Spin-orbit coupling in a 1D interacting boson gas

E. Orignac

Laboratoire de Physique de l'ENS de Lyon - 46 Allée d'Italie - 69364 Lyon Cedex 7

11 Mars 2015

Coworkers

- Roberta Citro (Università degli Studi di Salerno)
- Maria Luisa Chiofalo (Università di Pisa)
- Stefania De Palo, Mario Di Dio (Università degli Studi di Trieste)

Outline

1. Spin-orbit coupling and artificial gauge fields
2. Chain of spin-1/2 bosons with spin-orbit coupling
3. analytic treatment
4. DMRG study
5. conclusions

Spin orbit coupling in first quantization

Spin-orbit term for spin-1/2 fermions

$$H = \frac{\mathbf{p}^2}{2m} + \frac{\hbar}{4m^2c^2}\sigma \cdot [\nabla V(\mathbf{r}) \times \mathbf{p}] + V(\mathrm{r})$$
$$\mathbf{p} = \frac{\hbar}{i}\nabla$$

Relativistic correction, derived from Dirac equation.

Under time-reversal T

$$T^{-1}\sigma T = -\sigma \quad ; \quad T^{-1}\mathbf{p}T = -\mathbf{p}$$
$$T^{-1}\sigma^a p^b T = +\sigma^a p^b$$

In solid-state physics: parabolic band

Perturbation theory in the spin-orbit term near a dispersion minimum.

example 1: Dresselhaus term [Phys. Rev. 100]

$$H_D = \rho_D(\sigma_x p_x - \sigma_y p_y)$$

if no center of inversion

example 2: Rashba term [JETP Lett. 39, 78 (1984)]

$$H_R = \rho_R \cdot (\sigma \times \mathbf{p}),$$

ρ_R : along an axis of at least 3-fold symmetry

In solid-state physics: tight-binding bands (I)

General spin-orbit coupling

$$H_{ab}(\mathbf{k}) = \epsilon_0(\mathbf{k})\delta_{ab} + \sum_j h_j(\mathbf{k})\sigma^j_{ab},$$

with $a, b =$ indices of the spin representation; $\mathbf{k}$ quasi-momentum

Symmetries

- Time reversal: $T^{-1}h_j(\mathbf{k})T = -h_j(\mathbf{k})$
- Translation symmetry: $h_j(\mathbf{k}+\mathbf{K}) = h_j(\mathbf{k})$ ($\mathbf{k} \in$ R. L.)
- Point group symmetry

In solid state physics: tight-binding bands (II)

Example in one dimension

$$H_{ab}(k) = -2t\cos(k) + 2\Delta\sin(k)(\sigma^z)_{ab}$$

Tight-binding model (2nd quantized)

$$\begin{aligned} H &= -\sum_{j,a,b} c^\dagger_{j,a}(t + i\Delta\sigma^z)_{ab} c_{j+1,b} + \mathrm{H.c.} \\ &= -\bar{t}\sum_{j,a,b} c^\dagger_{j,a} e^{i\lambda\sigma_z} c_{j+1,b} + +\mathrm{H.c.} \end{aligned}$$

$t + i\Delta = \bar{t}e^{i\lambda}$.

Spin-orbit coupling as a static gauge field

General tight-binding Hamiltonian for spin 1/2

$$H = \sum_{ij} t_{ij}[c^\dagger_{i,a} U^{ij}_{ab} c_{j,b} + c^\dagger_{j,b} (U^{ij}_{ab})^* c_{i,a}]$$

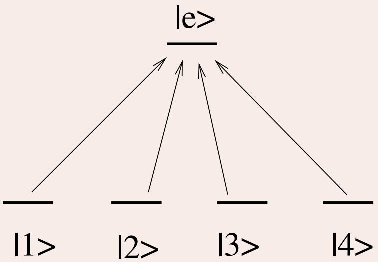
Under a local gauge transformation $c_j \to V_j c_j$, $U^{ij} \to V_i^\dagger U^{ij} V_j$. If $U^{ij} = V_i V_j^\dagger$ we can eliminate the spin-orbit coupling. This requires:

necessary condition

$$\prod_{\text{loop}} U^{ij} = 1$$

Artificial gauge fields with cold atoms

Geometric potentials



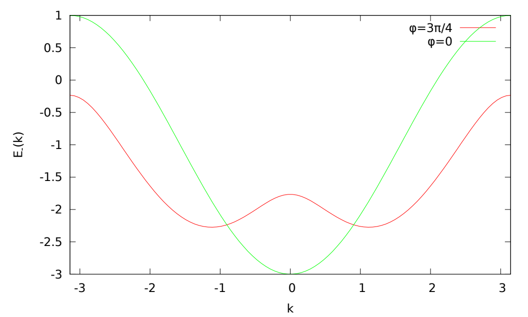
$$|\psi >= \sum_j \psi_j(\mathbf{r})|j(\mathbf{r})\rangle \; ; \; \mathbf{A}_{jk} = \frac{\hbar}{i}\langle j(\mathbf{r})|\nabla|k(\mathbf{r})\rangle$$

