Is your automaton good for playing games ?

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Solving an ω-regular game

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Question: Who wins G ? How ?

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Definition (Henzinger, Piterman 2006)

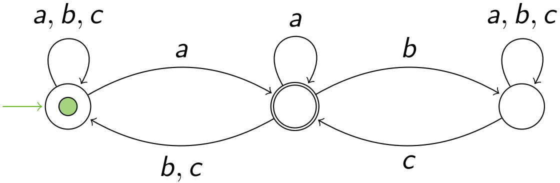
$\mathcal{A}$ is Good-for-Games (GFG) if $G \circ \mathcal{A}$ has same winner as G, for any game G with winning condition $L(\mathcal{A})$.

Definition of GFG via a game

$\mathcal{A}$ Parity automaton, we associate to it a **GFG game**:

Adam plays letters:

Eve: resolves non-deterministic choices for transitions



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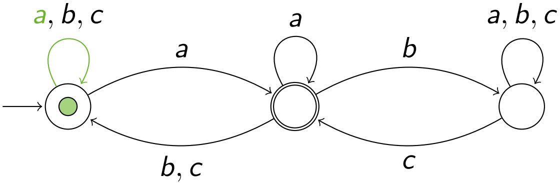

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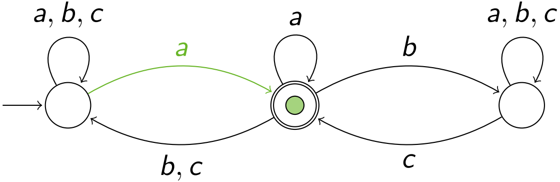

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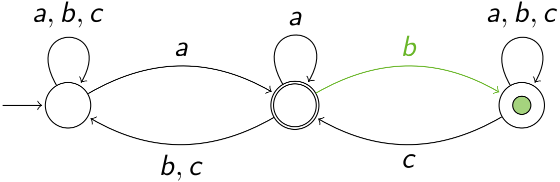

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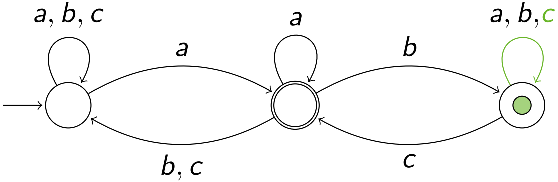

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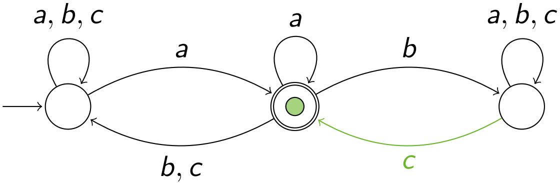

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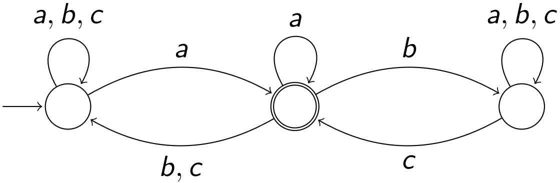

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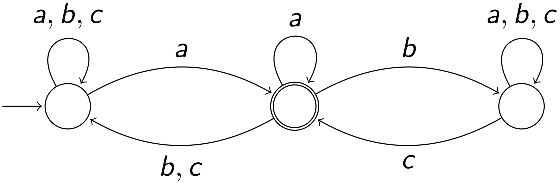

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$\mathcal{A}$ **GFG** $\Leftrightarrow$ Eve wins the GFG game on $\mathcal{A}$.

Recognizing GFG automata

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Theorem (Löding)

The GFGness problem is in P for reachability/safety automata.

Theorem (K., Skrzypczak 2015)

The GFGness problem is in P for coBüchi automata.
Parity Games $\equiv$ GFGness for universal Parity automata.

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This talk:

Theorem (Bagnol, K. (unpublished))

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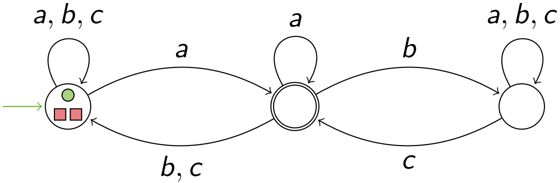
Abstracting the GFG game

The game G_2:

Adam plays letters:

Eve: moves one token Adam: moves two tokens



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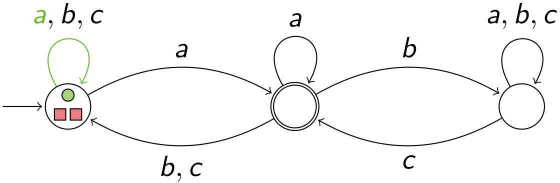

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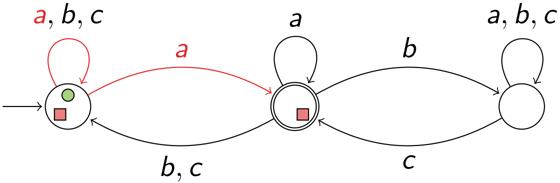

