need to deal with 'local' clocks.

A Usual Automata-Based Program Analysis

```
int lek = 10;
Foo()
int x, y;
⋮
if x > y then Foo();
⋮
```

A Usual Automata-Based Program Analysis

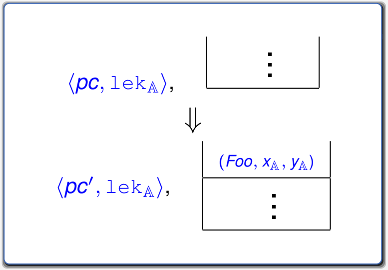
int $lek = 10$;
`Foo()`

int x, y;

⋮

if $x > y$ **then** `Foo();`

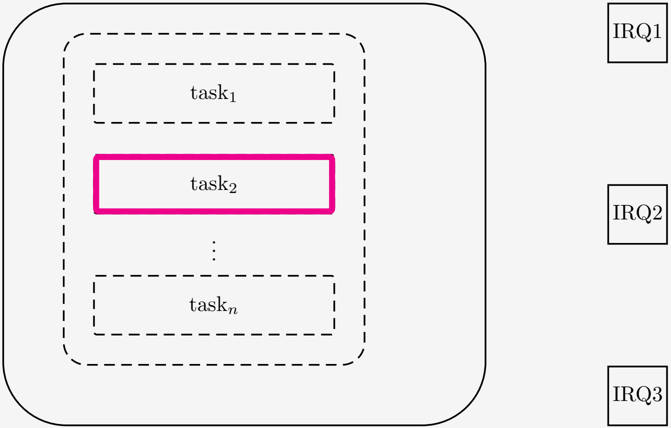
⋮



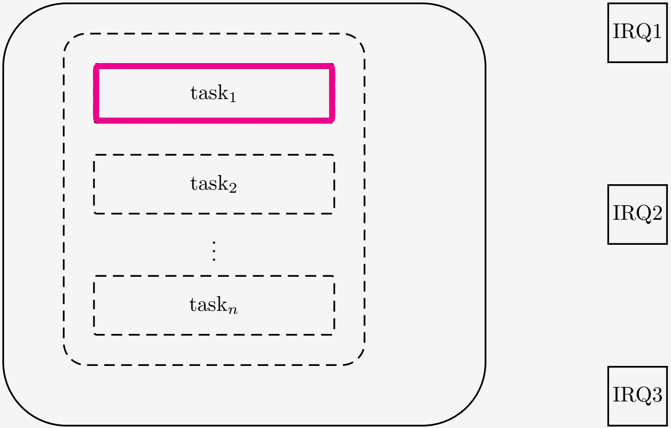
Procedure with Local Clocks

```
Tfoo() {
 clock x, y;
    ⋮
 reset(y);
    ⋮
 if x < 10 && y <= 5 then Tfoo();
 else return;
  ...
}
```

Multilevel (Nested) Interrupts

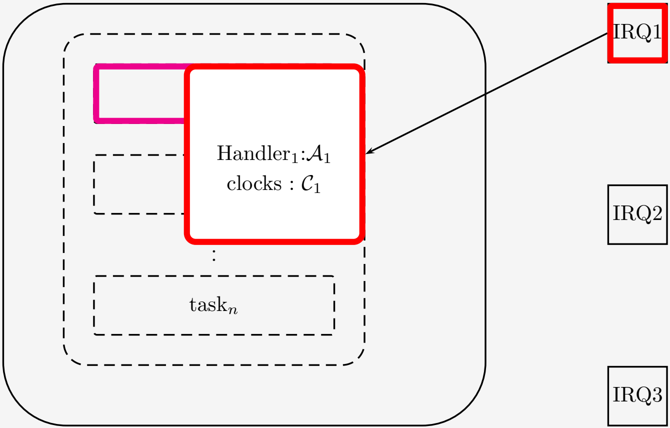


Multilevel (Nested) Interrupts



Multilevel (Nested) Interrupts

- Interrupt handlers override the behavior by $\mathcal{A}_i$.



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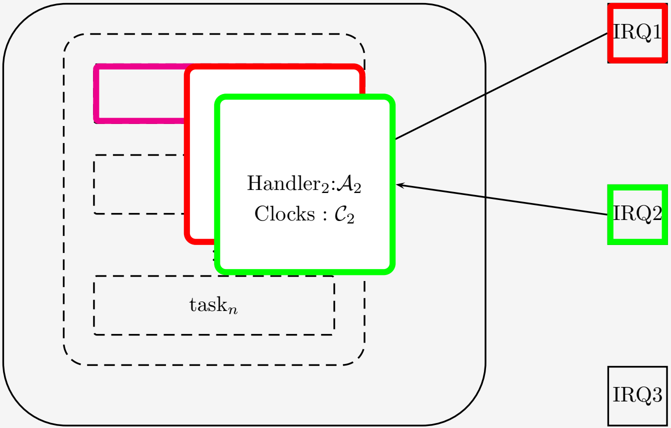

