Sensing cost for automata and synthesis

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Framework

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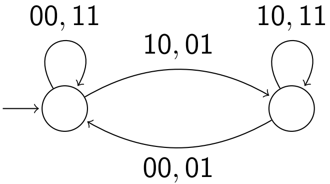
- **Input:** S set of signals, $\Sigma = 2^S$ alphabet of the automaton.
- **New approach:** Reading signals via sensors costs energy.
- **Goal:** Minimize the energy consumption in an average run.

Sensing cost of a deterministic automaton

Deterministic automaton $\mathcal{A}$ on 2 signals.



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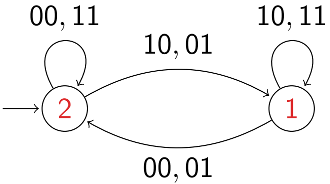

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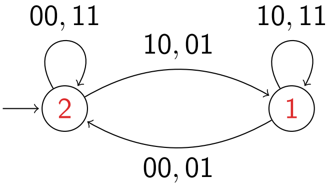

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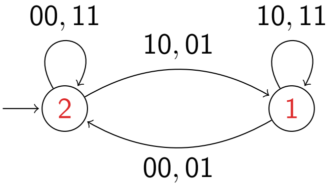

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w **word** : $scost(w)$ = average cost of states in the run of $\mathcal{A}$ on w.

$$scost(\mathcal{A}) = \lim_{m \to \infty} |\Sigma|^{-m} \sum_{|w|=m} scost(w)$$

Always converge.

Computing the cost

Remarks on the definition of sensing cost:

- Initial state plays a role but not acceptance condition.
- Works on finite or infinite words.
- Cost is deduced from the transition structure.
- Signals can be weighted with different probabilities or sensing cost.

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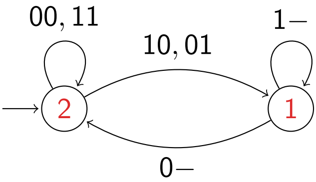
Theorem

Sensing cost of an automaton is computable in polynomial time.

By computing the stationary distribution of the induced Markov chain.

Back to the example



Stationary distribution: $\frac{1}{2}, \frac{1}{2}$

Sensing cost: $\frac{3}{2}$.

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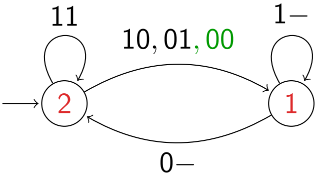

Stationary distribution: $\frac{2}{5}, \frac{3}{5}$

Sensing cost: $\frac{7}{5}$.

Sensing cost of a regular language

Sensing cost as a measure of complexity of regular languages.

$$scost(L) := \inf\{scost(\mathcal{A}) | L(\mathcal{A}) = L\}.$$

Can we compute the sensing cost of a language ? How hard is it ?

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On finite words, the optimal sensing cost of a language is always reached by its minimal automaton.

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Theorem

The sensing cost of an ω-regular language is the one of its residual automaton.

Corollary

*Computing the sensing cost of an ω-regular language is in **P**.*

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- Proof uses lemma of [Niwinski, Walukiewicz '98] on the structure of automata of optimal parity index.
- Trade-off between sensing cost and size.
- No trade-off between sensing cost and parity rank.
- Idea of the proof of general interest: one can "ignore" the input for arbitrary long periods and still recognize the language.

Safety Setting

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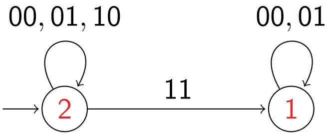
Two options:
Word average:

$$wcost(\mathcal{A}) = \lim_{m\to\infty} \frac{1}{L \cap |\Sigma|^m} \sum_{w \in L \cap |\Sigma|^m} scost(w)$$

Letter-by-letter: $lcost(\mathcal{A})$ via Markov chain induced by $\mathcal{A}$: letters are randomly picked step-by-step to stay in L.

Equivalent when all words were considered, different if we restrict attention to L.

Two variants of safety cost



After 11, no more 1 on the first component.

$$lcost(\mathcal{A}) = 1,\ wcost(\mathcal{A}) = 2.$$

Remark: *wcost* takes into account the transient components: if left self-loop has 2 labels, then $wcost(\mathcal{A}) = 3/2$.

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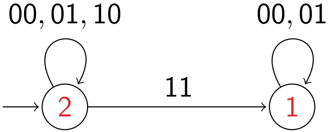

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Theorem

$\mathit{lcost}(\mathcal{A})$ and $\mathit{wcost}(\mathcal{A})$ are computable in polynomial time. Their minimal is reached on the minimal automaton.

For *wcost*: generating series, algorithms on algebraic numbers.

Synthesis

Cost of synthesis of a I/O specificiation L: Infimum of costs of transducers realizing L.

Computational problem:
Input: Deterministic automaton for $L \subseteq (I \cup O)^\omega$.
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Theorem

For a safety specification, the problem is ***EXPTIME****-complete, and an optimal transducer always exists.*

Remarks

- Optimal transducer can be exponential in the input deterministic automaton.
- $\rightsquigarrow$ Membership in **EXPTIME** by a game argument.
- Hardness by reduction from Tree Automata Intersection.
- For general languages as inputs, decidability open.

Conclusion

Results

- General definition of sensing cost for finite and infinite words.
- Optimal cost computable in **P**.
- Refined definitions for safety languages, well-behaved.
- EXPTIME-completeness of optimal safety synthesis.
- Minimally-sensing transducer for safety specifications (exponential)

Future work:

- Decidability of cost of synthesis for parity specifications
- Precise study of the trade-off between different complexity measures
- Refining the model: cost of switching,. . .