Spectral Functions in a Two-Component Tomonaga-Luttinger Liquid

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Retardated Fermion Green's function

$$G_a(x,t) = -i\theta(t)\langle\{\psi_a(x,t),\psi_a^\dagger(0,0)\}\rangle$$

$$A_a(q,\omega) = -\frac{1}{\pi}\Im\int dxdt G_a(x,t)e^{-i(qx-\omega t)}$$

$$\psi_a(x,t) = \sum_{r=\pm} e^{irk_{F,a}x}\psi_{r,a}(x,t)$$

$$\Rightarrow A_a(k_{F,a}+q,\omega) = I_a(q,\omega) + I_a(-q,-\omega)$$

$$I_a(q,\omega) = \frac{1}{2\pi}\int dxdt\langle\psi_{+,a}(x,t)\psi_{+,a}^\dagger(0,0)\rangle e^{-i(qx-\omega t)}$$

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$SU(2)$ invariant case: $\gamma_\rho = (K_\rho + K_\rho^{-1} - 2)/8$.

$$
\begin{aligned}
I_{\uparrow/\downarrow}(q,\omega) = {} & \frac{C\alpha^{2\gamma_\rho}}{(2\pi)^2} \int dxdt\, e^{i(\omega t - qx)} \\
& \times \frac{1}{[\alpha + i(u_\rho t - x)]^{\gamma_\rho + 1/2}} \frac{1}{[\alpha + i(u_\sigma t - x)]^{1/2}} \\
& \times \frac{1}{[\alpha + i(u_\rho t + x)]^{\gamma_\rho}},
\end{aligned}
$$

$C =$ Non-universal constant.

Feynman identity

$$\frac{1}{A_1^{\nu_1} A_2^{\nu_2}} = \frac{\Gamma(\nu_1+\nu_2)}{\Gamma(\nu_1)\Gamma(\nu_2)} \int_0^1 du \frac{u^{\nu_1-1}(1-u)^{\nu_2-1}}{(A_1 u + A_2(1-u))^{\nu_1+\nu_2}}$$

Integral representation

$$I_{\uparrow/\downarrow}(q,\omega) = \frac{C\alpha^{2\gamma_\rho}}{\Gamma(\gamma_\rho+1/2)\Gamma(\gamma_\rho)\Gamma(1/2)} |\omega + u_\rho q|^{\gamma_\rho}$$
$$\times \int_0^1 dv \frac{v^{\gamma_\rho-1/2}}{(1-v)^{1/2}} \frac{|\omega - (vu_\rho + (1-v)u_\sigma)q|^{\gamma_\rho-1}}{|u_\rho(1+v) + u_\sigma(1-v)|^{2\gamma_\rho}}$$
$$\times\Theta(\omega + u_\rho q)\Theta(\omega - [vu_\rho + (1-v)u_\sigma]q),$$

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$\omega > u_\rho q$

$$A(q,\omega) \propto \frac{\alpha^{2\gamma_\rho}(\omega + u_\rho q)^{\gamma_\rho}(\omega - u_\sigma q)^{\gamma_\rho - 1}}{\Gamma(\gamma_\rho)\Gamma(\gamma_\rho + 1)(2u_\rho)^{\frac{2\gamma_\rho+1}{2}}(u_\rho + u_\sigma)^{\frac{2\gamma_\rho-1}{2}}} \times {}_2F_1\left(1-\gamma_\rho, \gamma_\rho + \frac{1}{2}; \gamma_\rho + 1; \frac{u_\rho - u_\sigma}{2u_\rho}\frac{\omega + u_\rho q}{\omega - u_\sigma q}\right)$$

$u_\sigma q < \omega < u_\rho q$

$$A(q,\omega) \propto \frac{\pi^{-\frac{1}{2}}\alpha^{2\gamma_\rho}(\omega + u_\rho q)^{-\frac{1}{2}}(\omega - u_\sigma q)^{\frac{2\gamma_\rho-1}{2}}}{\Gamma(\frac{2\gamma_\rho+1}{2})(u_\rho + u_\sigma)^{\frac{2\gamma_\rho-1}{2}}(u_\rho - u_\sigma)^{\frac{2\gamma_\rho+1}{2}}} \times {}_2F_1\left(\frac{1}{2}, \gamma_\rho + \frac{1}{2}; 2\gamma_\rho + \frac{1}{2}; \frac{2u_\rho}{u_\rho - u_\sigma}\frac{\omega - u_\sigma q}{\omega + u_\rho q}\right)$$

See Gogolin et al., *Bosonization and Strongly Correlated Systems* (Cambridge University Press, 1999)

$\omega < -u_\rho q$

$$A(q,\omega) \propto \frac{\alpha^{2\gamma_\rho}|\omega - u_\sigma q|^{\gamma_\rho - 1}|\omega + u_\rho q|^{\gamma_\rho}}{\Gamma(\gamma_\rho)\Gamma(\gamma_\rho+1)(u_\rho+u_\sigma)^{\frac{2\gamma_\rho-1}{2}}(2u_\rho)^{\frac{2\gamma_\rho+1}{2}}} \times {}_2F_1\left(1-\gamma_\rho, \gamma_\rho + \frac{1}{2}; \gamma_\rho + 1; \frac{u_\rho - u_\sigma}{2u_\rho}\frac{\omega + u_\rho q}{\omega - u_\sigma q}\right)$$

