Good-for-Games Automata.

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Introduction

Deterministic automata on words are a central tool in automata theory:

- Polynomial algorithms for inclusion, complementation.
- Safe composition with games, trees.
- Solutions of the synthesis problem (verification).
- Easily implemented.

Problems :

- exponential state blow-up
- technical constructions (Safra)

Can we weaken the notion of determinism while preserving some good properties?

Good-for-Games automata

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Introduced independently in

- symbolic representation (Henzinger, Piterman '06)
  $\rightarrow$ simplification
- quantitative models (Colcombet '09) $\rightarrow$ replace determinism

Applications

- synthesis
- branching time verification
- tree languages (Boker, K, Kupferman, S '13)

Evaluating a game

Finite alphabets I for inputs and O for outputs.
Synthesis : design a system responding to environment, while satisfying a constraint $\varphi \subseteq (IO)^\omega$ (regular language).

Environment: I_1
System:

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Wrong approach: $\varphi \rightsquigarrow \mathcal{A}_{non-det}$: no player can guess the future.

A trivial synthesis example

Trivial instance of the synthesis problem:

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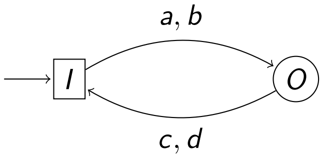

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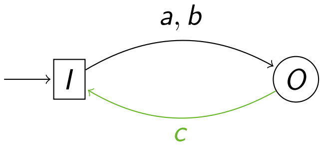

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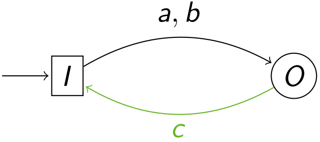

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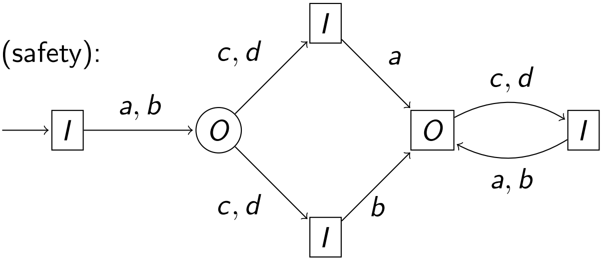

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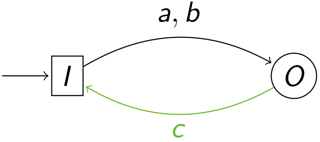

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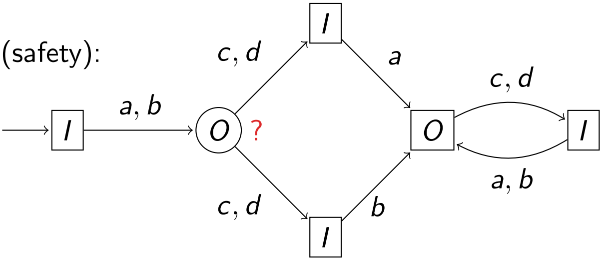
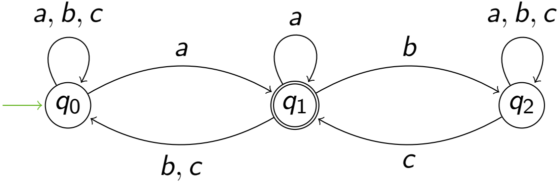


Definition of GFG via a game

$\mathcal{A}$ automaton on finite or infinite words.

Environment plays letters:

GFG Prover: controls transitions



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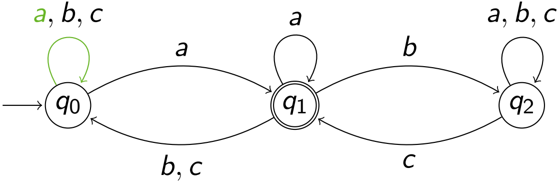

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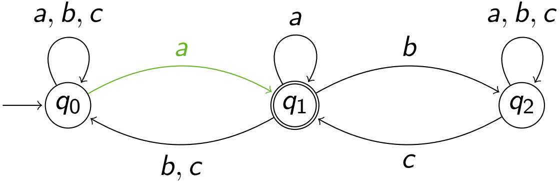

