Ising and Berezinskii-Kosterlitz-Thouless phase transitions of a two-leg boson ladder in flux

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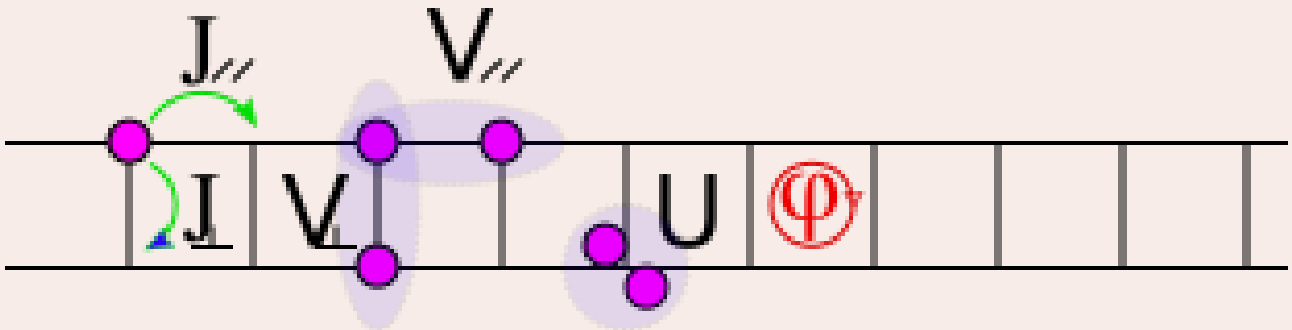
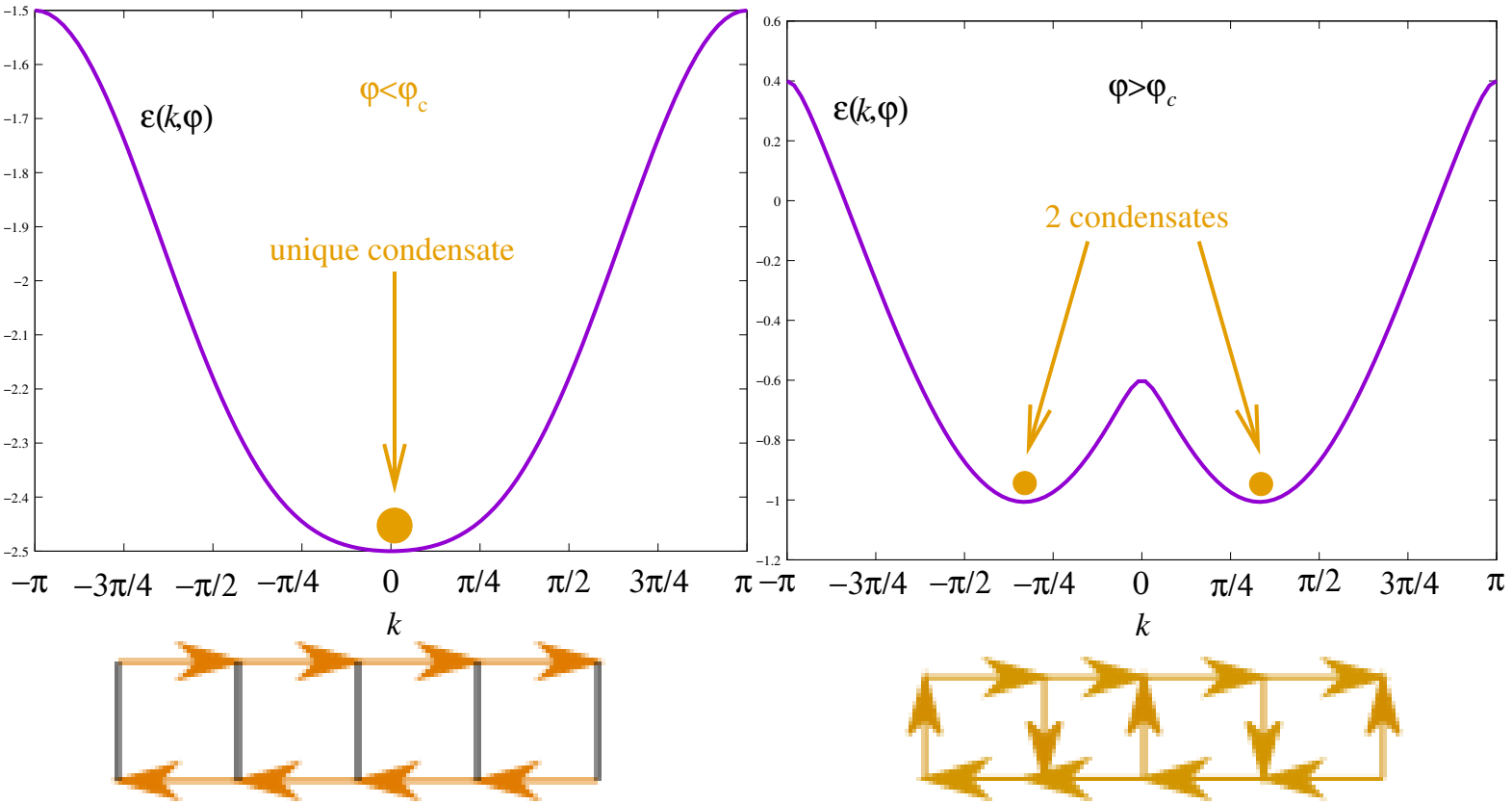
Two-leg boson ladder in a in flux: model