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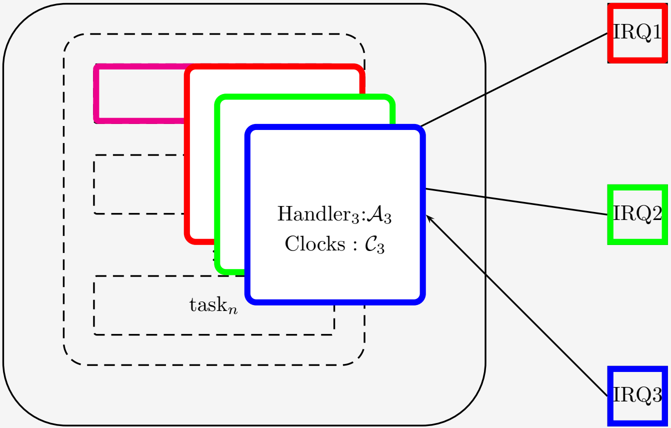
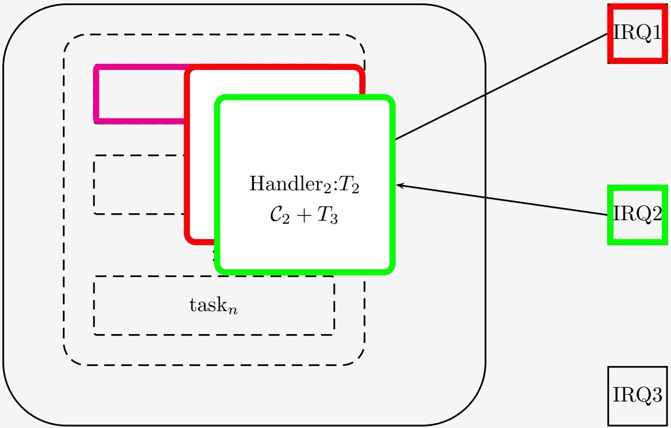


Multilevel (Nested) Interrupts

- The behavior is resumed after the handlers terminate.

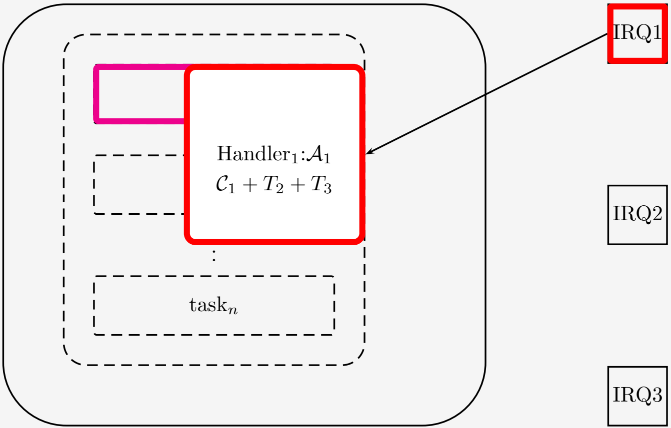
Clock values of $\mathcal{C}_2$ are changed.



Multilevel (Nested) Interrupts

- The behavior is resumed after the handlers terminate.

Clock values of $\mathcal{C}_1$ are changed.



Multilevel (Nested) Interrupts

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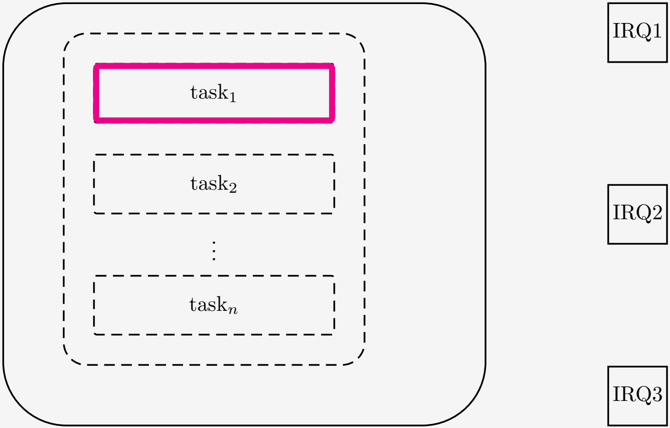


Behavioral Model

- A nested timed automaton is a pushdown system whose stack symbols are *timed automata*.
- It either behaves as the top TA in the stack, or switches from one TA to another by *pushing*, *popping*, and *altering* the top TA.
- When time passage happens, all clocks of these TAs in the stack *elapse uniformly*.

Contents

- Timed Automata
- Nested timed automata (NeTA)
- State reachability is decidable via translation into DTPDA (dense timed pushdown automata [Abdulla et.al. LICS2012])
- Correctness of the translation.
- Conclusion

Timed Automata (TA)

$\mathcal{A} = (Q, q_0, F, X, \Delta)$, where

- Q is a finite set of control locations, with the initial location $q_0 \in Q$,
- $F \subseteq Q$ is the set of final locations,
- X is a finite set of clocks,
- $\Delta \subseteq Q \times \mathcal{O} \times Q$, where $\mathcal{O}$ is a set of operations. A transition $q_1 \xrightarrow{\phi} q_2$, where ϕ is either of

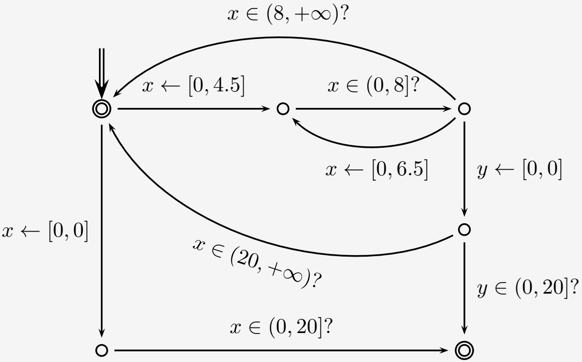
  Local ϵ,

  Test $x \in I?$,

  Assignment $x \leftarrow I$.