$-u_\rho q < \omega < u_\sigma q$

$$A(q,\omega) = 0$$

See Gogolin et al., *Bosonization and Strongly Correlated Systems* (Cambridge University Press, 1999)

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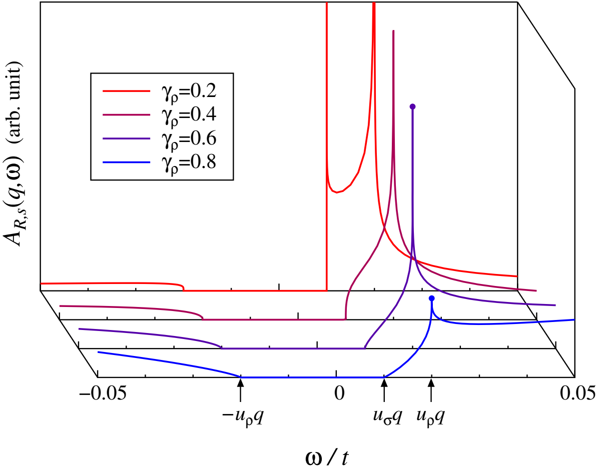

Singularities

- $A(q, \omega \simeq u_\sigma q) \sim (\omega - u_\sigma q)^{2\gamma_\rho - 1/2}$
- $A(q, \omega \simeq u_\rho q) \sim |\omega - u_\sigma q|^{\gamma_\rho - 1/2}$

Non $SU(2)$ invariant case

$$I_a(q,\omega) = \frac{C_a \alpha^{\nu_{a,1}+\nu_{a,2}+\nu'_{a,1}+\nu'_{a,2}}}{2\pi} \times \int \frac{e^{i(\omega t - qx)}}{(\alpha + i(v_1 t - x))^{\nu_{a,1}}(\alpha + i(v_2 t - x))^{\nu_{a,2}}} \times \frac{dxdt}{(\alpha + i(v_1 t + x))^{\nu'_{a,1}}(\alpha + i(v_2 t + x))^{\nu'_{a,2}}}$$

$\Rightarrow$ Apply Feynman identity twice.

Integral representation

$$I_a(q,\omega) = \int_{[0,1]^2} \frac{C_a \alpha^{\bar{\nu}_a - 1} dw_1 dw_2}{\Gamma(\nu_{a,+})\Gamma(\nu_{a,-})\Gamma(\nu'_{a,+})\Gamma(\nu'_{a,-})}$$
$$\times w_1^{\nu_{a,+}-1}(1-w_1)^{\nu_{a,-}-1}[\omega - u(w_1)q]^{\nu'_{a,+}+\nu'_{a,-}-1}$$
$$\times w_2^{\nu'_{a,+}-1}(1-w_2)^{\nu'_{a,-}-1}[\omega + u(w_2)q]^{\nu_{a,+}+\nu_{a,-}-1}$$
$$\times \frac{\Theta[\omega - u(w_1)q]\Theta[\omega + u(w_2)q]}{[u(w_1)+u(w_2)]^{\bar{\nu}_a - 1}},$$

$u(w) = wu_+ + (1-w)u_-$, $\bar{\nu}_a = \nu_{a,+} + \nu_{a,-} + \nu'_{a,+} + \nu'_{a,-}$

Threshold $(u_- < u_+)$

$I(q>0, \omega < u_- q) = 0 \Rightarrow A(q>0, |\omega| < v_2 q) = 0$

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$u_- q < \omega < u_+ q$

$$A_a(q,\omega) \propto \frac{(\alpha/\Delta u)^{\bar\nu_a - 1}}{\Gamma(\nu_{a,+} + \nu'_{a,+} + \nu'_{a,-})\Gamma(\nu_{a,-})}$$
$$\times \frac{(|\omega| - u_- q)^{\nu_{a,+}+\nu'_{a,-}+\nu'_{a,+}-1}}{(|\omega| + u_- q)^{\nu'_{a,-}}}$$
$$\times \frac{(-|\omega| + u_+ q)^{\nu_{a,-}+\nu'_{a,-}+\nu'_{a,+}-1}}{(|\omega| + u_+ q)^{\nu'_{a,+}}}$$
$$\times F_1\left(\bar\nu_a - 1; \nu'_{a,+}, \nu'_{a,-}; \nu_{a,+} + \nu'_{a,+} + \nu'_{a,-}; z_1, z_2\right)$$
$$z_1 = \frac{2u_+(|\omega| - u_- q)}{\Delta u(|\omega| + u_+ q)}; z_2 = \frac{2\bar u(|\omega| - u_- q)}{\Delta u(|\omega| + u_- q)}$$

Definition

$F_1(\alpha;\beta,\beta';\gamma;x,y)$: first Appell hypergeometric function of two variables.

