Chiral Order in Spin Glasses

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Canonical SG

CuMn, AuFe, etc.

Heisenberg system with weak random magnetic anisotropy

Frustration & randomness

Experimentally, thermodynamic SG transition and a SG ordered state has been established.

The true nature of the SG transition and the SG order state? → still at issue

[Canella and Mydosh, 1972]
Model of spin glasses

3D Edwards-Anderson model

\[ H = - \sum_{ij} J_{ij} S_i \cdot S_j \]

\[ S_i = S_{iz} : \text{Ising spin} \quad (\text{FeMnTiO}_3) \]

\[ S_i = (S_{ix}, S_{iy}, S_{iz}) : \text{Heisenberg spin} \quad (\text{canonical SG}) \]

\[ J_{ij} : \text{nearest-neighbor random coupling} \]

with zero mean and variance \( J^2 \)

Gaussian or binary (\( \pm J \))
Numerical results on the 3D EA SG models

• **3D Ising SG**

The existence of a finite-temperature SG transition established in zero field. The nature of the ordered state still under debate.

• **3D isotropic Heisenberg SG**

Earlier studies suggested no finite-$T$ transition

[Olive, Young, Sherrington, ’86, F. Matsubara et al ‘91]

The possibility of a finite-$T$ transition in the *chiral* sector was suggested [H.K., ’92] --- chiral glass state

\[ T_{SG} < T_{CG} \]

Spin-chirality decoupling

Chirality scenario of experimental SG transition [H.K. ’92]
Heisenberg SG possesses a new degree of freedom which is absent in Ising SG

\[ \chi = S_i \cdot S_j \times S_k \]

Unconventional anomalous Hall effect

R-handed

L-handed

\[ \mathbb{Z}_2 \times \text{SO}(3) \]

chirality

spin rotation

scalar chirality
Chirality scenario of SG transition

* Isotropic Heisenberg SG in 3D exhibits a spin-chirality decoupling, with the chiral-glass ordered phase not accompanying the standard SG order.

* Chirality is a hidden order parameter of real SG transitions. Experimental SG transition is a “disguized” chiral-glass transition: The spin is recoupled to the chirality, i.e., mixed into the chirality via the random magnetic anisotropy.
Spin-chirality decoupling in the 3D isotropic Heisenberg SG

How the spin and chiral correlations grow?

\( T_{CG} > T_{SG} \)
Recent controversy on the 3D Heisenberg SG

Due to the progress in the computer ability and simulation technique, significant numerical study now becomes possible for the 3D Heisenberg SG.

Consensus in recent numerical studies:
The 3D Heisenberg SG exhibits a finite-\(T\) transition. However, its nature has still been largely controversial.

Spin & chirality are decoupled or not?

* Yes, decoupling occurs \((T_{SG} < T_{CG})\)
  
  H.K.’98, K.Hukushima & H.K.’00 ’05

* No decoupling \((T_{SG} = T_{CG} > 0)\)
  
  F.Matsubara, T.Shirakura et al,
B.W.Lee & A.P.Young ’03; ’07
I.Campos et al ’06
L.Fernandez et al ‘09

Chirality scenario has been contested!
Our MC simulation on the 3D Gaussian Heisenberg SG

[D.X.Viet and H.K, PRL102, 027202 ('09); PRB80, 064418 ('10)]

* Heat bath and over-relaxation method, combined with the temperature exchange technique
* System as large as $L=32$ equilibrated well below $T_g$
* Sample # large ($\sim 10^3$ samples averaged)

Various independent quantities including the correlation length ratio $\xi/L$, Binder ratio $g$, and the glass order parameter $q^{(2)}$ are calculated,
Spin and chirality correlation length ratio $\xi/L$

--- a quantity most intensively studied

The transition temperature can be estimated from the size and temperature dependence of the dimensionless ratio $\xi/L$

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In reality ...
Correlation length ratios $\xi/L$