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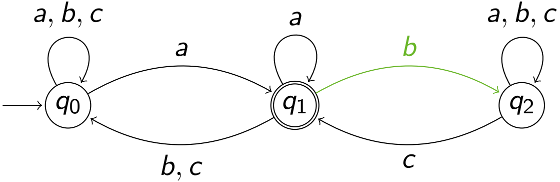

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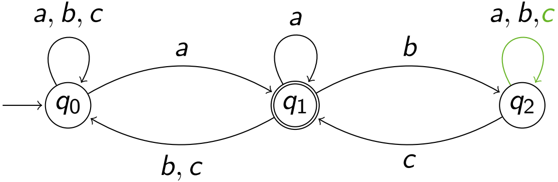

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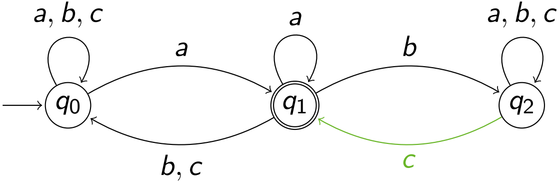

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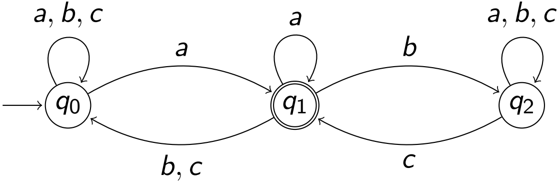

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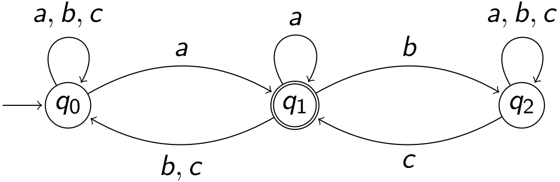

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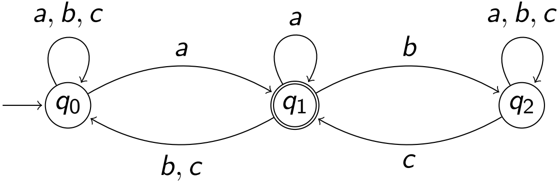

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How close is this to determinism?

Why Good-for-games

Composing a game with an automaton:

Input:

- Game G with complex winning condition L.
  A alphabet of actions in G.
- Automaton $\mathcal{A}_L$ recognizing L, on alphabet A.
  Simple accepting condition C.

Output:

Game $\mathcal{A}_L \circ G$, with winning condition C.
Straightforward construction, arena of size $|\mathcal{A}_L| \cdot |G|$.

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Theorem (Sound Composition)

$\mathcal{A}_L$ is **GFG** *if and only if*
for all G with condition L, $\mathcal{A}_L \circ G$ has same winner as G.

Some properties of GFG automata

GFG Automata:

- "$\mathcal{A} \subseteq \mathcal{B}$?": in **P** if $\mathcal{B}$ GFG (**PSPACE**-complete for ND)
- But Complementation $\sim$ Determinisation.
- Size of GFG strategy $\sigma \cong$ Size of deterministic automaton.

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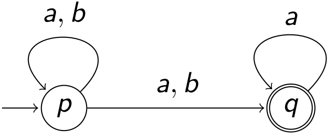
What about infinite words ? *Colcombet's conjecture*: **GFG** $\approx$ Det.

An automaton that is not GFG

This automaton for $L = (a+b)^* a^\omega$ is not **GFG**:



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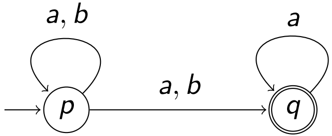

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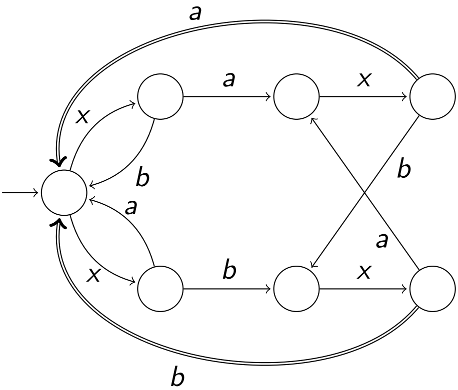
Fact

GFG *automata with condition C have same expressivity as deterministic automata with condition C.*

Therefore, **GFG** could improve succinctness but not expressivity.

A GFG Büchi example

Büchi condition: Run is accepting if infinitely many Büchi transitions are seen.