$\omega > u_+ q$

$$A_a(q,\omega) \propto \frac{(\alpha/2u_+)^{\bar\nu_a - 1}}{\Gamma(\nu_{a,+}+\nu_{a,-})\Gamma(\nu'_{a,+}+\nu'_{a,-})}$$
$$\times \frac{(|\omega| - u_+ q)^{\nu_{a,-}+\nu'_{a,+}+\nu'_{a,-}-1}}{(|\omega| - u_- q)^{\nu_{a,-}}}$$
$$\times \frac{(|\omega| + u_+ q)^{\nu_{a,+}+\nu_{a,-}+\nu'_{a,-}-1}}{(|\omega| + u_- q)^{\nu'_{a,-}}}$$
$$\times F_2\left(\bar\nu_a - 1;\, \nu_{a,-},\, \nu'_{a,-},\, \nu_{a,+}+\nu_{a,-},\, \nu'_{a,+}+\nu'_{a,-};\, z3, z4\right)$$
$$z_3 = \frac{\Delta u(|\omega| + u_+ q)}{2u_+(|\omega| - u_- q)}, z_4 = \frac{\Delta u(|\omega| - u_+ q)}{2u_+(|\omega| + u_- q)}$$

Definition

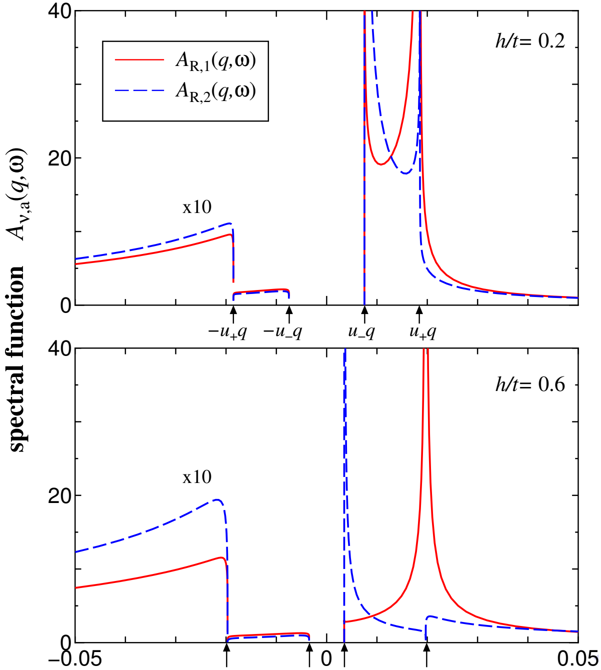
$F_2(\alpha;\beta,\beta';\gamma,\gamma';x,y)$: second Appell hypergeometric function

$-u_+q < \omega < -u_-q$

$$A_a(q,\omega) \propto \frac{(\alpha/\Delta u)^{\bar{\nu}_a-1}}{\Gamma(\nu_{a,+}+\nu_{a,-}+\nu'_{a,+})\Gamma(\nu'_{a,-})}$$
$$\times \frac{(|\omega|-u_-q)^{\nu_{a,+}+\nu_{a,-}+\nu'_{a,+}-1}}{(|\omega|+u_-q)^{\nu_{a,-}}}$$
$$\times \frac{(-|\omega|+u_+q)^{\nu_{a,+}+\nu_{a,-}+\nu'_{a,-}-1}}{(|\omega|+u_+q)^{\nu_{a,+}}}$$
$$\times F_1\left(\bar{\nu}_a-1;\nu_{a,+},\nu_{a,-};\nu_{a,+}+\nu_{a,-}+\nu'_{a,+};z1,z2\right)$$

$\omega < -u_+ q$

$$A_a(q,\omega) \propto \frac{(\alpha/2u_+)^{\bar{\nu}_a - 1}}{\Gamma(\nu'_{a,+} + \nu'_{a,-})\Gamma(\nu_{a,+} + \nu_{a,-})}$$
$$\times \frac{(|\omega| - u_+ q)^{\nu_{a,+} + \nu_{a,-} + \nu'_{a,-} - 1}}{(|\omega| - u_- q)^{\nu'_{a,-}}}$$
$$\times \frac{(|\omega| + u_+ q)^{\nu_{a,-} + \nu'_{a,+} + \nu'_{a,-} - 1}}{(|\omega| + u_- q)^{\nu_{a,-}}}$$
$$\times F_2\left(\bar{\nu}_a - 1;\, \nu'_{a,-},\, \nu_{a,-};\, \nu'_{a,+} + \nu'_{a,-},\, \nu_{a,+} + \nu_{a,-};\, z3, z4\right),$$



spectral function $A_{\nu,a}(q,\omega)$
$A_{R,1}(q,\omega)$
$A_{R,2}(q,\omega)$
$h/t = 0.2$
x10
$-u_+q$
$-u_-q$
u_-q
u_+q
$h/t = 0.6$
x10
40
20
0
−0.05
0
0.05