Extended Bose-Hubbard Hamiltonian

$$H = -J_{\parallel} \sum_{j,\sigma=\pm 1/2} (b^{\dagger}_{j,\sigma} e^{i\sigma\varphi} b_{j+1,\sigma} + \text{H.c.}) - J_{\perp} \sum_{j} (b^{\dagger}_{j,1/2} b_{j,-1/2} + \text{H.c.})$$
$$+ U \sum_{j,\sigma} n_{j,\sigma}(n_{j,\sigma} - 1) + V_{\parallel} \sum_{j,\sigma} n_{j,\sigma} n_{j+1,\sigma} + V_{\perp} \sum_{j} n_{j,\uparrow} n_{j,\downarrow}$$

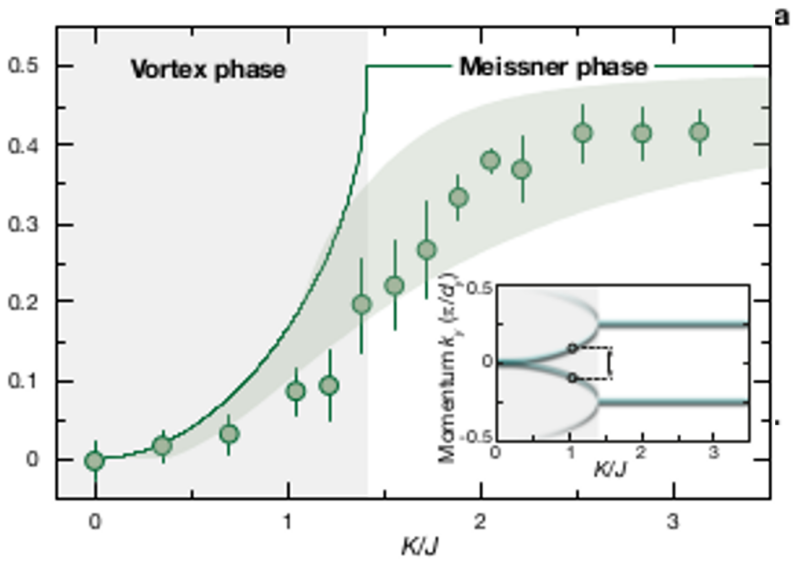


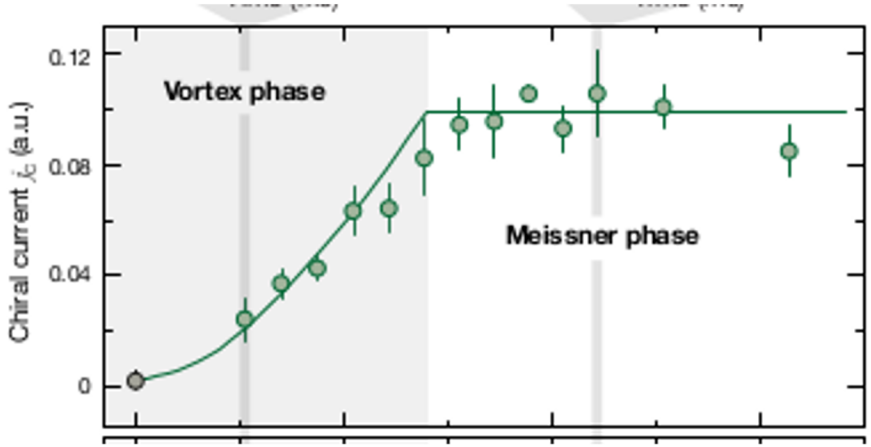
Without interaction



- Single Condensate: Commensurate, Meissner-like currents
- 2 condensates: Incommensurate, Vortex-like currents

Experiment with ^{87}Rb ladder in artificial flux





φ fixed, φ_c increased by increasing $J_\perp/J$.
From Atala et al. Nat. Phys. **10**, 588 (2014).

With intrachain interaction only ($V_\perp = 0$)

Bosonization approach [EO, T. Giamarchi 2001 & A. Tokuno, A. Georges 2014]

- Symmetric modes are always gapless
- Antisymmetric modes show a commensurate-incommensurate transition at $\varphi = \varphi_c$.
  - At $\varphi < \varphi_c$ gapped, commensurate Meissner-like phase.
  - At $\varphi > \varphi_c$ gapless, incommensurate Vortex-like phase.

$\varphi < \varphi_c$: Commensurate Meissner-like phase

Observables

Current difference along the legs:

$$J_s = \langle j_\uparrow - j_\downarrow \rangle \quad = \quad \mathbb{D}\varphi$$

Momentum distribution:

$$\sum_\sigma \langle n_\sigma(k) \rangle \quad \sim \quad |k|^{\frac{1}{2K_c}-1}$$

Instead of a true condensate, a quasi-condensate at $k = 0$.
Vanishing $\langle j_\perp \rangle = 0$.

$\varphi > \varphi_c$: Incommensurate Vortex like phase

Observables

Current difference along the legs:

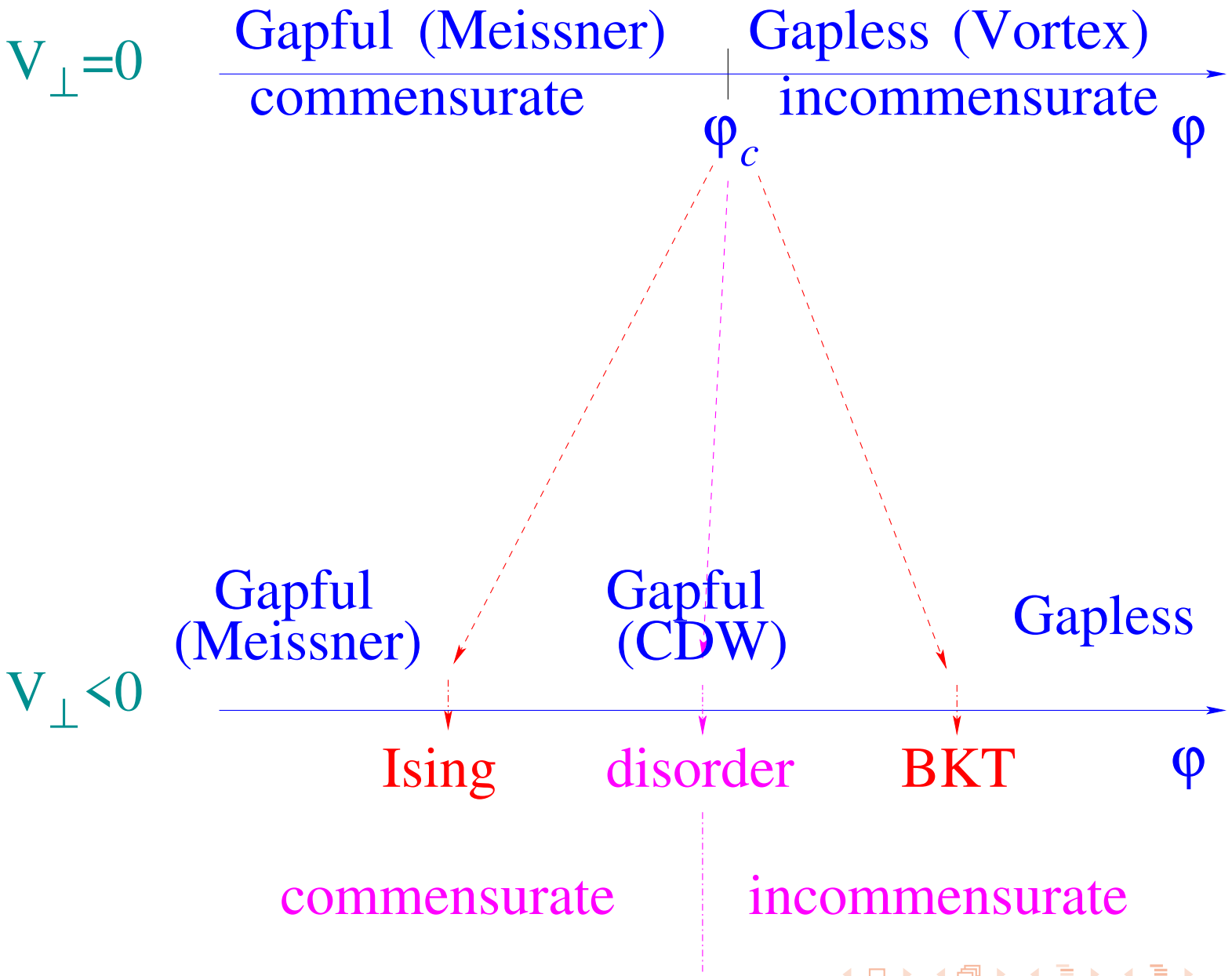
$$J_s = \langle j_\uparrow - j_\downarrow \rangle \quad = \quad \mathbb{D}(\varphi - \sqrt{\varphi^2 - \varphi_c^2})$$

Momentum distribution:

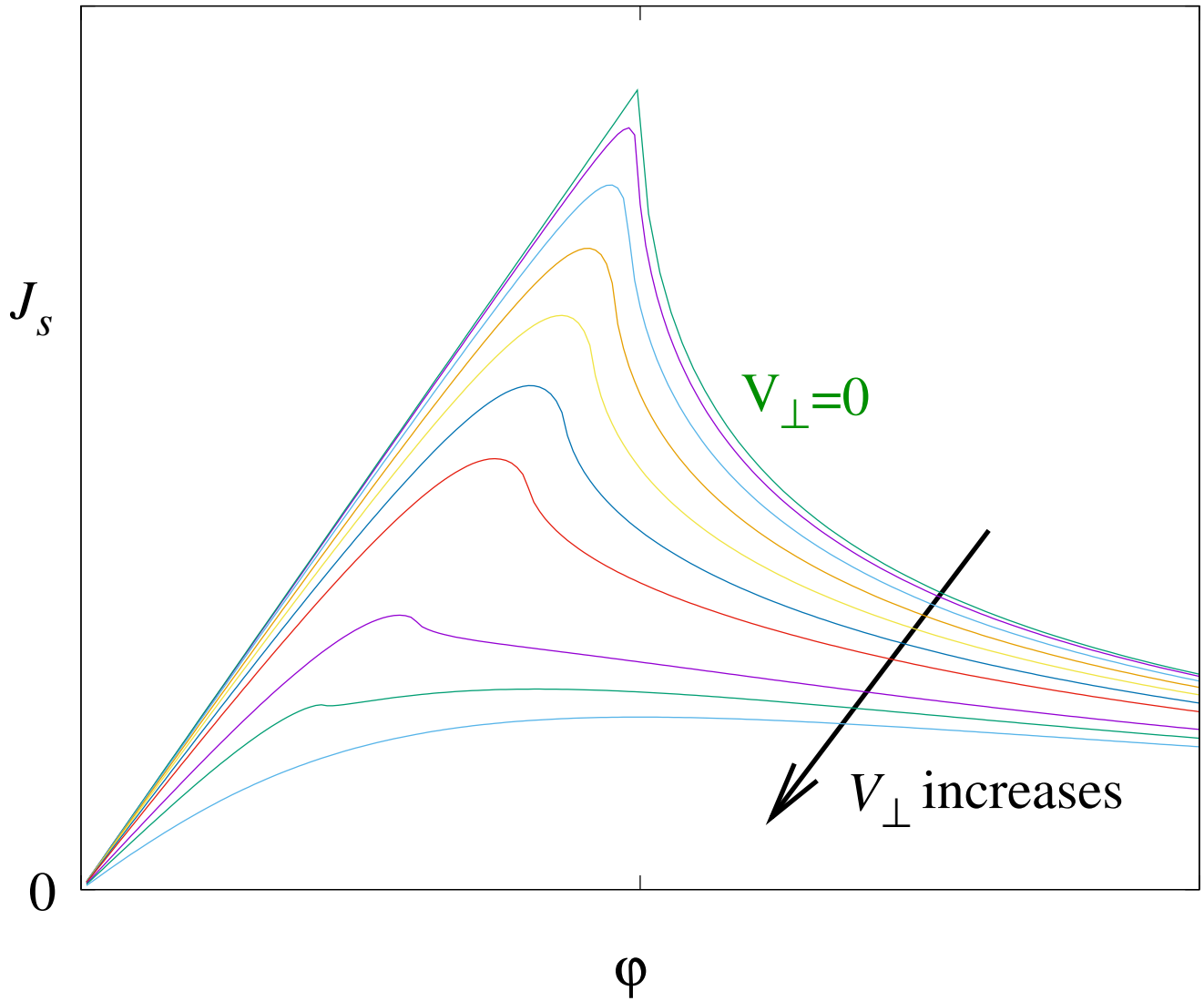
$$\sum_\sigma \langle n_\sigma(k) \rangle \quad \sim \quad \sum_{r=\pm} \left| k + rC\sqrt{\varphi^2 - \varphi_c^2} \right|^{1/(2K_c)+2/(2K_s^*)-1}$$

Two quasi-condensates at $k = \pm C\sqrt{\varphi^2 - \varphi_c^2}$.