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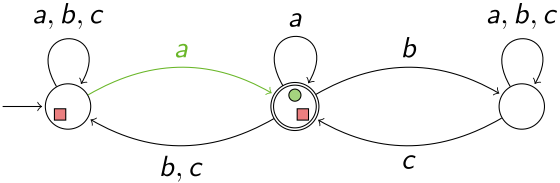

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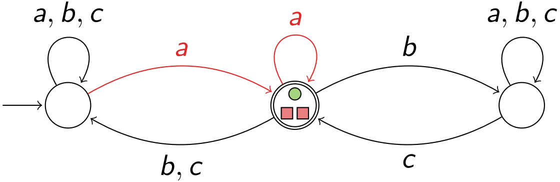

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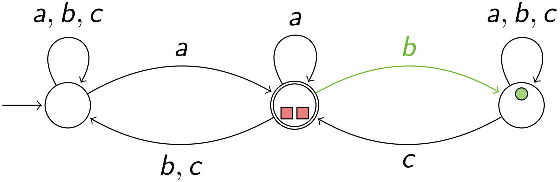

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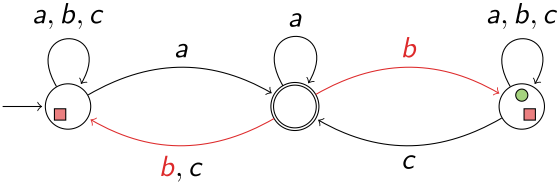

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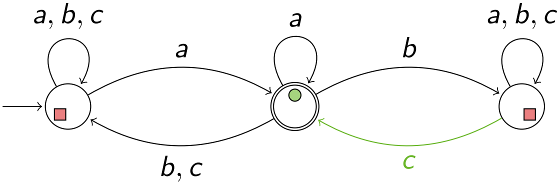

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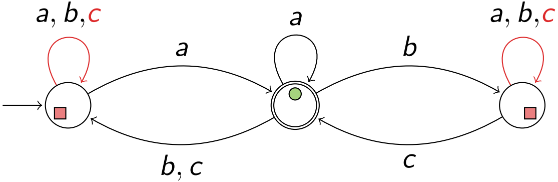

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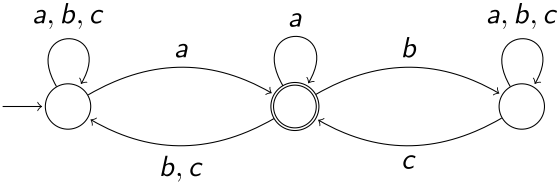

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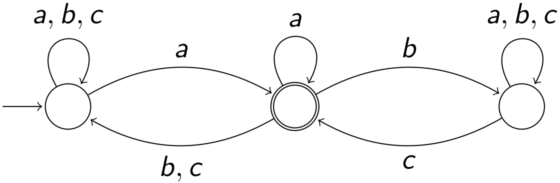

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Goal: Show that Eve wins G_2 ⇔ Eve wins the GFG game.

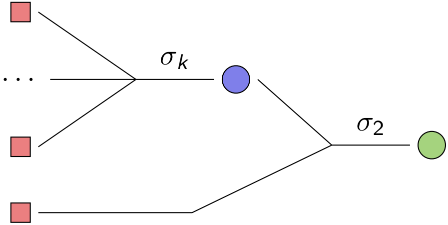
Proof sketch

Lemma

Eve wins $G_2 \Leftrightarrow$ Eve wins G_k for all k.

Proof sketch: how to win against $k+1$ tokens:

- play a *virtual token* ● against the first k tokens
- play the G_2 strategy against the virtual token and the remaining token.



Main proof sketch for $G_2 \Leftrightarrow \mathrm{GFG}$

Assume:

- Adam wins the GFG game with finite-memory strategy τ_{GFG}.
- Eve wins $G_2 \Rightarrow$ wins G_k with strategy σ_k, for a big k.

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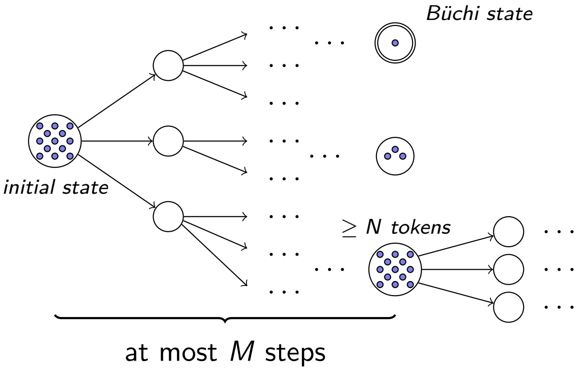
Build strategy for Eve against τ_{GFG}:

- move k virtual tokens ● against τ_{GFG}
- play σ_k against these k tokens



Conclusion

Results

- Characterisation of Büchi GFG automata via G_2.
- $\rightarrow$ Büchi GFGness $\in$ P, actually in $O(n^4 m(n+m)|\Sigma|^2)$.

Perspectives

- is G_2 equivalent to GFG for coBüchi, Parity ?
- if Yes, Parity GFGness $\in$ P for any fixed parity condition.
- recognizing GFG automata $\rightarrow$ building them.