Clock updates, Diagnal-free and convex constraints, No invariants

Timed Automata (TA) [An Example]



Nested Timed Automata

Nested Timed Automata

$\mathcal{N} = (T, \mathcal{A}_0, \Delta)$, where

- T is a finite set of TA, with the initial timed automaton $\mathcal{A}_0 \in T$,
- $\Delta \subseteq T \times \mathcal{P} \times (T \cup \{\varepsilon\})$, where $\mathcal{P} = \{push, pop, internal\}$.
- A rule $(\mathcal{A}_i, \Phi, \mathcal{A}_j) \in \Delta$ is written as $\mathcal{A}_i \xrightarrow{\Phi} \mathcal{A}_j$, where

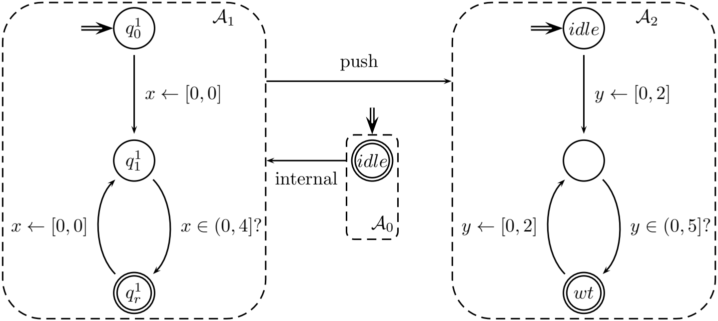
$$\begin{aligned} \text{Push} \quad & \mathcal{A}_i \xrightarrow{push} \mathcal{A}_j, \\ \text{Pop} \quad & \mathcal{A}_i \xrightarrow{pop} \varepsilon, \text{ and} \\ \text{Internal} \quad & \mathcal{A}_i \xrightarrow{internal} \mathcal{A}_j. \end{aligned}$$

Semantics of NeTA

Given an NeTA $(T, \mathcal{A}_0, \Delta)$, a configuration is a stack, and the stack alphabet is a tuple $\langle \mathcal{A}, q, \nu \rangle$, The transition of NeTA is represented as follows:

- Progress transitions: $c \xrightarrow{t}_{\mathcal{N}} c + t$.
- Discrete transitions: $c \xrightarrow{\phi}_{\mathcal{N}} c'$
  - **Intra-action** $\langle \mathcal{A}, q, \nu \rangle c \xrightarrow{\phi}_{\mathcal{N}} \langle \mathcal{A}, q', \nu' \rangle c$
  - **Push** $\langle \mathcal{A}, q, \nu \rangle c \xrightarrow{push}_{\mathcal{N}} \langle \mathcal{A}', q_0(\mathcal{A}'), \nu'_{\mathbf{0}} \rangle \langle \mathcal{A}, q, \nu \rangle c$
  - **Pop** $\langle \mathcal{A}, q, \nu \rangle c \xrightarrow{pop}_{\mathcal{N}} c$ if $q \in F(\mathcal{A})$.
  - **Inter-action** $\langle \mathcal{A}, q, \nu \rangle c \xrightarrow{internal}_{\mathcal{N}} \langle \mathcal{A}', q_0(\mathcal{A}'), \nu'_{\mathbf{0}} \rangle c$ if $q \in F(\mathcal{A})$.

Nested Timed Automata [An Example]



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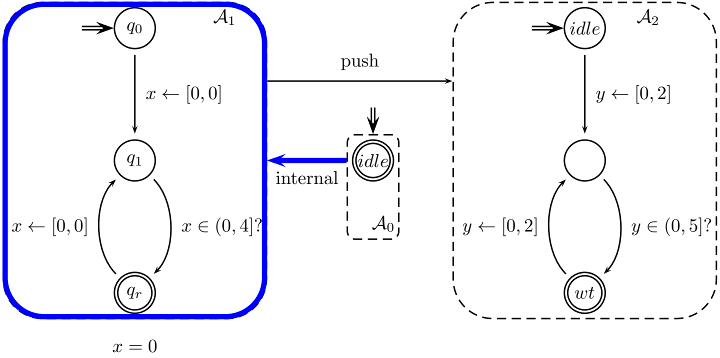

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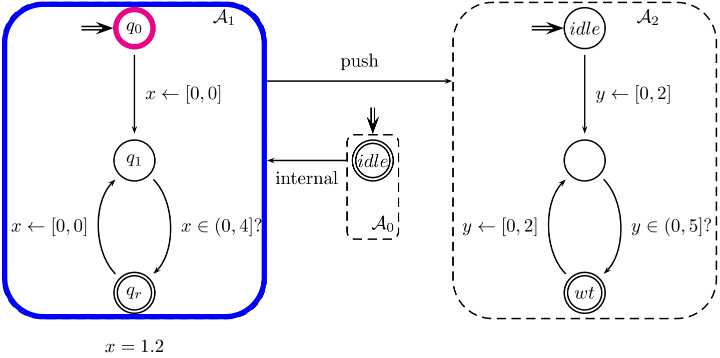