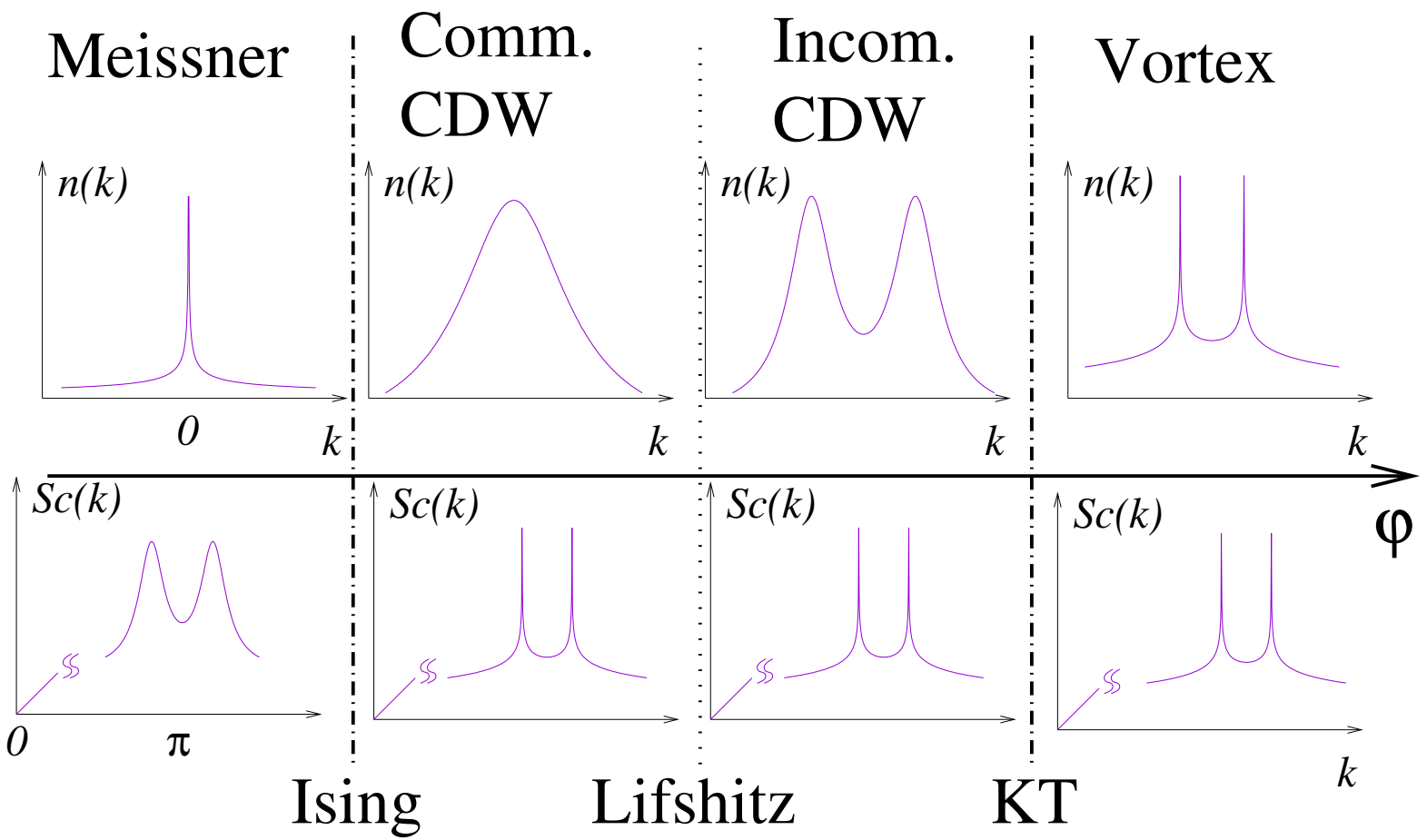
Ground state phase diagram with $V_\perp \neq 0$



