Density of states of a two-band one-dimensional conductor

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Hubbard model in magnetic field

$$
\begin{aligned}
H &= -t\sum_{j,a}(c^{\dagger}_{j,a}c_{j+1,a} + \text{H.c.}) + U\sum_{j} n_{j,\uparrow}n_{j,\downarrow} \\
&- \sum_{j,a}(\mu + g\mu_B Ba)n_{j,a}
\end{aligned}
$$

Retarded Green's function

$$
\begin{aligned}
G_a(j,t) &= -i\theta(t)\langle\{c_{j,a}(t), c^{\dagger}_{0,a}(0)\}\rangle \\
A(q,\omega) &= \sum_{j}\int dt G_a(j,t)e^{-i(qj-\omega t)}
\end{aligned}
$$

- map to two bands of interacting spinless fermions.
- bosonization method

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Definitions for bosonization

$$
\begin{aligned}
x &= j\alpha \\
[\phi_a(x), \partial_x\theta_b(y)] &= i\pi\delta_{ab}\delta(x-y) \\
c_{j,a} &\sim e^{ik_{F,a}x+\theta_a-\phi_a} + e^{-ik_{F,a}x+\theta_a+\phi_a}
\end{aligned}
$$

Bosonized Hamiltonian

$$
H = \int \frac{dx}{2\pi}\left[{}^t(\partial_x\theta)M\partial_x\theta + {}^t(\partial_x\phi)N\partial_x\phi\right]
$$

perturbative expressions

$M_{ab} = v_a\delta_{ab}$
$N_{ab} = v_a\delta_{ab} + (1-\delta_{ab})U\alpha/\pi$
$v_a = 2t\alpha\sin(k_{F,a}\alpha)$

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Diagonalization of bosonized Hamiltonian

$$
\begin{aligned}
\Delta_1 &= {}^tR_1 M R_1 \\
\Delta_2 &= {}^tR_2 \Delta_1^{1/2} {}^tR_1 N R_1 \Delta_1^{1/2} R_2 \\
\theta &= R_1 \Delta_1^{-1/2} R_2 \Delta_2^{1/4} \vartheta = Q\vartheta \\
\phi &= R_1 \Delta_1^{1/2} R_2 \Delta_2^{-1/4} \varphi = P\varphi \\
H &= \int \frac{dx}{2\pi} \left[{}^t(\partial_x \vartheta) \Delta_2^{1/2} \partial_x \vartheta + {}^t(\partial_x \varphi) \Delta_2^{1/2} \partial_x \varphi \right]
\end{aligned}
$$

$\Delta_2^{1/2}$=diagonal matrix. Its eigenvalues are the velocities of the modes.

Correlation functions

$$\langle e^{i(\theta_a-\phi_a)(x,t)}e^{-i(\theta_a-\phi_a)(0,0)}\rangle =$$
$$= e^{\langle(\theta_a(x,t)-\phi_a(x,t))(\theta_a(0,0)-\phi_a(0,0))\rangle_{conn.}}$$
$$\langle(\theta_a(x,t)-\phi_a(x,t))(\theta_a(0,0)-\phi_a(0,0))\rangle_{conn.}$$
$$= \sum_b \nu_b \ln\left(\frac{\alpha}{\alpha+i(v_b t - x)}\right) + \nu'_b \ln\left(\frac{\alpha}{\alpha+i(v_b t + x)}\right)$$
$$\nu_b = (P_{ab}+Q_{ab})^2/4$$
$$\nu'_b = (P_{ab}-Q_{ab})^2/4$$

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$T = 0$ spectral functions

$$
\begin{aligned}
A(k_F + q, \omega) &= I(q,\omega) + I(-q,-\omega) \\
I(q,\omega) &= \frac{C\alpha^{\nu_1+\nu_2+\nu_1'+\nu_2'}}{2\pi} \\
&\times \int \frac{e^{i(\omega t - qx)}}{(\alpha + i(v_1 t - x))^{\nu_1}(\alpha + i(v_2 t - x))} \\
&\times \frac{dxdt}{(\alpha + i(v_1 t + x))^{\nu_1'}(\alpha + i(v_2 t + x))^{\nu_2'}}
\end{aligned}
$$