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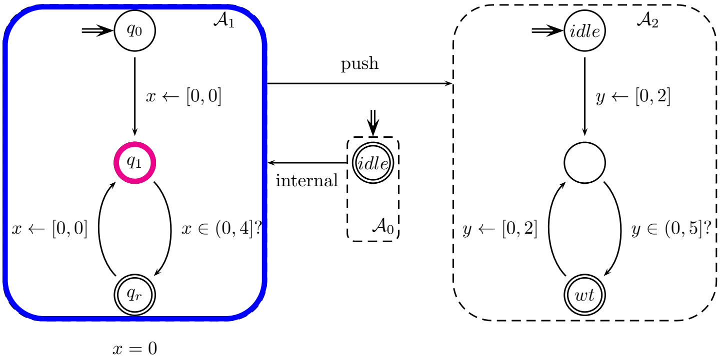

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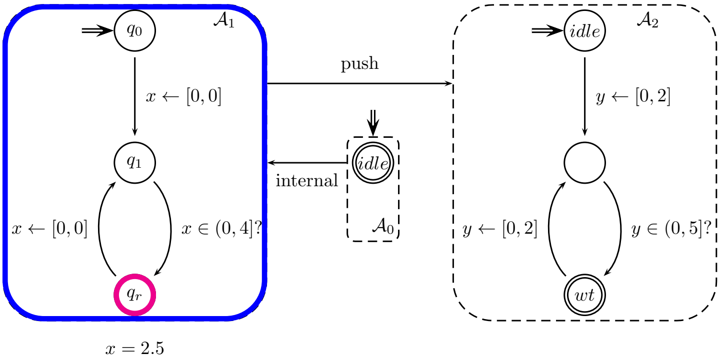

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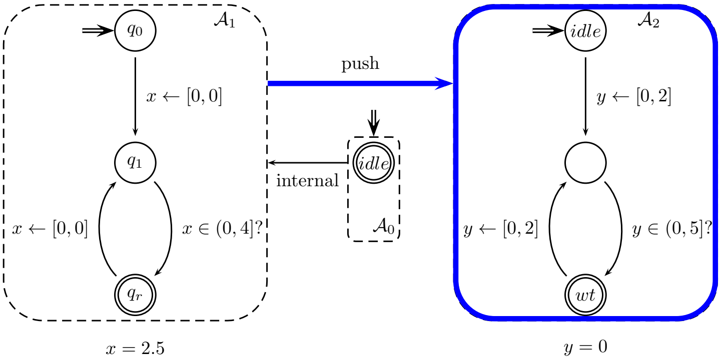

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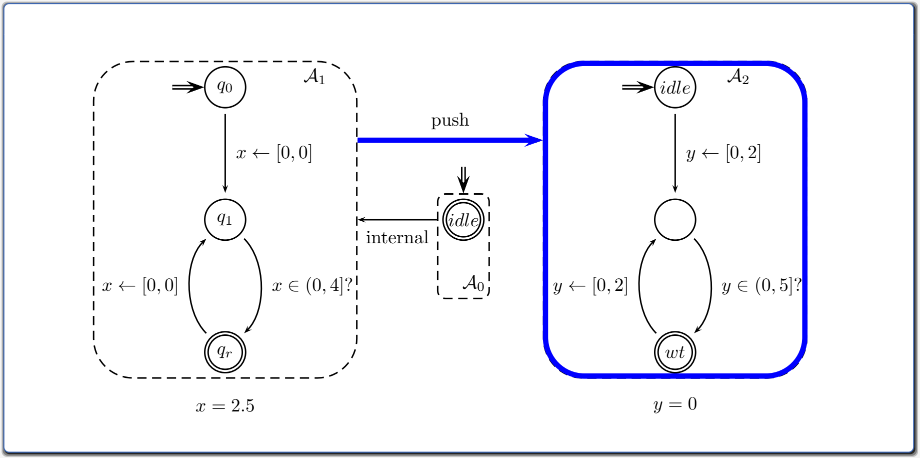

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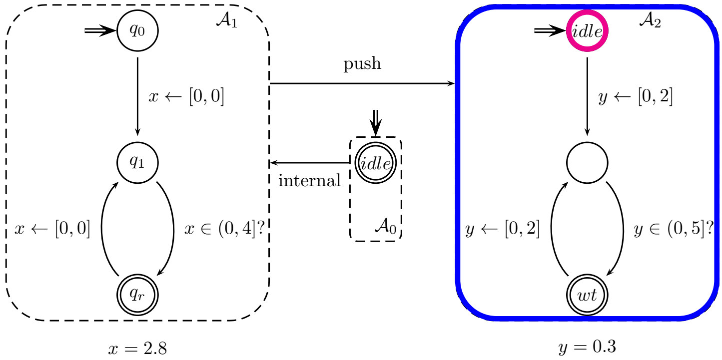