- J. Dalibard et al. Rev. Mod. Phys. **83**, 1523 (2011).
- N. Goldman et al. Rep. Prog. Physics **77**, 126401 (2014).

Non-interacting spin-1/2 bosons with s. o. c.

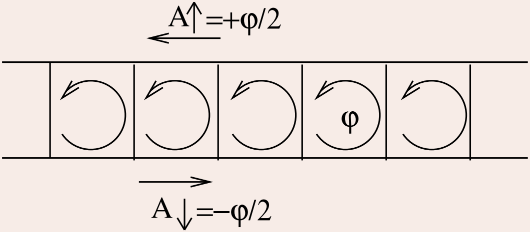
Hamiltonian and dispersion

$$H = -J_{\parallel} \sum_{j,\sigma} (b^\dagger_{j,\sigma} e^{i\sigma\varphi/2} b_{j+1,\sigma} + \mathrm{H.c.}) - J_{\perp} \sum_{j} (b^\dagger_{j,\uparrow} b_{j,\downarrow} + \mathrm{H.c.})$$



Equivalence with a two-leg ladder in a flux

Spinless bosons on a ladder with flux φ



Landau gauge: $\vec{A} \parallel$ chains

Equivalences

Definition of the currents in the ladder

$$j_{\parallel} = -iJ_{\parallel} \sum_{j,\sigma} \sigma (b^{\dagger}_{j,\sigma} e^{i\sigma\varphi/2} b_{j+1,\sigma} - \mathrm{H.c.})$$

$$j_{\perp} = -iJ_{\perp} \sum_{j,\sigma} \sigma b^{\dagger}_{j,\sigma} b_{j,-\sigma}$$

Mapping to spin-1/2 bosons with s. o. c.

- $j_{\parallel} \to$ spin current
- $j_{\perp} \to$ local magnetization along $\hat{\mathbf{y}}$

Zero temperature phases

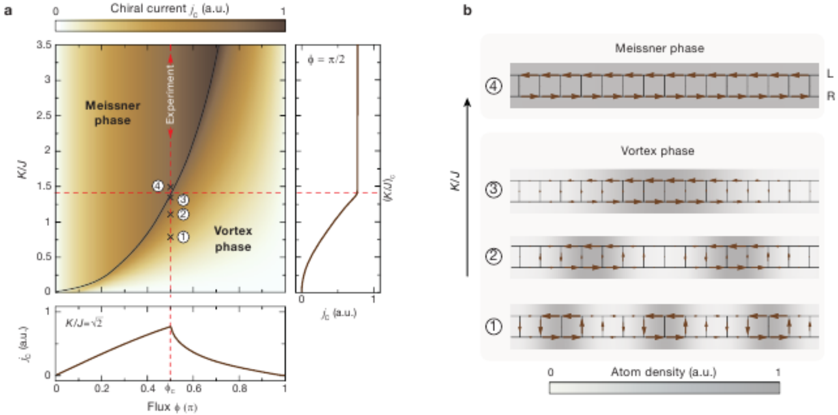
Meissner phase: single minimum in dispersion

$$j_{\|}(j) = J_{\|} \sin(\varphi/2) \; ; \; j_{\perp}(j) = 0.$$

Vortex phase: two minimas at $\pm k_c$ in dispersion

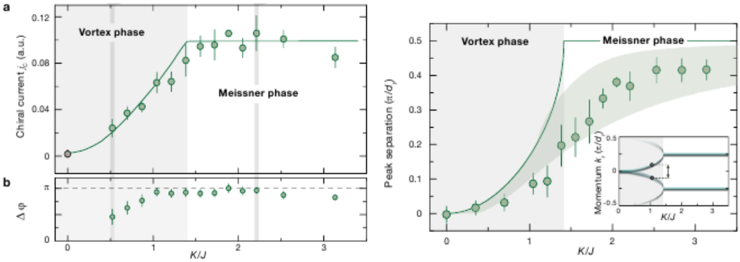
$$\begin{aligned} j_{\|}(j) &= \frac{J_{\perp}^2 \cos(\varphi/2)}{2\sin^2(\varphi/2)\sqrt{J_{\perp}^2 + 4J_{\|}^2 \sin^2(\varphi/2)}} + C_1(\varphi)\cos(2k_c j) \\ j_{\perp}(j) &= C_2(\varphi)\sin(2k_c j) \end{aligned}$$

