Sensing cost for automata and synthesis

Shaull Almagor¹, Denis Kuperberg², Orna Kupferman¹

¹Hebrew University of Jerusalem
²TU Munich

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Deterministic automata scanning the environment and checking a specification.
Framework

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- **Input:** \( S \) set of signals, \( \Sigma = 2^S \) alphabet of the automaton.
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- **New approach:** Reading signals via sensors costs energy.
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New approach: Reading signals via sensors costs energy.

Goal: Minimize the energy consumption in an average run.
Deterministic automaton $A$ on 2 signals.

$q$ state: $s\text{cost}(q) =$ number of relevant signals in $q$. 
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$q$ state: $\text{scost}(q) =$ number of relevant signals in $q$.

$w$ word: $\text{scost}(w) =$ average cost of states in the run of $\mathcal{A}$ on $w$.

$$\text{scost}(\mathcal{A}) = \lim_{m \to \infty} |\Sigma|^{-m} \sum_{|w|=m} \text{scost}(w)$$

Always converge.
Remarks on the definition of sensing cost:

- Initial state plays a role but not acceptance condition.
- Works on finite or infinite words.
- Cost is *deduced* from the transition structure.
- Signals can be weighted with different probabilities or sensing cost.
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**Theorem**

*Sensing cost of an automaton is computable in polynomial time.*

By computing the stationary distribution of the induced Markov chain.
Stationary distribution: \( \frac{1}{2}, \frac{1}{2} \)

Sensing cost: \( \frac{3}{2} \).
Back to the example

Stationary distribution: \( \frac{2}{5}, \frac{3}{5} \)

Sensing cost: \( \frac{7}{5} \).
Sensing cost as a measure of complexity of regular languages.

\[ \text{scost}(L) := \inf \{ \text{scost}(A) | L(A) = L \} . \]

Can we compute the sensing cost of a language? How hard is it?
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**Theorem**

*On finite words, the optimal sensing cost of a language is always reached by its minimal automaton.*
Sensing cost of $\omega$-regular languages

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- Computing the minimal number of states is NP-complete [Schewe ’10].
- Third complexity measure of $\omega$-languages: parity rank.

**Theorem**

*The sensing cost of an $\omega$-regular language is the one of its residual automaton.*

**Corollary**

*Computing the sensing cost of an $\omega$-regular language is in P.*
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- Trade-off between sensing cost and size.
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- Idea of the proof of general interest: one can “ignore” the input for arbitrary long periods and still recognize the language.
Safety Setting

Limitation of the probabilistic model: Safety automata always have cost 0. Only ergodic components matter in the long run.

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Solution: Average only on accepted words.

Two options:
Word average:

\[ wcost(A) = \lim_{m \to \infty} \frac{1}{L \cap |\Sigma|^m} \sum_{w \in L \cap |\Sigma|^m} scost(w) \]

Letter-by-letter: \( lcost(A) \) via Markov chain induced by \( A \): letters are randomly picked step-by-step to stay in \( L \).

Equivalent when all words were considered, different if we restrict attention to \( L \).
Two variants of safety cost

After 11, no more 1 on the first component.

$$l_{\text{cost}}(A) = 1, \ w_{\text{cost}}(A) = 2.$$  

Remark: $w_{\text{cost}}$ takes into account the transient components: if left self-loop has 2 labels, then $w_{\text{cost}}(A) = 3/2.$
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Theorem

\( lcost(A) \) and \( wcost(A) \) are computable in polynomial time. Their minimal is reached on the minimal automaton.

For \( wcost \): generating series, algorithms on algebraic numbers.
Cost of synthesis of a I/O specification $L$: Infimum of costs of transducers realizing $L$.

**Computational problem:**

**Input:** Deterministic automaton for $L \subseteq (I \cup O)^\omega$.

**Output:** Cost of synthesis of $L$. 

**Theorem**

For a safety specification, the problem is \textsc{Exptime}-complete, and an optimal transducer always exists.

**Remarks**

Optimal transducer can be exponential in the input deterministic automaton. Membership in \textsc{Exptime} by a game argument. Hardness by reduction from Tree Automata Intersection. For general languages as inputs, decidability open.
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Conclusion

Results

- General definition of sensing cost for finite and infinite words.
- Optimal cost computable in $\mathbf{P}$.
- Refined definitions for safety languages, well-behaved.
- EXPTIME-completeness of optimal safety synthesis.
- Minimally-sensing transducer for safety specifications (exponential)

Future work:

- Decidability of cost of synthesis for parity specifications
- Precise study of the trade-off between different complexity measures
- Refining the model: cost of switching,...