Determinisation of history-deterministic automata

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History-deterministic automata

- non-deterministic
- parity ω-word automata
- there exists $\sigma\colon \mathbf{A}^* \to \mathbf{Q}$ satisfying:

  for every word $w \in \mathrm{L}(\mathcal{A}) \subseteq A^\omega$
  the sequence of states $\sigma(w) \in Q^\omega$
  is an accepting run of $\mathcal{A}$

Open problem

What is the state blow-up when determinising a history-deterministic automaton?

Solution: exponential for co-Büchi automata
polynomial for Büchi automata

Applications

- synthesis (Büchi, Landweber '69)
- branching time verification (Emerson, Sistla '84)
- derived languages (Niwiński, Walukiewicz '98)
- symbolic representation (Henzinger, Piterman '06)
- quantitative models (Colcombet, Löding '10)
- ...

Co-Büchi automata

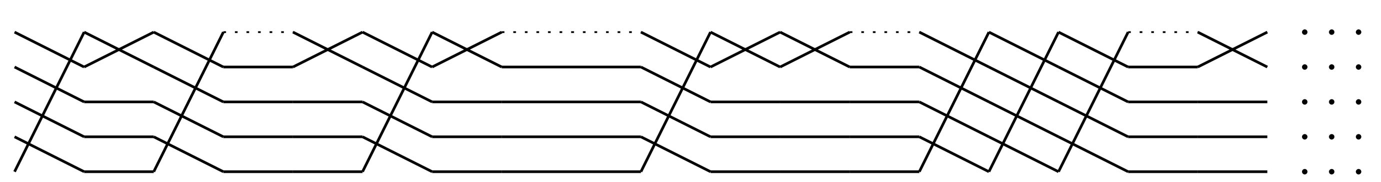
Theorem 1. *There are* **H-D** *automata $\mathcal{A}_n$:*

- $|\mathcal{A}_n| = \Theta(n)$ *and $\mathcal{A}_n$ is co-Büchi*
- *if $\mathcal{B}$ is deterministic and $\mathrm{L}(\mathcal{B}) = \mathrm{L}(\mathcal{A}_n)$ then $|\mathcal{B}| \geq 2^n$*

History-deterministic automata are **succinct**!

Proof scheme: Language of permutations:

$\sigma\ \pi\ \sigma\ \sharp\ \pi\ \sigma\ \pi\ \sharp\ \sharp\ \sigma\ \pi\ \pi\ \sharp\ \sigma\ \sigma\ \sigma\ \sharp\ \pi\ \ldots$



+ compactness for pumping the limitary behaviour.

Is $\mathcal{A}$ history-deterministic?

New complexity bounds:

- for co-Büchi: in **P**
- for Büchi: in **NP**
- for parity: at least as hard as parity games ($\mathbf{NP} \cap$ co-$\mathbf{NP}$)

Technical details at **arXiv.org** soon!

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Büchi automata

Theorem 2. *Every* **H-D** *Büchi automaton $\mathcal{A}$ admits $\mathcal{B}$:*

- *$\mathcal{B}$ is deterministic and Büchi*
- $\mathrm{L}(\mathcal{A}) = \mathrm{L}(\mathcal{B})$ *and* $|\mathcal{B}| \leq |\mathcal{A}|^2$

Polynomial determinisation procedure!

Proof scheme:

- residual languages and brutal powerset determinisation
- rank signatures of Walukiewicz
- iterative normalization of $\mathcal{A}$
- dependency graph over the automaton

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