CR15: Logics, Automata and Games for Advanced Verification

Midterm homework

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In all the document, Σ denotes an arbitrary finite alphabet.

Exercise 1

1. Is it decidable whether a NFA recognize a finite language ?
2. Same question with NBA instead of NFA.

Exercise 2
Given an ω-word $w \in \{0,1\}^\omega$, we note $|w|_1$ the number of 1's in w, so $|w|_1 \in \mathbb{N} \cup \{\infty\}$.
If $w = a_0a_1\dots$ and $w' = b_0b_1\dots$ are in $\{0,1\}^\omega$, we note $\langle w, w'\rangle := (a_0,b_0)(a_1,b_1)\dots \in \{0,1\}^2$.

1. Is the language $\{\langle w, w'\rangle \mid |w|_1 = |w'|_1\}$ ω-regular ?
2. Is the language $\{w \mid |w|_1 \text{ is a power of } 2\}$ ω-regular ?

Exercise 3
If $L \subseteq \Sigma^*$ is a language, we define $Sqrt(L) = \{u \mid uu \in L\}$.

1. Show that if L is regular then $Sqrt(L)$ is regular *(hint: use finite monoids)*.
2. Show that if L is FO-definable then $Sqrt(L)$ is FO-definable.
3. Same questions with $Root(L) = \{u \mid u^n \in L \text{ for some } n \in \mathbb{N}\}$.

Exercise 4
We add to MSO on finite words a quantifier $\exists^n$ for each $n \in \mathbb{N}$. The formula $\exists^n x.\varphi(x)$ is true if and only if the number of positions x such that $\varphi(x)$ is true is a multiple of n. Let us call MSO(Mod) this new logic.

1. Show that MSO(Mod) is expressively equivalent to MSO(ModPrime), where only quantifiers $\exists^p$ with p prime are used.
2. Show that MSO(Mod) is expressively equivalent to MSO.

Exercise 5
An NFA $\mathcal{A} = (\Sigma, Q, q_0, F, \Delta)$ is said history-deterministic if there is a function $\tau : \Sigma^* \to Q$ such that for any word $u = a_1a_2\dots a_n \in L(\mathcal{A})$, the sequence $\tau(\epsilon)\tau(a_1)\tau(a_1a_2)\dots\tau(u)$ is an accepting run of $\mathcal{A}$ on u. We will assume that $\mathcal{A}$ is complete, i.e. that for any $(p,a) \in Q \times \Sigma$ there is at least one $q \in Q$ such that $(p,a,q) \in \Delta$, by addink a rejecting sink state if necessary.

1. Show that if an NFA $\mathcal{A}$ is history-deterministic, its strategy τ can be replaced by a strategy $\sigma : Q \times \Sigma \to Q$, needing only the current state and letter to choose the next state, but ignoring the rest of the history. *(hint: associate to each state q the language $L(q)$ accepted from this state).*
2. We informally describe the safety game $G_{\mathcal{A}}$, played on $\mathcal{A} \times \mathcal{A}$, as follows:

Start in position (q_0, q_0), and at each round from (p, q):
-Adam chooses a letter $a \in \Sigma$
-Eve chooses a transition $(p, a, p') \in \Delta$
-Adam chooses a transition $(q, a, q') \in \Delta$
-The game moves to position (p', q')
The safe region for Eve is $\{(p, q) \mid q \in F \Rightarrow p \in F\}$.

- Give a formal description of the game $G_{\mathcal{A}}$ as $(V_{Eve} \cup V_{Adam}, E)$.
- Show that an NFA A is history-deterministic if and only if Eve wins $G_{\mathcal{A}}$.

3. Show that it is in PTIME to decide whether an NFA is history-deterministic
4. Show that if an NFA is history-deterministic, it can be determinized in PTIME.