Evolution of leg current with $V_\perp$

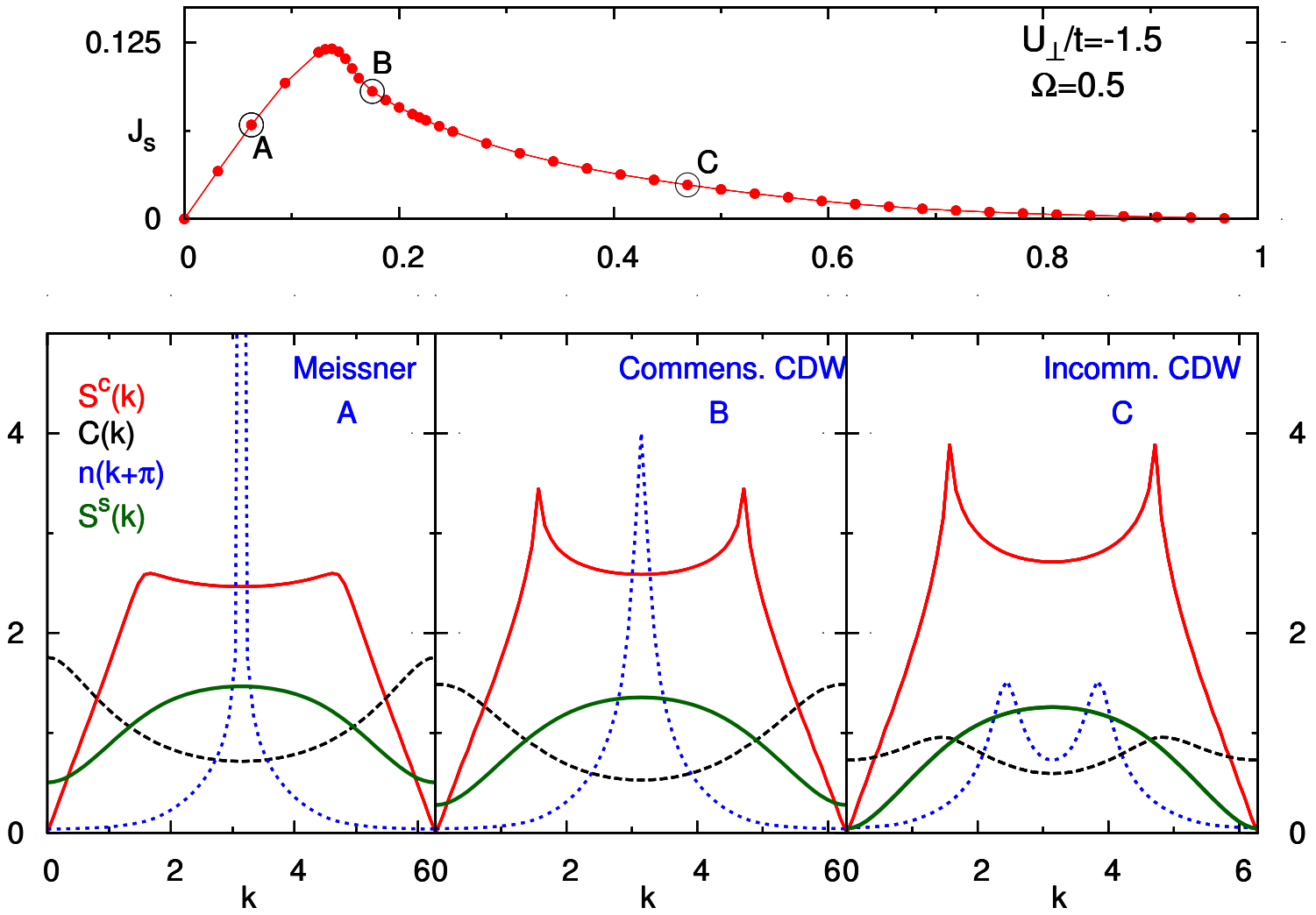


Physical observables in the ground state



$$S_c(k) = F.T.\left[\sum_{\sigma,\sigma'}\langle n_{j\sigma} n_{0\sigma'}\rangle\right]$$

DMRG computations (De Palo & Di Dio)



Conclusions

Summary

1. Spin-orbit coupling/artificial gauge field experimentally accessible with cold atoms, trapped ions, Josephson junctions.
2. Meissner/Vortex transition = C-IC transition for $V_\perp = 0$
3. $V_\perp \neq 0$ Meissner/CDW/Vortex

References

1. Phys. Rev. B **96**, 014518 (2017)
2. Phys. Rev. B **97**, 174523 (2018)

Open questions

1. In finite chains, edge states ?
2. Effect of harmonic trapping for bosonic atoms ?
3. Effect of dissipation in Josephson ladders ?