Zero temperature phase diagram



From Atala et al. Nat. Phys. (2014)

Experimental measurements



From Atala et al. Nat. Phys. (2014)

What is the effect of interaction ?

Model Hamiltonian

$$H = -J_{\parallel} \sum_{j,\sigma} (b^{\dagger}_{j,\sigma} e^{i\sigma\varphi/2} b_{j+1,\sigma} + \mathrm{H.c.}) - J_{\perp} \sum_{j} (b^{\dagger}_{j,\uparrow} b_{j,\downarrow} + \mathrm{H.c.}) + U \sum_{j,\sigma} n_{j,\sigma}(n_{j,\sigma} - 1)$$

Limit $J_\perp \ll J_\parallel, U$

Bosonization approach [EO, T. Giamarchi PRB **64**, 144515 (2001)]

$$
\begin{aligned}
H &= H_0 + V \\
H_0 &= \sum_\sigma \int \frac{dx}{2\pi} \left[uK(\pi\Pi_\sigma - \sigma\varphi/(2a))^2 + \frac{u}{K}(\partial_x\phi_\sigma)^2 \right] \\
\partial_x\theta_\sigma &= \pi\Pi_\sigma \, ; [\phi_\sigma(x), \Pi_{\sigma'}(x')] = i\delta_{\sigma\sigma'}\delta(x-x') \\
b_{j,\sigma} &= e^{i\theta_\sigma}(ja) \sum_m A_m cos(2m\phi_\sigma - 2m\pi\rho_\sigma x) \\
n_{j,\sigma}/a &= \rho_\sigma - \frac{1}{\pi}\partial_x\phi_\sigma + \sum_m B_m cos(2m\phi_\sigma - 2m\pi\rho_\sigma x) \\
V &= 2J_\perp \int dx A_0^2 \cos(\theta_1 - \theta_2) + \ldots
\end{aligned}
$$

Spin Charge separation

Change of variables

$$\Pi_c = \frac{1}{\sqrt{2}}(\Pi_\uparrow + \Pi_\downarrow) \; ; \; \Pi_s = \frac{1}{\sqrt{2}}(\Pi_\uparrow - \Pi_\downarrow)$$

$$\phi_c = \frac{1}{\sqrt{2}}(\phi_\uparrow + \phi_\downarrow) \; ; \; \phi_s = \frac{1}{\sqrt{2}}(\phi_\uparrow - \phi_\downarrow)$$

$$H = H_c + H_s$$

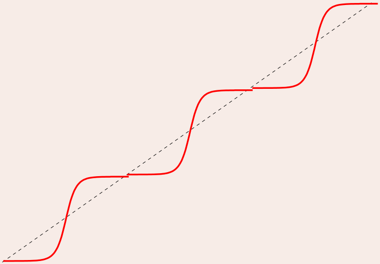
$$H_c = \int \frac{dx}{2\pi}\left[u_c K_c (\pi \Pi_c)^2 + \frac{u_c}{K_c}(\partial_x \phi_c)^2\right]$$

$$H_s = \int \frac{dx}{2\pi}\left[u_s K_s (\pi \Pi_s - \varphi/(\sqrt{8}a))^2 + \frac{u_s}{K_s}(\partial_x \phi_s)^2\right] - 2J_\perp A_0^2 \int dx \cos \sqrt{2}\theta_s$$

Commensurate-Incommensurate transition

Semiclassical picture

- $\cos\sqrt{2}\theta_s$ is imposing $\langle\theta_s\rangle = n\pi\sqrt{2}$
- φ is imposing $\langle\theta_s\rangle = \varphi x/(\sqrt{8}a)$
- low φ : stay with $\langle\theta_s\rangle = 0$ (Meissner)
- large φ : form solitons in θ_s (Vortex)



