Lieb-Liniger variational ansatz and one-dimensional dipolar gases

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Abstract

We introduce a variational ansatz using the ground state wavefunction of the integrable Lieb-Liniger model as trial wavefunction. The expectation value of the potential energy is obtained from an approximation of the static structure factor of the Lieb-Liniger model introduced by Cherny and Brand. We apply the variational ansatz to the calculation of the ground state energy of a quasi-one dimensional gas of dipolar bosons as a function of density and tilt angle of the dipoles. We discuss the breathing modes of the dipolar gas.

Dipolar bosons trapped in one dimension

$$H_{Q1D} = -\frac{\hbar^2}{2m}\sum_i \frac{\partial^2}{\partial x_i^2} + \sum_{i<j} V_{Q1D}(x_i - x_j)$$
$$+ g_{VdW}\sum_{i<j}\delta(x_i - x_j)$$

Dipolar interactions in the single mode approximation

$$V_{Q1D}(x) = V(\theta)\left[V_{DDI}^{1D}\left(\frac{x}{l_\perp}\right) - \frac{8}{3}\delta\left(\frac{x}{l_\perp}\right)\right],$$
$$V(\theta) = \frac{\mu_0\mu_D^2}{4\pi}\frac{1-3\cos^2\theta}{4l_\perp^3},$$
$$V_{DDI}^{1D}\left(\frac{x}{l_\perp}\right) = -2\left|\frac{x}{l_\perp}\right| + \sqrt{2\pi}\left[1+\left(\frac{x}{l_\perp}\right)^2\right]$$
$$\times e^{\frac{1}{2}\left(\frac{x}{l_\perp}\right)^2}\mathrm{erfc}\left[\left|\frac{x}{\sqrt{2}l_\perp}\right|\right],$$

F. Deuretzbacher, J. C. Cremon, and S. M. Reimann, Phys. Rev. A **81**, 063616 (2010); **87**, 039903(E) (2013).

Variational method

$$H_{1D} = \mathcal{K} + \mathcal{V},$$
$$H_{\mathrm{var}} = \mathcal{K} + g\mathcal{U},$$
$$H_{\mathrm{var}}|\psi_0(g)\rangle = E_0(g)|\psi_0(g)\rangle$$
$$E_{var}(g) = E_0(g) - g\frac{\partial E_0(g)}{\partial g} + \langle\psi_0(g)|\mathcal{V}|\psi_0(g)\rangle.$$

The best approximation is obtained by minimizing with respect to g.

Choice of variational Hamiltonian

$$\mathcal{U} = \sum_{1\le i<j\le N}\delta(x_i - x_j),$$

H_{var} is the Hamiltonian of the Lieb-Liniger model, integrable by the Bethe Ansatz. $E_0(g)$ is obtained from the solution of an integral equation.
E. H. Lieb and W. Liniger, Phys. Rev. **130**, 1605 (1963)

Variational energy as a function of the structure factor

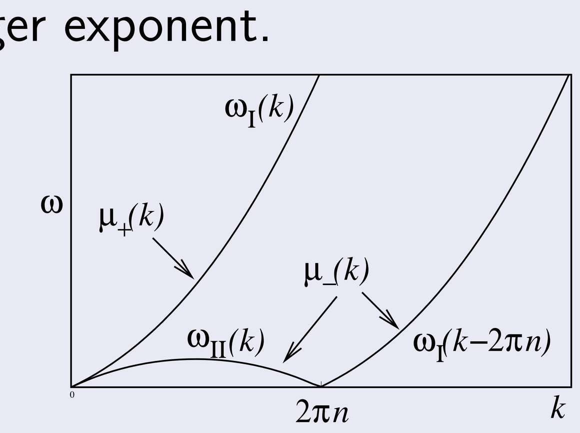
$$\frac{\langle\psi_0(g)|\mathcal{V}|\psi_0(g)\rangle}{L} = \frac{n}{2}\int_0^{+\infty}\frac{dk}{\pi}\hat{v}(k)[S(k)-1] + \frac{n^2}{2}\hat{v}(k=0),$$

where $S(k)$ is the static structure factor of the Lieb-Liniger model.

Cherny Brand Ansatz for the structure factor of the Lieb-Linger gas

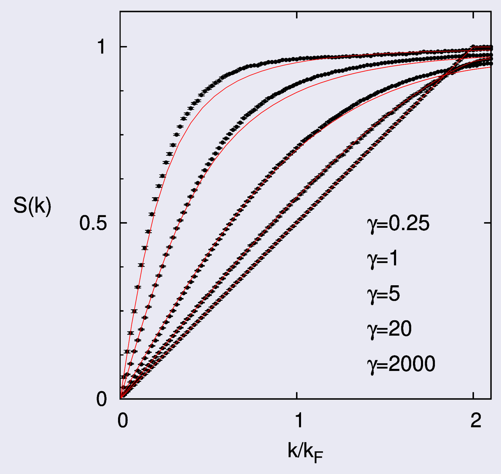
$$S(k) = \frac{k^2}{2m\omega_{II}(k)}$$
$$\times\frac{{}_2F_1\left(1+\frac{\sqrt{K}}{1+\sqrt{K}} + \mu_-(k) + \mu_+(k), 1+\mu_-(k); 2+\mu_-(k)-\mu_+(k), 1-\frac{\omega_{II}^2(k)}{\omega_I^2(k)}\right)}{{}_2F_1\left(1+\frac{2\sqrt{K}}{1+\sqrt{K}} + \mu_-(k) + \mu_+(k), 1+\mu_-(k); 2+\mu_-(k)-\mu_+(k), 1-\frac{\omega_{II}^2(k)}{\omega_I^2(k)}\right)}$$

- $\omega_I(k), \mu_+(k)$ dispersion and threshold exponent of Type-I excitations.
- $\omega_{II}(k), \mu_-(k)$ dispersion and threshold exponent of Type-II excitations.
- K Tomonaga-Luttinger exponent.