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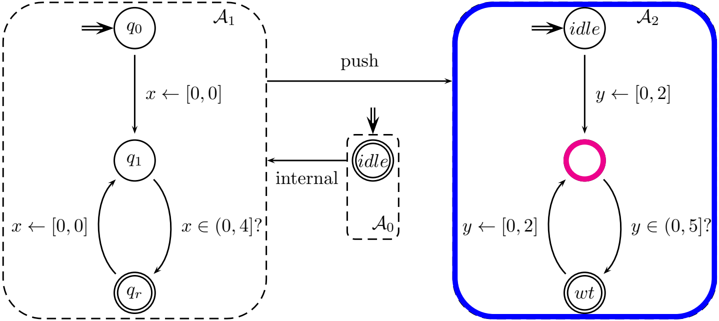
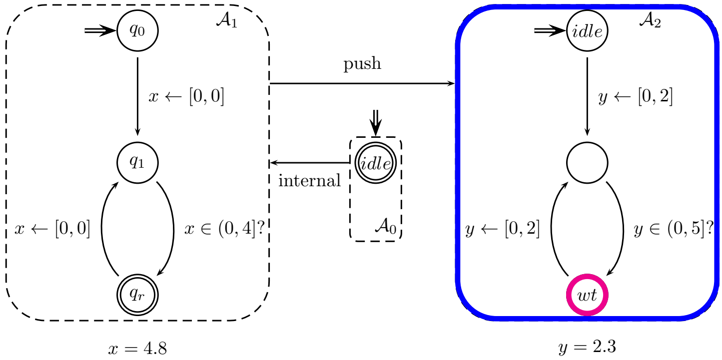


$x = 3.5$

$y = 1$

Nested Timed Automata [An Example]



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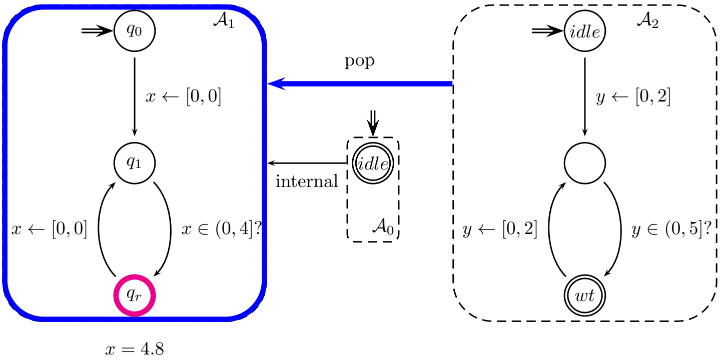

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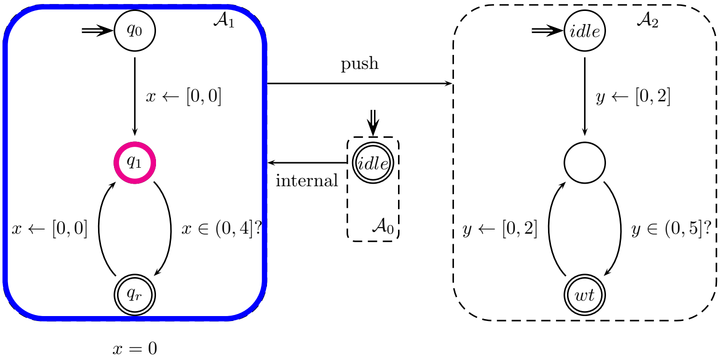
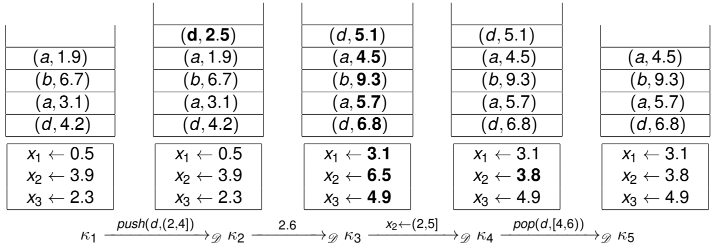


TRANSLATION TO DTPDA

- State: $S = \{\bullet\}$
- clocks: $\mathcal{C} = \{x_1, x_2, x_3\}$,
- Stack symbols: $\Gamma = \{a, b, d\}$



Dense Timed PDA [Abdulla et.al. 2012]

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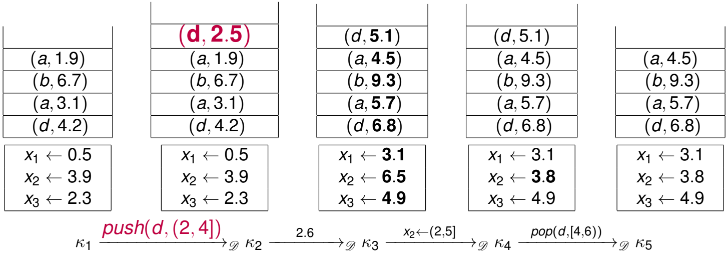

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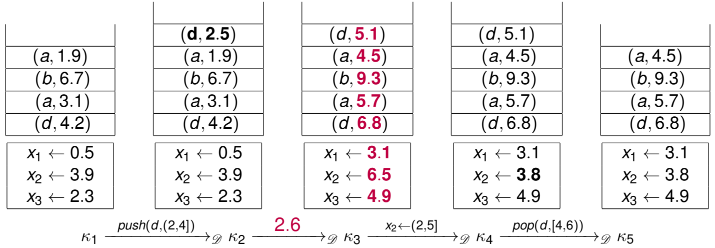