$\xi_{CG}/L \approx \frac{1}{T_{CG}}$

$\xi_{SG}/L \approx \frac{1}{T_{SG}}$

$T_{cross} \text{ extrapolated to } L=\infty$

$T_{CG} = 0.145 \pm 0.004$

$T_{SG} = 0.120 \pm 0.006$

$T_{CG} > T_{SG}$ by $\sim 15\%$

[D.X. Viet and H.K., ‘10]
Binder ratio

[D.X. Viet and H.K., ’10]

Growing negative dip consistent with 1-step RSB

chirality

spin

T_{CG}

T_{cross}

T_{SG}

T_{dp}
Critical properties of the chiral-glass transition of the 3D Gaussian Heisenberg SG

Finite-size scaling plots with the leading correction-to-scaling

chiral-glass exponents

\[ \nu_{\text{CG}} = 1.4(2) \]
\[ \eta_{\text{CG}} = 0.6(2) \]

differ from the 3D Ising values!

\[ \nu \sim 2.5 \]
\[ \eta \sim -0.40 \]
Chirality hypothesis [H.K. 1992]

Isotropic (ideal) system

→ “spin-chirality decoupling”

Real system is weakly anisotropic!
Spin is “recoupled” to the chirality due to the weak random magnetic magnetic anisotropy $D$.

$\mathbb{Z}_2$ [chiral] $\times$ $SO(3)$ [spin-rotation]

The chiral-glass transition now appears as the SG transition.

“spin-chirality recoupling”

experimental SG exponents = CG exponents
## Critical properties of canonical SG

<table>
<thead>
<tr>
<th></th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\nu$</th>
<th>$\eta$</th>
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</thead>
<tbody>
<tr>
<td>CuMn &amp; AgMn</td>
<td>1.0±0.1</td>
<td>2.2±0.1</td>
<td>≈ 1.4</td>
<td>≈ 0.4</td>
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<tr>
<td>[de Courtenary et al]</td>
<td>1.0±0.1</td>
<td>2.2±0.2</td>
<td>≈ 1.4</td>
<td>≈ 0.4</td>
</tr>
<tr>
<td>AgMn</td>
<td>0.9±0.2</td>
<td>2.1±0.1</td>
<td>≈ 1.3</td>
<td>≈ 0.4</td>
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<tr>
<td>[Bouchiat]</td>
<td>≈ 1.0</td>
<td>≈ 1.9</td>
<td>≈ 1.3</td>
<td>≈ 0.5</td>
</tr>
<tr>
<td>CuAlMn</td>
<td>0.9±0.15</td>
<td>2.0±0.2</td>
<td>≈ 1.3</td>
<td>≈ 0.4</td>
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<tr>
<td>[Simpsons]</td>
<td>1.0±0.2</td>
<td>2.0±0.2</td>
<td>≈ 1.3</td>
<td>≈ 0.5</td>
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<tr>
<td>PdMn</td>
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<tr>
<td>[Coles and Williams]</td>
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<tr>
<td>AuFe</td>
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<tr>
<td>[Taniguchi &amp; Miyako]</td>
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<tr>
<td>CdCr$<em>{2.08}$In$</em>{2.08}$S$_4$</td>
<td>0.75±0.10</td>
<td>2.3±0.4</td>
<td>≈ 1.3</td>
<td>≈ 0.2</td>
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<tr>
<td>[Vincent et al]</td>
<td></td>
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<tr>
<td>$\pm J$</td>
<td>≈ 0.82</td>
<td>≈ 6.5</td>
<td>2.72±0.08</td>
<td>−0.40±0.04</td>
</tr>
<tr>
<td>Campbell et al.</td>
<td></td>
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<tr>
<td>$\pm J$</td>
<td>≈ 0.77</td>
<td>≈ 5.8</td>
<td>2.45±0.15</td>
<td>−0.375±0.010</td>
</tr>
<tr>
<td>Hasenbusch et al.</td>
<td></td>
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<tr>
<td>Fe$<em>{0.5}$Mn$</em>{0.5}$TiO$_3$</td>
<td>≈ 0.54</td>
<td>4.0±0.3</td>
<td>≈ 1.7</td>
<td>≈ −0.35</td>
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<tr>
<td>[Gunnarsson et al]</td>
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### Chiral (Heisenberg)