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Feynman identity

$$\frac{1}{A_1^{\nu_1}A_2^{\nu_2}} = \frac{\Gamma(\nu_1+\nu_2)}{\Gamma(\nu_1)\Gamma(\nu_2)}\int_0^1 du \frac{u^{\nu_1-1}(1-u)^{\nu_2-1}}{(A_1 u + A_2(1-u))^{\nu_1+\nu_2}}$$

Master formula

$$I(q,\omega) = \frac{\pi^2}{\Gamma(\nu_1)\Gamma(\nu_2)\Gamma(\nu'_1)\Gamma(\nu'_2)}\int_0^1 du_1 u_1^{\nu_1-1}(1-u_1)^{\nu_2}$$

$$\times(\omega - v(u_1)q)^{\nu'_2+\nu'_1-1}\int_0^1 du_2 u_2^{\nu'_1-1}(1-u_2)^{\nu'_2-1}$$

$$\times|\omega + v(u_2)q|^{\nu_2+\nu_1-1}\frac{\Theta(\omega - v(u_1)q)\Theta(\omega+v(u_2)q)}{(v(u_1)+v(u_2))^{\nu'_2+\nu'_1\nu_2+\nu_1-1}}$$

$$v(u) = uv_2 + (1-u)v_1$$

Threshold ($v_2 < v_1$)

$$I(q>0,\omega<v_2 q) = 0 \Rightarrow A(q>0,|\omega|<v_2 q) = 0$$

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$0 < v_2 q < \omega < v_1 q$

$$I(q,\omega) = \frac{\pi^2}{\Gamma(\nu_2)\Gamma(\nu_1')\Gamma(\nu_2')\Gamma(\nu_1+\nu_1'+\nu_2')} \times \frac{(\omega - v_2 q)^{\nu_2'+2\nu_1'-1/2}(v_1 q-\omega)^{2\nu_2'+\nu_1'-1/2}}{(v_1-v_2)^{2\nu_1'+2\nu_2'}(\omega+v_2 q)^{\nu_2'}(\omega+v_2 q)^{\nu_1'}} \times \int_0^1 t^{\nu_2'-1}(1-t)^{\nu_1'-1}$$

$${}_2F_1\left(2(\nu_1'+\nu_2'), \nu_1'+\nu_2'; \nu_2'+2\nu_1'+1/2; 1+\frac{\omega - v_1 q}{(v_1-v_2)q} \times \frac{(\omega+v_2 q)(v_1+v_2)+(v_1-v_2)(v_2 q-\omega)t}{(\omega+v_2 q)(\omega+v_1 q)}\right)$$

${}_2F_1$ is Gauss' hypergeometric function.

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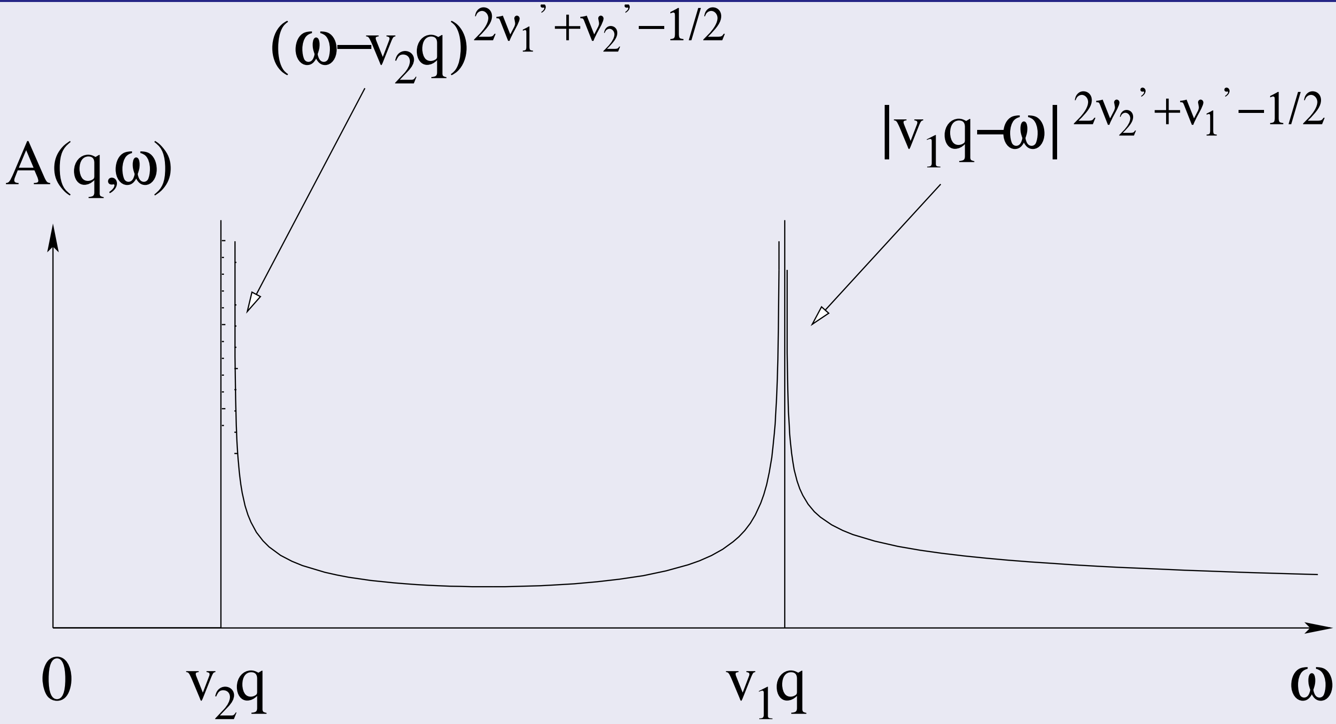
spectral function for $\omega > 0$: peaks



