Nondeterminism in the Presence of Diverse or Unknown Future.

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Séminaire Graphes et Logique, LaBRI, Talence
07-01-2014

ω-words

$$w = \quad (a)-(a)-(b)-(a)-(c)-(b)-(b)-(a)-\cdots \quad \in \Sigma^\omega$$

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$$\exists_X \ \forall_{x \in X} \ a(x) \ \wedge \ \ldots$$

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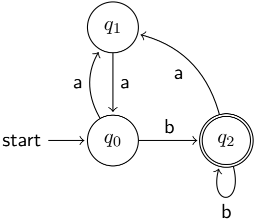
Finite automata



Model checking

formula $\rightsquigarrow$ automaton

$$w \models \varphi \quad \text{iff} \quad \mathcal{A} \text{ accepts } w$$

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(we focus on parity automata)

This work

Three classes in-between deterministic and non-det.:

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Containment???

Size???

Determinisation???

Synthesis problem

$\forall$ — environment
$\exists$ — system
φ — specification

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I_0

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- [Henzinger, Piterman]

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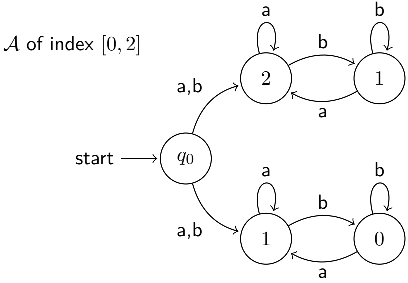
- [Henzinger, Piterman]
- also known as **History Determinism**
- applications to counter automata [Colcombet]

A non-example



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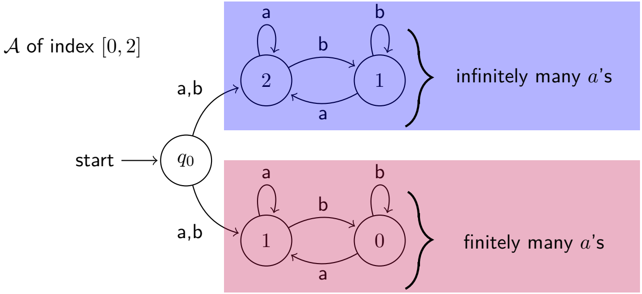

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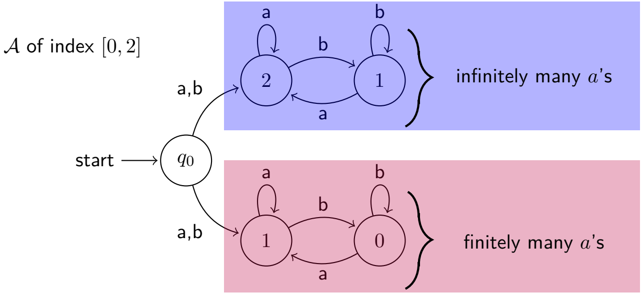

$$\mathrm{L}(\mathcal{A}) = \{a,b\}^\omega$$ but $\mathcal{A}$ is **not** GFG

Good For Trees — CTL* model checking

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A subproblem:

Given	:	$\mathcal{A}$ for an LTL path formula φ
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ω-word automaton for $L \rightsquigarrow$ tree automaton for $\mathrm{der}(L)$

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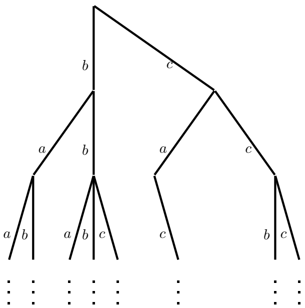
$\Sigma = \{a, b, c\}$



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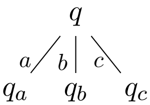

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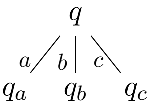

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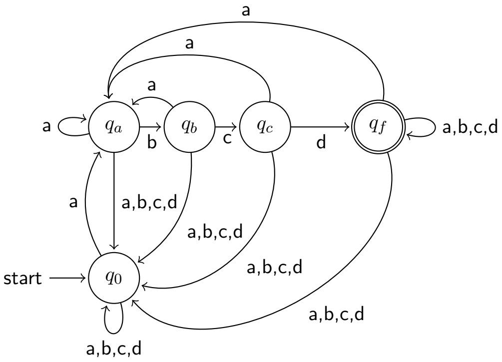
$\mathcal{A}$ is Good For Trees if $\mathbf{L}(\widehat{\mathcal{A}}) = \mathbf{der}(L)$

Determinisable By Pruning



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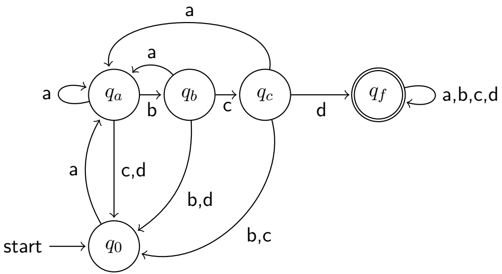