Language: $[(xa + xb)^*(xaxa + xbxb)]^\omega$

Determinization of Büchi GFG

Theorem (K, Skrzypczak '15)

Let $\mathcal{A}$ a **GFG** *Büchi automaton. There exists a deterministic automaton $\mathcal{B}$ with $L(\mathcal{B}) = L(\mathcal{A})$ and $|\mathcal{B}| \leq |\mathcal{A}|^2$.*

Proof scheme:

- Brutal powerset determinisation,
- Use is as a guide to normalize $\mathcal{A}$.

Conclusion: the automaton can use itself as memory structure $\Rightarrow$ quadratic blow-up only.

Is it true for all ω-regular conditions?

The coBüchi jump

CoBüchi condition: must see finitely many rejecting states.

Fact (Miyano-Hayashi '84)

Nondeterministic CoBüchi automata are easier to determinise than Büchi ones: 2^n instead of $2^{n \log n}$ and much simpler construction.

Are CoBüchi **GFG** simpler to determinize than Büchi **GFG** ?

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Theorem (K, Skrzypczak '15)

For all $n \geq 2$, there exists a language L_n on 3 letters such that

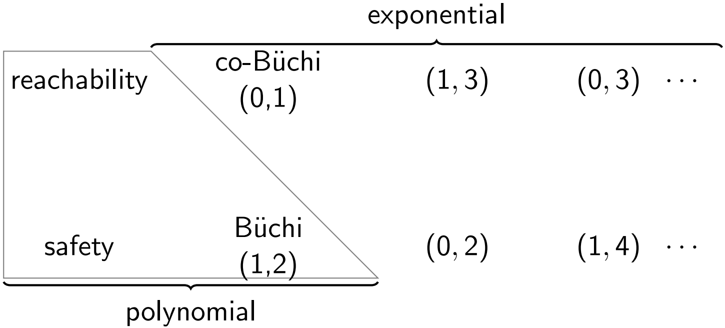
- *There is a n-state CoBüchi* **GFG** *automaton for L_n,*
- *any deterministic automaton for L_n has $\Omega(2^n)$ states.*

CoBüchi (and parity) **GFG** automata can provide both succinctness and sound behaviour with respect to games.

General picture

(i, j)-Parity condition: Each state has a color in $\{i, i+1, \ldots, j\}$.
Accepting runs: Maximal color occuring infinitely often is even.

Blow-up GFG $\rightarrow$ Det:



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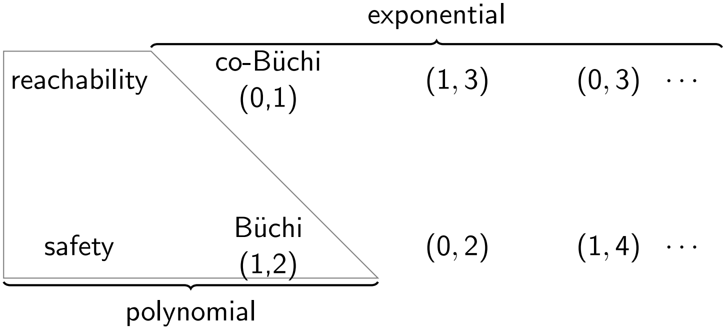

Question: How practical are these **GFG** ?

Recognizing GFG automata

Question: Given an automaton $\mathcal{A}$, is it **GFG**?

Theorem (K, Skrzypczak '15)

The complexity of deciding **GFG***-ness is in*

- *Upper bound:* **EXPTIME** *(even for* $(1,3)$*-parity)*
- **NP** *for Büchi automata*
- **P** *for coBüchi automata (surprising given blow-up result)*
- *at least as hard as solving parity games (***P** / **NP** $\cap$ *co***NP***) for parity automata.*

Open Problems

- Is it in **P** for any fixed acceptance condition?
- Is it equivalent to parity games for arbitrary condition?

Summary and conclusion

Results

- **GFG** automata capture good properties of deterministic automata.
- Inclusion is in **P**, but Complementation $\sim$ Determinisation.
- Conditions Büchi and lower: **GFG** $\approx$ Deterministic.
- Conditions coBüchi and higher: exponential succinctness.
- Recognizing **GFG** coBüchi is in **P**.

Open Problems

- Can we build small **GFG** automata in a systematic way?
- Complexity of deciding **GFG**-ness for parity automata? (gap **P** vs **EXPTIME**)