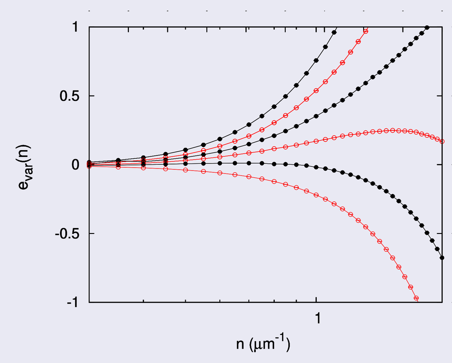


A. Y. Cherny and J. Brand, Phys. Rev. A **79**, 043607 (2009)

Comparison of the Ansatz for $S(k)$ with Quantum Monte Carlo



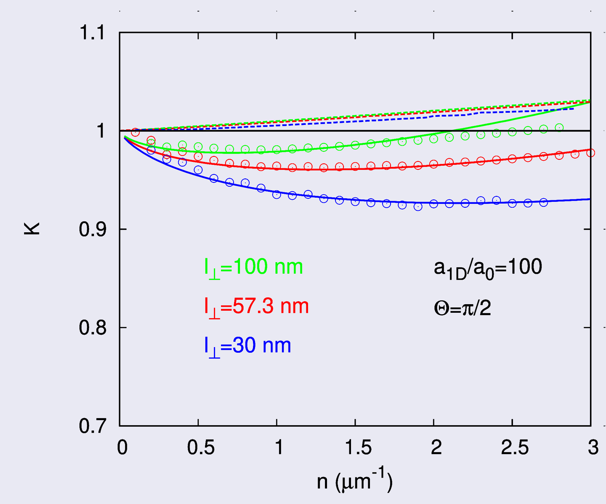
Variational energy per unit length for $\theta = 0$



Energy per unit length as a function of atom density for selected scattering lengths. Top to bottom $a_{1D}/a_0 = 5000, 6000, 7000, 8000, 9000$ and 10000

Tomonaga-Luttinger exponent K

$$\frac{u}{K} = \frac{1}{\pi\hbar}\frac{\partial^2 e_{GS}}{\partial n^2}, uK = \frac{\hbar\pi n}{m},$$

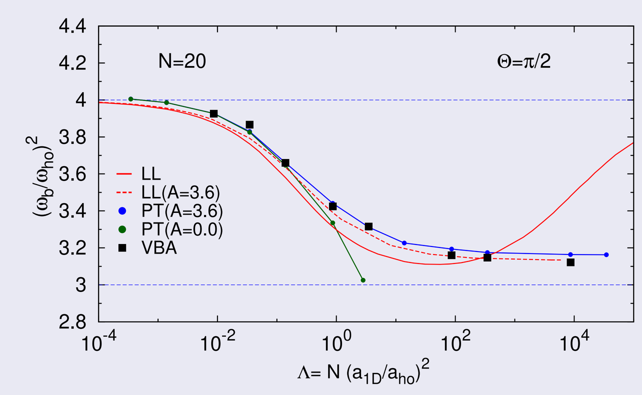


Breathing modes

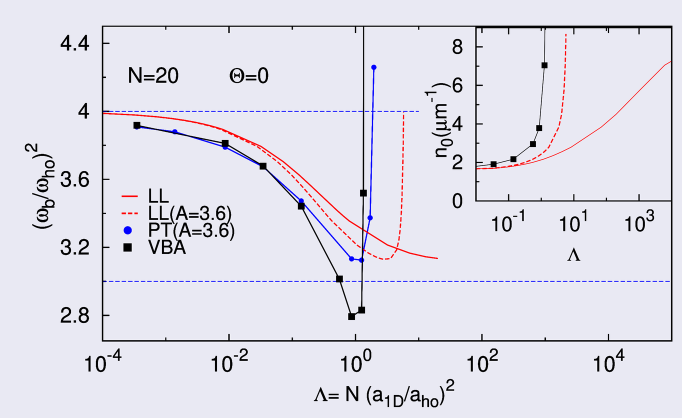
1. Add a longitudinal trapping potential $\frac{1}{2}m\omega_{ho}^2 xr$.
2. find density profile from stationary generalized Gross-Pitaveskii equation
3. obtain breathing mode from sum rule approach

$$\omega_B^2 = -2\langle\sum_{i=1}^N x_i^2\rangle\left[\frac{\partial\langle\sum_{i=1}^N x_i^2\rangle}{\partial\omega_{ho}^2}\right]^{-1}$$

Calculated breathing mode in the repulsive case



Calculated breathing mode in the attractive case



Perspectives

1. Other non-contact interactions: shoulder potential, etc...
2. Droplets
3. Super-Tonks type metastable states
4. Generalized time-dependent Gross-Pitaevskii equation