In the Meissner phase

Observables

$$
\begin{aligned}
\langle j_{\|} \rangle &= \frac{uK}{4\pi}\varphi \\
\langle S_j^z S_0^z \rangle &= O(e^{-|j|/\xi}) \\
\langle j_\perp(ja) j_\perp(0) \rangle &= O(e^{-|j|/\xi}) \\
\langle b_{j,\sigma} b_{0,\sigma'}^\dagger \rangle &\sim \frac{1}{j^{1/(2K_c)}}
\end{aligned}
$$

Mermin-Wagner $\Rightarrow$ no long-range order

In the vortex phase

Observables

$$\langle j_{\|}\rangle = \frac{uK}{4\pi}(\varphi - \sqrt{\varphi^2 - \varphi_c^2})$$

$$\langle S_j^z S_0^z\rangle = -\frac{K_s^*}{4\pi^2 j^2} + B_0^2 \frac{\cos(2\pi\rho_0 j)}{|j|^{K_s^*/2+K_c/2}}$$

$$Q \propto \sqrt{\varphi^2 - \varphi_c^2}$$

$$\langle j_\perp(ja) j_\perp(0)\rangle \sim \frac{\cos(2Qj)}{|j|^{1/K_s^*}}$$

$$\langle b_{j,\sigma} b^\dagger_{j,\sigma'}\rangle \sim \delta_{\sigma\sigma'} \frac{\cos(Qj/2)}{|j|^{1/(2K_c)+2/(2K_s^*)}}$$

Limit $U \ll J_\perp, J_\parallel$: Bogoliubov approximation

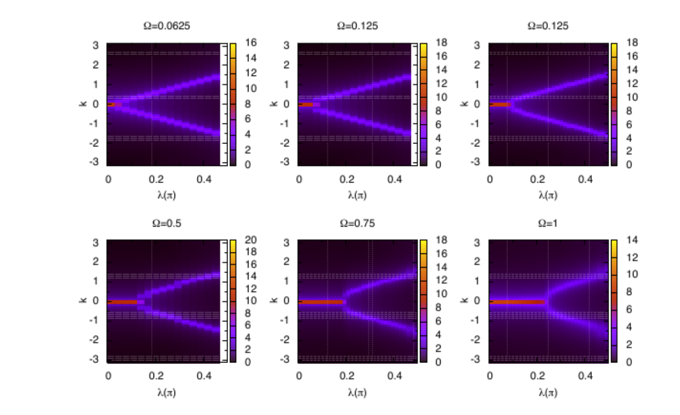
[A. Tokuno, A. Georges New J. Phys. **16**, 073005 (2014)]

- For $\varphi < \varphi_c$: reproduce H_c and Meissner state.
- For $\varphi > \varphi_c$: reproduces $H_c + H_s^*$ and the Vortex state.

Density Matrix Renormalization Group approach

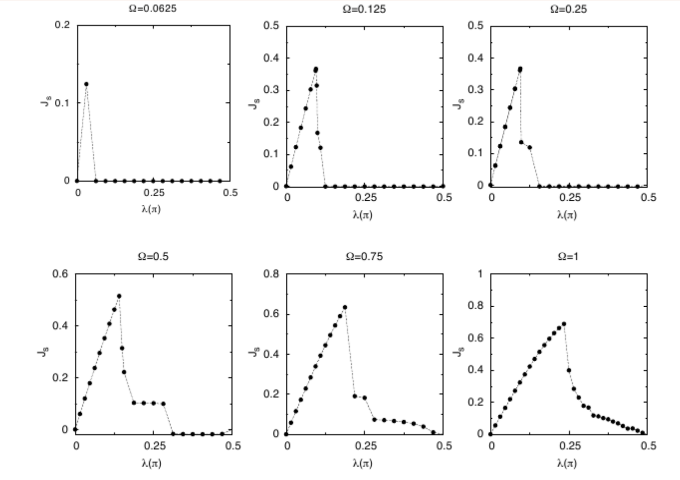
Momentum distribution [M. Di Dio *et. al.*, EO unpub. (2015)]

$n(k) = \sum_{k,\sigma} e^{-ikj} \langle b_{j\sigma} b^{\dagger}_{0,\sigma} \rangle \ \Omega \leftrightarrow J_{\perp} \ , \ \lambda \leftrightarrow \varphi \ , \ U = +\infty.$