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- State: $S = \{\bullet\}$
- clocks: $\mathcal{C} = \{x_1, x_2, x_3\}$,
- Stack symbols: $\Gamma = \{a, b, d\}$

κ_1	κ_2	κ_3	κ_4	κ_5
	$(\mathbf{d}, \mathbf{2.5})$	$(d, \mathbf{5.1})$	$(d, 5.1)$	
$(a, 1.9)$	$(a, 1.9)$	$(a, \mathbf{4.5})$	$(a, 4.5)$	$(a, 4.5)$
$(b, 6.7)$	$(b, 6.7)$	$(b, \mathbf{9.3})$	$(b, 9.3)$	$(b, 9.3)$
$(a, 3.1)$	$(a, 3.1)$	$(a, \mathbf{5.7})$	$(a, 5.7)$	$(a, 5.7)$
$(d, 4.2)$	$(d, 4.2)$	$(d, \mathbf{6.8})$	$(d, 6.8)$	$(d, 6.8)$
$x_1 \leftarrow 0.5$	$x_1 \leftarrow 0.5$	$x_1 \leftarrow \mathbf{3.1}$	$x_1 \leftarrow 3.1$	$x_1 \leftarrow 3.1$
$x_2 \leftarrow 3.9$	$x_2 \leftarrow 3.9$	$x_2 \leftarrow \mathbf{6.5}$	$x_2 \leftarrow \mathbf{3.8}$	$x_2 \leftarrow 3.8$
$x_3 \leftarrow 2.3$	$x_3 \leftarrow 2.3$	$x_3 \leftarrow \mathbf{4.9}$	$x_3 \leftarrow 4.9$	$x_3 \leftarrow 4.9$

$$\kappa_1 \xrightarrow{push(d,(2,4])}_{\mathscr{D}} \kappa_2 \xrightarrow{2.6}_{\mathscr{D}} \kappa_3 \xrightarrow{x_2 \leftarrow (2,5]}_{\mathscr{D}} \kappa_4 \xrightarrow{pop(d,[4,6))}_{\mathscr{D}} \kappa_5$$

Dense Timed PDA [Abdulla et.al. 2012]

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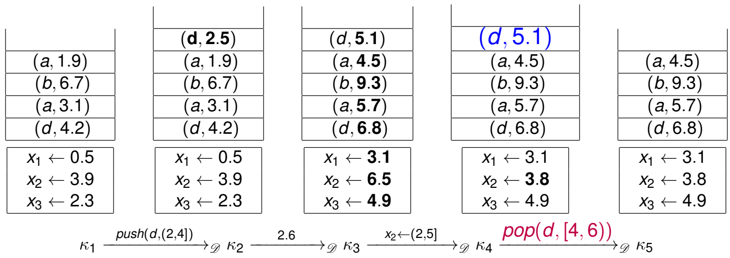

DTPDA

$\mathcal{D} = \langle S, s_0, \Gamma, C, \Delta \rangle$, where

- S is a finite set of states with the initial state $s_0 \in S$,
- Γ is a finite stack alphabet,
- C is a finite set of clocks, and
- $\Delta \subseteq S \times \mathcal{O} \times S$ is a finite set of transitions.

A transition $s_1 \xrightarrow{\phi} s_2$, where ϕ is either of

Local:	ϵ		
Test:	$x \in I?$	Assignment:	$x \leftarrow I$
Push:	$push(\gamma, I)$	Pop:	$pop(\gamma, I)$

A Variation of DTPDA for Encoding NeTA

Push *push*(γ, I) pushes γ to the top of the stack, with the age in the interval I.

Pop *pop*(γ, I) pops the top-most stack symbol provided that this symbol is γ and its age belongs to I.

Push$_A$ *push*(γ, x) pushes γ to the stack associated with a local age with the value of the x's value.

Pop$_A$ *pop*(γ, x) pops γ from a stack and assigns value of its local age to the global clock x.

An Example of the DTPDA Variant

- State: $S = \{\bullet\}$
- clocks: $\mathcal{C} = \{x_1, x_2, x_3\}$,
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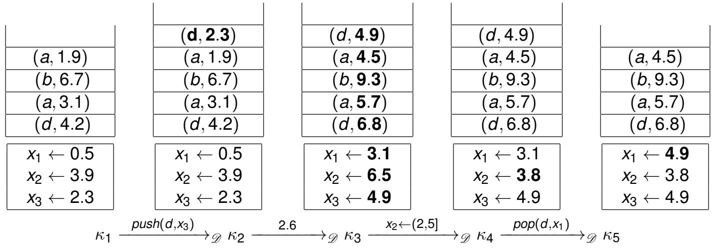