$\beta \sim 1$, $\gamma \sim 2$, $\nu \sim 1.4$, $\eta \sim 0.6$
Direct test of the chirality scenario

→ needs to measure the chirality directly

Use an anomalous Hall effect as a probe of chiral order!

Measurements of linear and nonlinear chiral susceptibilities, $X_\chi$ & $X_{\chi_{nl}}$, becomes possible via measurements of Hall coefficient $R_s$.

[G. Tatara & H.K. ’02, H.K. ’03]

$$R_s = \frac{\rho_{xy}}{M} = -A\rho - B\rho^2 - CD \left[ X_\chi + X_{\chi_{nl}} (DM)^2 + \ldots \right]$$
Anomalous Hall coefficient $R_s$ of SG

Kageyama et al (‘03) FeAl (RSG); T. Taniguchi et al (‘04) AuFe
P. Pureur et al (‘04) AuMn;
T. Taniguchi et al (‘06) AuMn

$\delta_{\chi} = 3.3$

$\delta_{\chi} = 1.7$
What is the nature of the chiral-glass ordered state?

Is there an RSB?
If so, what type?
SG ordered state might possess a complex phase-space structure

RSB (replica-symmetry breaking)

Possible multi-valley structure in SG

$q$: overlap

$q = (1/N) \sum_i S_i^{(a)} S_i^{(b)}$

Overlap distribution function: $P(q)$
Overlap distribution of the isotropic model

Chirality

Central peak

\(-q_{EA}\) peak

\(q_{EA}\) peak

Spin

\([T_{SG} < T = 0.133 < T_{CG}]\)

Chiral \(P(q_{\chi})\):

- Central peak
- \(\pm q_{EA}\) peak

\(\rightarrow 1\text{-step-like RSB?}\)

3D Ising SG

[K. Hukushima & HK '05]

[E. Marinari et al, '98]

\(\pm J\)
How to measure $P(q)$ experimentally?

In equilibrium, FDT holds

$$R(t_1, t_2); \text{ response function}$$

$$C(t_1, t_2); \text{ correlation function}$$

$$T; \text{ heat bath temperature}$$

$$R(t_1, t_2) = (1/k_B T) \left[ \frac{dC(t_1, t_2)}{dt_1} \right]$$

In off-equilibrium, FDT does not hold, but there is an off-equilibrium counterpart

$$R(t_1, t_2) = \left( \frac{X(t_1, t_2)}{k_B T} \right) \left[ \frac{dC(t_1, t_2)}{dt_1} \right]$$

In the limit of $t_1, t_2 \to \infty$, $1/k_B T_{\text{eff}}$

$$X(t_1, t_2) \to X(C(t_1, t_2))$$

[L.F. Kugliandolo and J. Kurchan ‘93]

$$P(q) = \frac{d X(q)}{d q}$$

$P(q); \text{ overlap distribution function}$
Susceptibility $\chi(t_1, t_2)$ vs. Correlation $C(t_1, t_2)$

(a) No RSB
(b) full-step RSB
(c) 1-step RSB
(d) combination of (b)+(c)
Off-equilibrium MC simulation of the isotropic 3D Gaussian Heisenberg SG

Chiral autocorrelations $C_{\chi}(t,t_w)$ at $T=0.05$

$q_{EA}$ plateau

quasi-equilibrium regime

exhibits strong superaging!