Density Matrix Renormalization Group approach

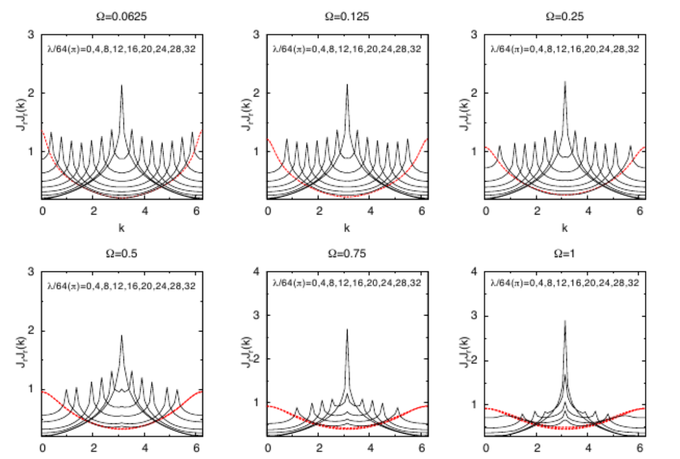
Leg current [M. Di Dio *et. al.*, EO unpub. (2015)]



Density Matrix Renormalization Group approach

Rung current [M. Di Dio *et. al.*, EO unpub. (2015)]

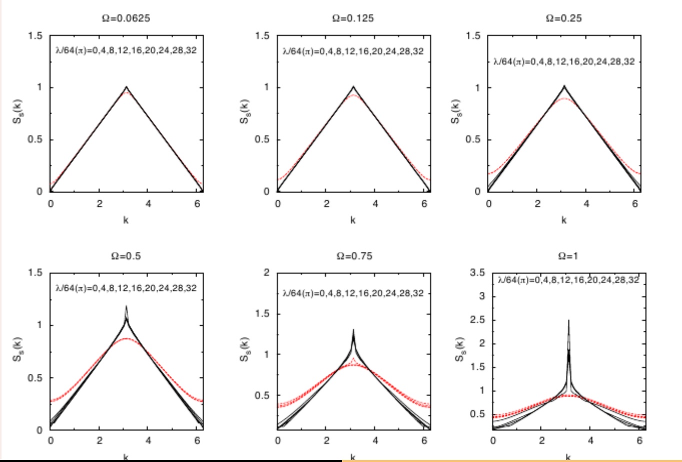
$$\langle J_r J_r \rangle(k) = \sum_j \langle j_\perp(ja) j_\perp(0) \rangle e^{ikj}$$



Density Matrix Renormalization Group approach

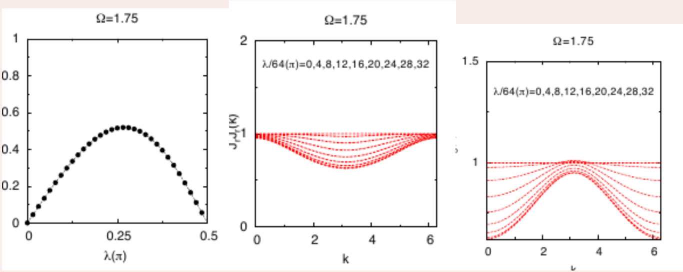
density wave correlations [M. Di Dio *et. al.*, EO unpub. (2015)]

$$S_s(k) = \sum_j \langle S_j^z S_0^z \rangle e^{ikj}$$



Density Matrix renormalization group

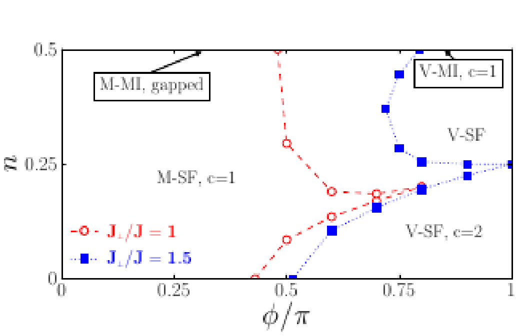
Loss of commensurate-incomensurate transition



$\Rightarrow$ Breakdown of field theory for large Ω.

Flux-filling phase diagram

M. Piraud et al. arXiv:1409.7016



Conclusions

Summary

1. Spin-orbit coupling/artificial gauge field experimentally accessible with cold atoms.
2. Meissner/Vortex transition obervable
3. Interactions modify the phases (quasi long range order)
4. critical exponents modified
5. qualitative agreement bosonization/DMRG at weak coupling

Perspectives

1. Commensurate filling and Mott insulating state
2. Analogs of Fractional Quantum Hall Effect (Le Hur &amp Petrescu 2014)
3. Interchain interaction and Ising transition
4. Case of $\varphi = \pi$