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	$(d, 2.3)$	$(d, 4.9)$	$(d, 4.9)$	
$(a, 1.9)$	$(a, 1.9)$	$(a, 4.5)$	$(a, 4.5)$	$(a, 4.5)$
$(b, 6.7)$	$(b, 6.7)$	$(b, 9.3)$	$(b, 9.3)$	$(b, 9.3)$
$(a, 3.1)$	$(a, 3.1)$	$(a, 5.7)$	$(a, 5.7)$	$(a, 5.7)$
$(d, 4.2)$	$(d, 4.2)$	$(d, 6.8)$	$(d, 6.8)$	$(d, 6.8)$
$x_1 \leftarrow 0.5$	$x_1 \leftarrow 0.5$	$x_1 \leftarrow 3.1$	$x_1 \leftarrow 3.1$	$x_1 \leftarrow 4.9$
$x_2 \leftarrow 3.9$	$x_2 \leftarrow 3.9$	$x_2 \leftarrow 6.5$	$x_2 \leftarrow 3.8$	$x_2 \leftarrow 3.8$
$x_3 \leftarrow 2.3$	$x_3 \leftarrow 2.3$	$x_3 \leftarrow 4.9$	$x_3 \leftarrow 4.9$	$x_3 \leftarrow 4.9$

$$\kappa_1 \xrightarrow{push(d, x_3)}_{\mathscr{D}} \kappa_2 \xrightarrow{2.6}_{\mathscr{D}} \kappa_3 \xrightarrow{x_2 \leftarrow (2,5]}_{\mathscr{D}} \kappa_4 \xrightarrow{pop(d, x_1)}_{\mathscr{D}} \kappa_5$$

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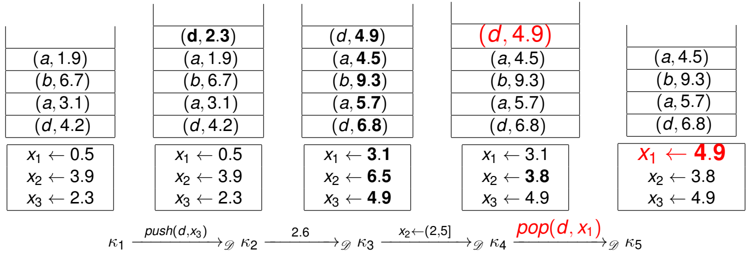

State Reachability of DTPDA

State reachability

s is reachabile if for some w' and ν',
$\langle s_0, w, \nu\rangle \rightarrow^* \langle s, w', \nu'\rangle$
where $\rightarrow = \xrightarrow{\text{Local}} \cup \xrightarrow{\text{Test}} \cup \xrightarrow{\text{Assignment}} \cup \xrightarrow{\text{Push}} \cup \xrightarrow{\text{Pop}}$

Theorem

The state reachability of DTPDA is decidable.

[Abdulla et.al. LICS2012]
Region construction by fractional parts of ages
Rotation at poping with shadow variables consistency

State Reachability of DTPDA Variant

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Theorem

The state reachability of DTPDA variant is decidable.

Small modification to LICS2012 proof works for the variant.

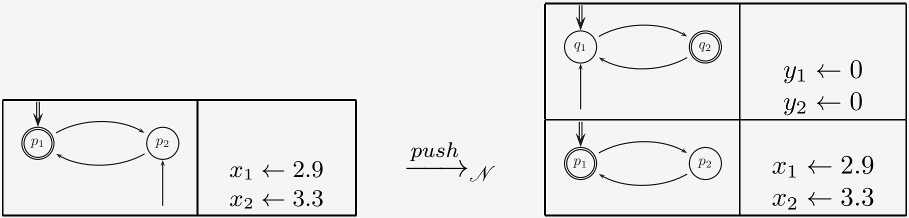
Another proof idea via WSPDS: [Cai et.al. 2013, 2014]

Encoding to DTPDA Variant

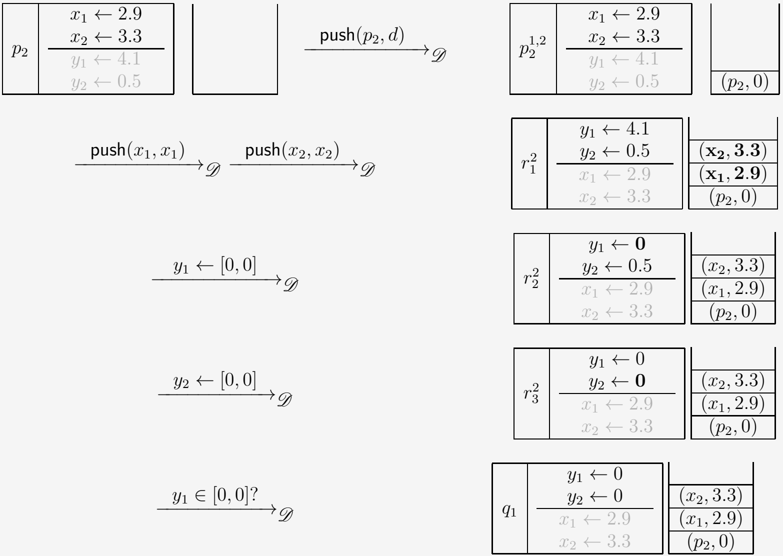
$\mathcal{A}_i \xrightarrow{push} \mathcal{A}_j$	$q_i \xrightarrow{push(q_i,d)} p_1^{i,j} \xrightarrow{push(x_1^i,x_1^i)} \cdots p_{m_i}^{i,j} \xrightarrow{push(x_{m_i}^i,x_{m_i}^i)} p_{m_i+1}^{i,j} \xrightarrow{\epsilon}$ $r_1^j \xrightarrow{x_1^j \leftarrow [0,0]} r_2^j \cdots r_{m_j+1}^j \xrightarrow{\epsilon} t^j \xrightarrow{x_1^j \in [0,0]?} q_0(\mathcal{A}_j)$
$\mathcal{A}_i \xrightarrow{pop} \epsilon$	$q_i \xrightarrow{\epsilon} o \xrightarrow{pop(x_{m_i+1}^i,x_{m_i+1}^i)} \cdots \xrightarrow{pop(x_1^i,x_1^i)} o \xrightarrow{pop(q,d)} q$
$\mathcal{A}_i \xrightarrow{internal} \mathcal{A}_j$	$q_i \xrightarrow{\epsilon} r_1^j \xrightarrow{x_1^j \leftarrow [0,0]} r_2^j \cdots r_{m_j+1}^j \xrightarrow{\epsilon} t^j \xrightarrow{x_1^j \in [0,0]?} q_0(\mathcal{A}_j)$