$log t$

$log t/w$

Chiral EA parameter $q_{EA}$

$log t$

$log t_w$

$q_{EA}$ plateaus

aging regime

superaging

$log t/w$

$T_{CG} \sim 0.15$

$\beta \sim 1$

$tw$: waiting time
Off-equilibrium simulation of the weakly anisotropic 3D $\pm J$ Heisenberg SG

$\chi-C$ plot for the spin

$T_g \sim 0.21 \quad D=0.01$

1-step-like behavior

$T_{eff} \sim 2T_g$

irrespective of $T$

common $T_{eff}$

c.f. 3D Ising SG

$[E. \text{Marinari et al, '00}]$
Experiment on Heisenberg-like SG

\[ \text{CdCr}_{1.7}\text{In}_{0.3}\text{S}_4 \]

[D.Herisson and M.Ocio, ’02]

1-step-like?

\[ T_{\text{eff}} \approx 1.9 T_g \]

(measured at \( T=0.8T_g \))

c.f. Lennard-Jones fluid simulation

[J.-L. Barrat and W. Kob, ’99]

\[ T_{\text{eff}} \approx 1.5T_{mc} \] (irrespective of \( T \))

\( T_{mc} \): mode-coupling temperature
SG (chiral-glass) ordered state of canonical SG might exhibit a one-step-like RSB

**Analogy to molecular glasses**
Dynamical equations describing structural glass are similar to MF SG models exhibiting a 1-step RSB

[T.E. Kirkpatrick, D. Thirumalai, P.G. Wolynes ’87]

1. **Discontinuous 1-step RSB** (discontinuous $q_{EA}$ at $T=T_g$)
   
   $p>2$-spin MF SG, $p>4$ state MF Potts SG
dynamical $T_D$ and static $T_g$ ($T_D > T_g$)

   → **structural glass**

2. **Continuous 1-step RSB** (continuous $q_{EA}$ at $T=T_g$)
   
   $2<p<4$ state MF Potts SG
No dynamical $T_D$

   → **Heisenberg-like SG**
Reference model

1D Heisenberg SG model with a long-range power-law interaction

\[ H = - \sum_{i,j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j \]

\[ J_{ij} = c(\sigma) \frac{\epsilon_{ij}}{\gamma_{ij}} \]

\( \epsilon_{ij} \): Gaussian with zero mean and standard deviation unity

\[ \sigma \cdot d \] correspondence expected

Varying \( \sigma \) of 1D LR model \( \sim \) varying \( d \) of SR modes

\( \sigma \rightarrow 0 \quad \Leftrightarrow \quad d = \infty \)

\( \sigma \text{ large} \quad \Leftrightarrow \quad d = 1 \)

\( \sigma = 2/3 \quad \Leftrightarrow \quad d = 6 \text{ (ucd)} \)

\( \sigma \approx 0.9 \quad \Leftrightarrow \quad d = 3 \)

\( \sigma = 1.0 \quad \Leftrightarrow \quad \text{between } d = 2 \text{ & } d = 3 \)
\[ \sigma = 0.9 \]

[correlation length ratio]

\[ \xi_{\text{CG}} / L \]

spin

chirality

\[ T_{\text{CG}} \]

\[ T_{\text{SG}} \]

\[ T_{\text{cross}} \]

\[ \sigma - T \text{ phase diagram} \]

[D.X. Viet & H.K. PRL 105, 097206 (2010); JPSJ 79, 104708 (2010)]
Another 1D LR model - randomly diluted model

Choose nonzero $J_{ij}$ with probability $z \times p_{ij}$ ($z=6$)

$\sigma = 0.85$

$T_{CG} > T_{SG}$!
Summary

* An intriguing “spin-chirality decoupling” phenomenon occurs in Heisenberg-like SGs.
* SG (chiral-glass) ordered state of canonical SG might exhibit a 1-step-like RSB.
* Chirality scenario might solve the long-standing puzzles of experimental SG ordering.

Chirality might be a missing link in spin glasses ?!