Regular Sensing

Shaull Almagor[1], Denis Kuperberg[2], Orna Kupferman[1]

[1]Hebrew University of Jerusalem

[2]University of Warsaw.

Highlights of Logic, Games and Automata
05-09-2014

Framework

- Deterministic automata scanning the environment and checking a specification.

Framework

- Deterministic automata scanning the environment and checking a specification.
- **Input:** S set of signals, $\Sigma = 2^S$ alphabet of the automaton.

Framework

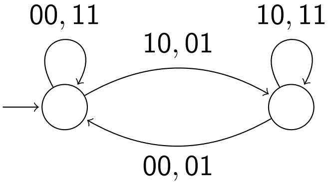
- Deterministic automata scanning the environment and checking a specification.
- **Input:** S set of signals, $\Sigma = 2^S$ alphabet of the automaton.
- **New approach:** Reading signals via sensors costs energy.

Framework

- Deterministic automata scanning the environment and checking a specification.
- **Input:** S set of signals, $\Sigma = 2^S$ alphabet of the automaton.
- **New approach:** Reading signals via sensors costs energy.
- **Goal:** Minimize the energy consumption in an average run.

Sensing cost of a deterministic automaton

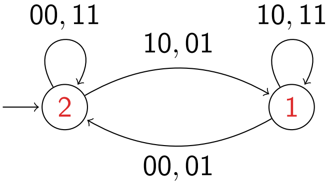
Deterministic automaton $\mathcal{A}$ on $\{00, 01, 10, 11\}$.



q **state** : $scost(q)$ = number of relevant signals in q.

Sensing cost of a deterministic automaton

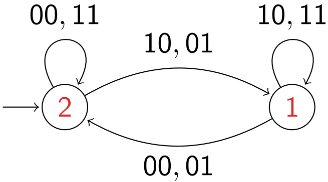
Deterministic automaton $\mathcal{A}$ on $\{00, 01, 10, 11\}$.



q **state** : $\color{blue}{scost(q)}$ = number of relevant signals in q.

Sensing cost of a deterministic automaton

Deterministic automaton $\mathcal{A}$ on $\{00, 01, 10, 11\}$.

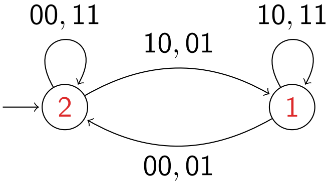


q **state** : $scost(q)$ = number of relevant signals in q.

w **word** : $scost(w)$ = average cost of states in the run of $\mathcal{A}$ on w.

Sensing cost of a deterministic automaton

Deterministic automaton $\mathcal{A}$ on $\{00, 01, 10, 11\}$.



q **state** : $scost(q)$ = number of relevant signals in q.

w **word** : $scost(w)$ = average cost of states in the run of $\mathcal{A}$ on w.

$$scost(\mathcal{A}) = \lim_{m\to\infty} |\Sigma|^{-m} \sum_{w:|w|=m} scost(w)$$

Computing the cost

Remarks on the definition of sensing cost:

- Initial state plays a role but not acceptance condition.
- Works on finite or infinite words.
- Cost is deduced from the transition structure.
- Signals can be weighted with different probabilities or sensing cost.

Computing the cost

Remarks on the definition of sensing cost:

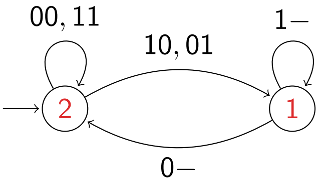
- Initial state plays a role but not acceptance condition.
- Works on finite or infinite words.
- Cost is deduced from the transition structure.
- Signals can be weighted with different probabilities or sensing cost.

Theorem

Sensing cost of an automaton is computable in polynomial time.

By computing the stationary distribution of the induced Markov chain.

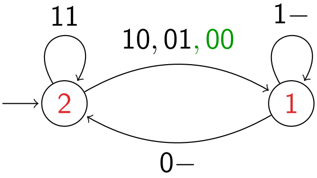
Back to the example



Stationary distribution: $\frac{1}{2}, \frac{1}{2}$

Sensing cost: $\frac{3}{2}$.

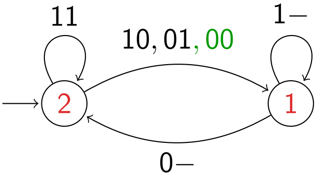
Back to the example



Stationary distribution: $\frac{2}{5}, \frac{3}{5}$

Sensing cost: $\frac{7}{5}$.

Back to the example



Stationary distribution: $\frac{2}{5}, \frac{3}{5}$

Sensing cost: $\frac{7}{5}$.

Limitation of the probabilistic model: Safety or Reachability automata always have cost 0. Only ergodic components matter in the long run.

Sensing cost of a regular language

Sensing cost as a measure of complexity of regular languages.

$$scost(L) := \inf\{scost(\mathcal{A}) | L(\mathcal{A}) = L\}.$$

Can we compute the sensing cost of a language ? How hard is it ?

Sensing cost of a regular language

Sensing cost as a measure of complexity of regular languages.

$$scost(L) := \inf\{scost(\mathcal{A})|L(\mathcal{A}) = L\}.$$

Can we compute the sensing cost of a language ? How hard is it ?

Theorem

On finite words, the optimal sensing cost of a language is always reached by its minimal automaton.

$\rightarrow$ Sensing as a complexity measure is not interesting on finite words, coincides with size.

Sensing cost of ω-regular languages

- On infinite words: deterministic parity automata.

Sensing cost of ω-regular languages

- On infinite words: deterministic parity automata.
- Computing the minimal number of states is **NP**-complete [Schewe '10].

Sensing cost of ω-regular languages

- On infinite words: deterministic parity automata.
- Computing the minimal number of states is **NP**-complete [Schewe '10].
- Third complexity measure of ω-languages: parity index.

Sensing cost of ω-regular languages

- On infinite words: deterministic parity automata.
- Computing the minimal number of states is **NP**-complete [Schewe '10].
- Third complexity measure of ω-languages: parity index.

Theorem

The sensing cost of an ω-regular language is the one of its residual automaton.

Corollary

*Computing the sensing cost of an ω-regular language is in **PTime**.*

Remarks

Remarks on the result:

- Optimal sensing cost might be reached only in the limit, not by a particular automaton.

Remarks

Remarks on the result:

- Optimal sensing cost might be reached only in the limit, not by a particular automaton.
- Proof uses lemma of [Niwinski, Walukiewicz '98] on the structure of automata of optimal parity index.

Remarks

Remarks on the result:

- Optimal sensing cost might be reached only in the limit, not by a particular automaton.
- Proof uses lemma of [Niwinski, Walukiewicz '98] on the structure of automata of optimal parity index.
- Trade-off between sensing cost and size.

Remarks

Remarks on the result:

- Optimal sensing cost might be reached only in the limit, not by a particular automaton.
- Proof uses lemma of [Niwinski, Walukiewicz '98] on the structure of automata of optimal parity index.
- Trade-off between sensing cost and size.
- No trade-off between sensing cost and parity rank.

Remarks

Remarks on the result:

- Optimal sensing cost might be reached only in the limit, not by a particular automaton.
- Proof uses lemma of [Niwinski, Walukiewicz '98] on the structure of automata of optimal parity index.
- Trade-off between sensing cost and size.
- No trade-off between sensing cost and parity rank.
- Idea of the proof of general interest: one can "ignore" the input for arbitrary long periods and still recognize the language.

Conclusion

Future work:

- Precise study of the trade-off between different complexity measures
- Generalize to transducers
- Modify the definition to account for transient states