- The key of the encoding is to synchronize the initial value of clocks, and
- Storing and restoring clocks values simultaneously when timed context switches.

Encoding Push of NeTA to DTPDA



Encoding Push of NeTA to DTPDA



Correctness

Lemma

Given an NeTA $\mathcal{N}$, its encoding $\mathcal{E}(\mathcal{N})$, and configurations c, c' of $\mathcal{N}$.

- **(Preservation)** *if* $c \longrightarrow c'$*, then* $[\![c]\!] \hookrightarrow^* [\![c']\!]$*;*
- **(Reflection)** *if* $[\![c]\!] \hookrightarrow^* \kappa$*,*
  1. *there exists* c' *such that* $\kappa = [\![c']\!]$ *and* $c \longrightarrow^* c'$*, or*
  2. κ *is not an encoded configuration, and there exists* c' *such that* $\kappa \hookrightarrow^* [\![c']\!]$ *by discrete transitions and* $c \longrightarrow^* c'$*.*

Correctness

Lemma

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- **(Preservation)** *if* $c \longrightarrow c'$, *then* $[\![c]\!] \hookrightarrow^* [\![c']\!]$;
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  1. *there exists* c' *such that* $\kappa = [\![c']\!]$ *and* $c \longrightarrow^* c'$, *or*
  2. κ *is not an encoded configuration, and there exists* c' *such that* $\kappa \hookrightarrow^* [\![c']\!]$ *by discrete transitions and* $c \longrightarrow^* c'$.

Theorem

The state reachability problem of NeTA is decidable.

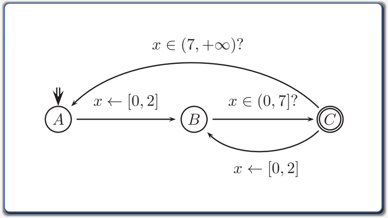
Deadline Analysis for multilevel interrupt handling

Deadline Analysis

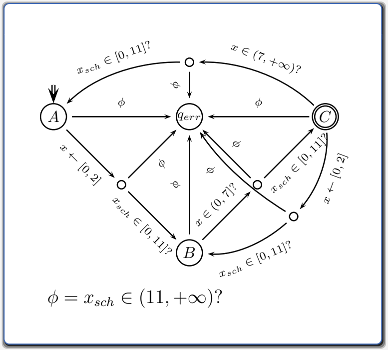
- Interrupt handlers as guarded timed automata.
- Interrupt request as $push_A$ operation.
- Return from interrupt as pop_A operation.
- Deadline violation as the reachability to *err* state.

Timed Automata with Deadline

Add a stopwatch to check deadline and an error state



$\Rightarrow$
Deadline
=11



Fall into q_{err} when deadline passed.

Deadline Analysis

$$\textit{Handler}_i \quad : \quad \text{guard}(\mathcal{A}_i, d_i)$$

guard($\mathcal{A}_i, d_i$) adds the deadline to $\mathcal{A}$

$$\textit{Interrupt} \quad : \quad \mathcal{A}_i \xrightarrow{\text{push}} \mathcal{A}_|$$

A_i may interrupt A_j

$$\textit{Initial} \quad : \quad \mathcal{A}_0 = \textit{Task}_1 \mid\mid \textit{Task}_2 \mid\mid \cdots \mid\mid \textit{Task}_n$$

Interrupt fails to be handled if q_{err} in some $\mathcal{A}_i (i > 0)$ is reachabile.

Ongoing Work

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- Reachability problem of NeTA with invariant is positive.

Future Implementation

- Develop a tool based on a restrictive class such that a pop action occurs only with an integer-valued age.
- This subclass can be encoded into UTPDA (without local age).
- Encode UTPDA to weighted pushdown system to gain the efficiency.

Related Work

- Timed PDA [Bouajjani et. al.,1994]
  PDA with Global clocks
- Timed recursive state machines [Benerecetti et.al, 2010]
  Extended PDA with two stacks for states and clocks
- Recursive timed automata [Trivedi et.al., 2010]
  Local clocks stop
- Hierarchical timed automata [David et.al., 2001]
  Static hierarchy

Conclusion

- An NeTA is a pushdown system with a finite set of TA as stack symbols.
- All clocks in the stack elapse uniformly.
- The state reachability is decidable by encoding to DTPDA with an extension of local clock assignment.

Thank you!

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