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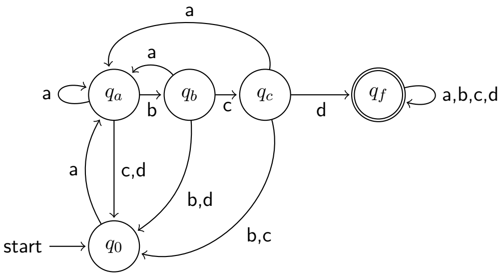

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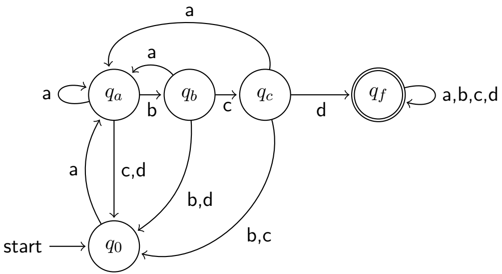

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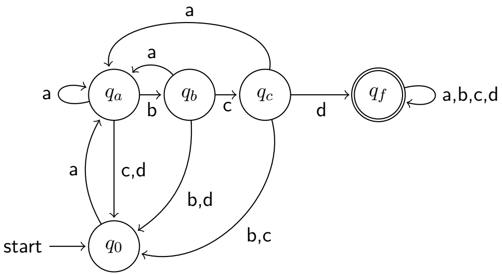

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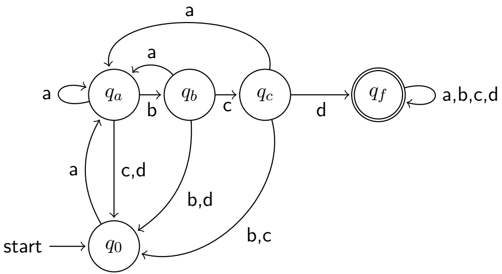

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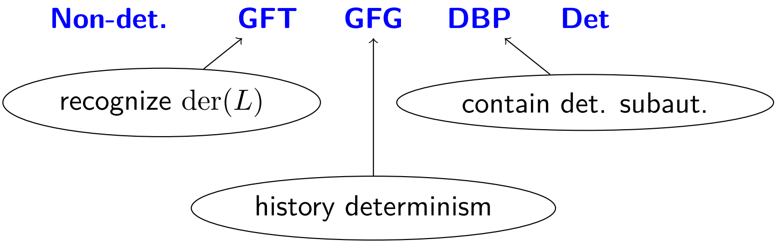

Containment

$$\textbf{Non-det.} \supset \textbf{GFT} \overset{?}{\supseteq} \textbf{GFG} \overset{?}{\supseteq} \textbf{DBP} \supset \textbf{Det}$$

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two examples

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Given automaton $\mathcal{A}$:

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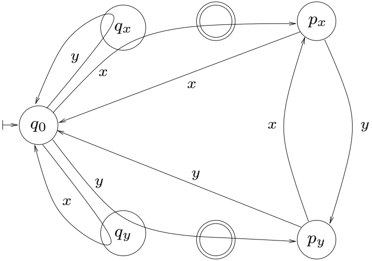

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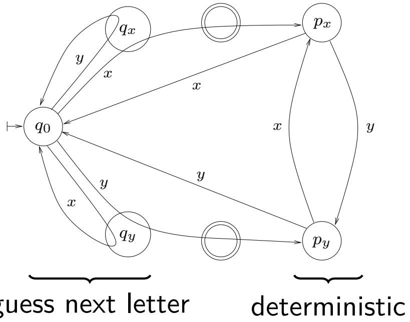

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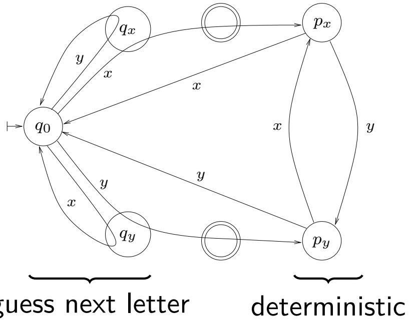

co-Büchi: follows from the *blow-up*

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Problem: what is the blow-up when determinising?

(given $\mathcal{A}$ of size n find equivalent deterministic one)

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- what about GFG?

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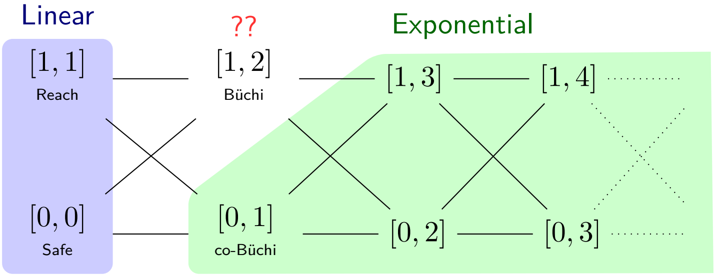
- determinisation $\sim$ size of memory of an advice σ



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What about Büchi GFG?