Erratum

In the papers [Kup11, Kup14], there is an error in the proof translating aperiodic stabilisation monoids to cost LTL.

Indeed, when decomposing words according to a letter $b$, we have to pay a constant attention to the fact that the monoid is ordered. When writing $b$ in a cost LTL formula, we actually capture all elements above $b$. We do take care of the fact that $\neg b$ captures the wanted elements, but the formula $b$ will capture elements that do not coincide with $b$, and in particular elements that are also captured by $\neg b$. Because of this phenomenon, we cannot directly describe the wanted decomposition using LTL formulas, as it is done in the classical case.

It was pointed out by Thomas Colcombet [Col] that even without considering the quantitative extension of cost functions, an ordered version of the equivalence between LTL (or first-order logic) and aperiodic ordered monoids constitutes an interesting open problem. Here is a clear formalization of this problem, that can be rephrased as a conjecture without mentioning monoids, using an ordered version of LTL.

Let $\Sigma$ be an alphabet equipped with a partial order $\leq$. A language $L \subseteq \Sigma^*$ is \textit{upwards-closed} if for any words $u, v \in \Sigma^*$ and letters $a \leq b$, we have $uav \in L \Rightarrow ubv \in L$.

Let $\text{LTL}_{\leq}$ be as in [Kup14] without quantitative operators. I.e. formulas of $\text{LTL}_{\leq}$ (on finite words on an alphabet $\Sigma$) are defined by the following grammar:

$$\varphi ::= a \mid \varphi \land \varphi \mid \varphi \lor \varphi \mid X\varphi \mid \varphi U\varphi \mid \Omega$$

Recall that $\Omega$ stands for the end of word. The only semantic difference with classical LTL is that an atomic formula $a \in \Sigma$ is true if the word starts with a letter $b \geq a$.

Example: Consider the alphabet $\{a, b, c\}$ with $a \leq c$ and $b \leq c$. Then the formula $\varphi = (a \land b) U \Omega$ of $\text{LTL}_{\leq}$ describes the language $c^*$. On the other hand, when interpreted classically as a LTL formula, the language of $\varphi$ is $\{\varepsilon\}$.

Notice that the logic $\text{LTL}_{\leq}$ can only describe upwards-closed languages.

Conjecture 1 [Col] If $L$ is an aperiodic upwards-closed language, then $L$ is recognized by a formula of $\text{LTL}_{\leq}$.

References

[Col] Thomas Colcombet. Personal communication.
