# Regular Sensing

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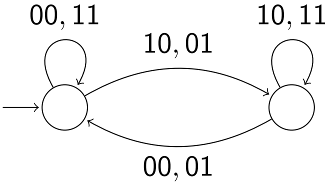
- **New approach:** Reading signals via sensors costs energy.
- **Goal:** Minimize the energy consumption in an average run.

# Sensing cost of a deterministic automaton

Deterministic automaton $\mathcal{A}$ on $\{00, 01, 10, 11\}$.

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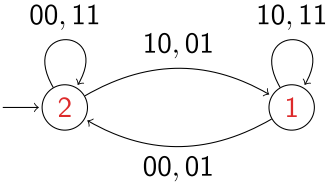


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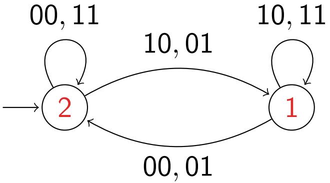


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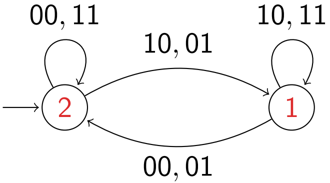


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$$scost(\mathcal{A}) = \lim_{m\to\infty} |\Sigma|^{-m} \sum_{w:|w|=m} scost(w)$$

# Computing the cost

Remarks on the definition of sensing cost:

- Initial state plays a role but not acceptance condition.
- Works on finite or infinite words.
- Cost is deduced from the transition structure.
- Signals can be weighted with different probabilities or sensing cost.

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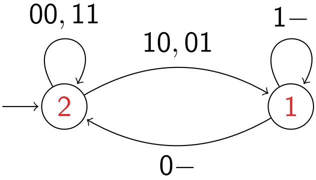
**Theorem**

*Sensing cost of an automaton is computable in polynomial time.*

By computing the stationary distribution of the induced Markov chain.

# Back to the example




Stationary distribution: $\frac{1}{2}, \frac{1}{2}$

Sensing cost: $\frac{3}{2}$.

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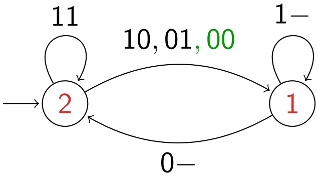


Stationary distribution: $\frac{2}{5}, \frac{3}{5}$

Sensing cost: $\frac{7}{5}$.

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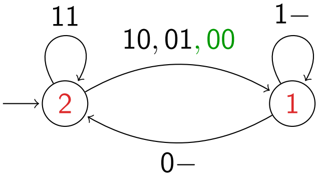


Stationary distribution: $\frac{2}{5}, \frac{3}{5}$

Sensing cost: $\frac{7}{5}$.

Limitation of the probabilistic model: Safety or Reachability automata always have cost 0. Only ergodic components matter in the long run.

# Sensing cost of a regular language

Sensing cost as a measure of complexity of regular languages.

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**Theorem**

*On finite words, the optimal sensing cost of a language is always reached by its minimal automaton.*

$\rightarrow$ Sensing as a complexity measure is not interesting on finite words, coincides with size.

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**Theorem**

*The sensing cost of an $\omega$-regular language is the one of its residual automaton.*

**Corollary**

*Computing the sensing cost of an $\omega$-regular language is in* ***PTime****.*

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- Trade-off between sensing cost and size.
- No trade-off between sensing cost and parity rank.
- Idea of the proof of general interest: one can "ignore" the input for arbitrary long periods and still recognize the language.

# Conclusion

On-going work

- Minimally-sensing transducer for safety specifications (exponential)
- Alternative definitions for
  - Safety languages
  - Transient components

Future work:

- Cost of realizing for parity specifications
- Precise study of the trade-off between different complexity measures