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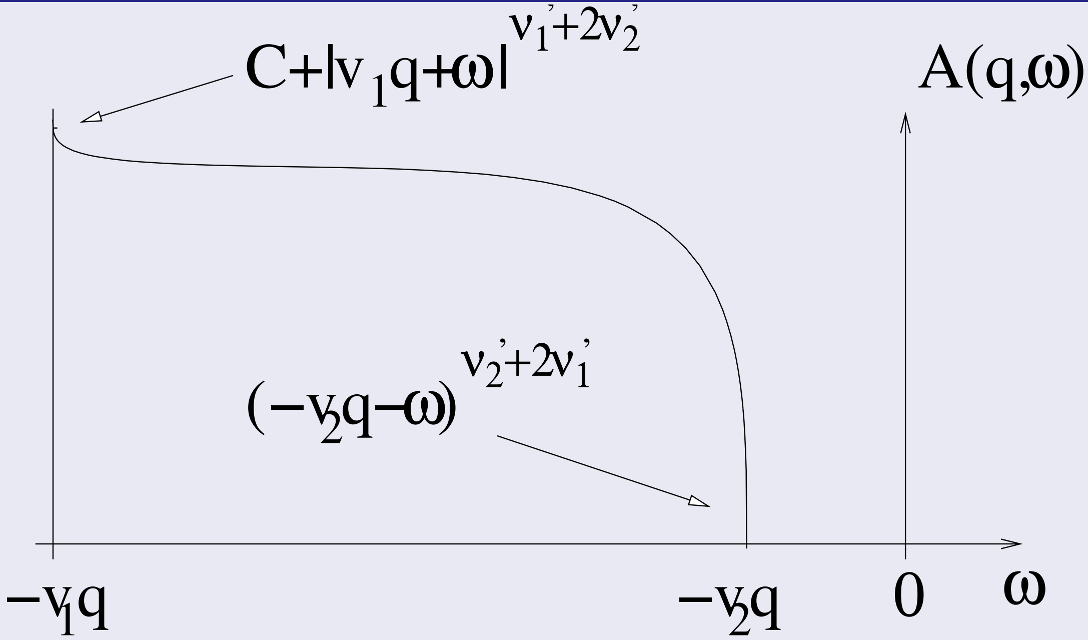
spectral function for $-v_1 q < \omega < 0$: cusps



Conclusion

1. Feynman identity used to obtain spectral functions of a 2-band spinless conductor
2. expressions in terms of integrals of Gauss hypergeometric functions
3. Cusps and peaks easily recovered from general formula.

Perspectives

1. More general case with multiple bands (such as 2-leg ladder).
2. case of positive temperature

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