Two leg spin ladders under magnetic field

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Collaboration

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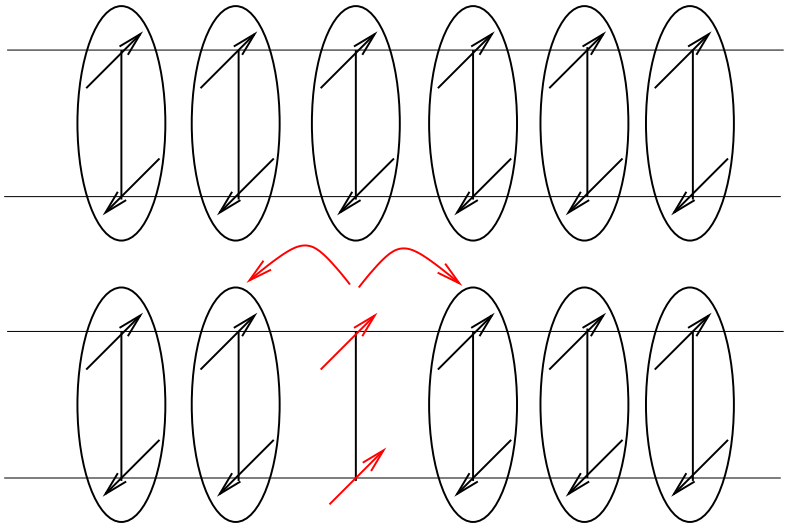
The antiferromagnetic spin-1/2 two-leg ladder

$$H = \sum_n J_{\|}(\vec{S}_{n,1} \cdot \vec{S}_{n+1,1} + \vec{S}_{n,2} \cdot \vec{S}_{n+1,2}) + J_{\perp} \vec{S}_{n,1} \cdot \vec{S}_{n,2}$$

Ground state is a spin singlet. Excited states are spin 1 triplets, separated from the ground state by a gap.

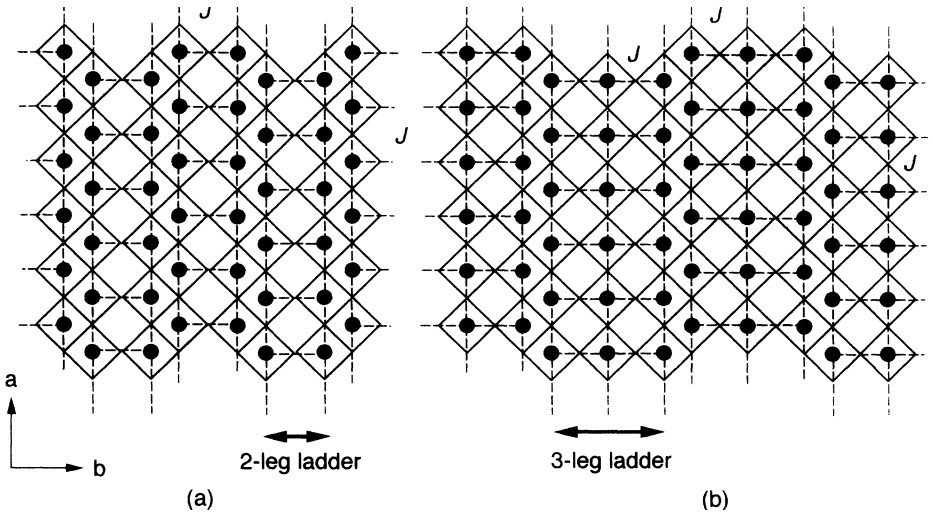


Experimental realizations

- $SrCu_2O_3$ M. Azuma et al., Phys. Rev. Lett. **73**, 3463 (1994).
- $Sr_{14}Cu_{24}O_{41}$ J. Akimitsu J. Phys. Soc. Jpn. (1996).
- CaV_2O_5 J. Phys. Soc. Jpn. **67**, 755 (1998).
- $(C_5H_{12}N)_2CuBr_4$ (BPCP) Phys. Rev. Lett. **86**, 5168 (2001).

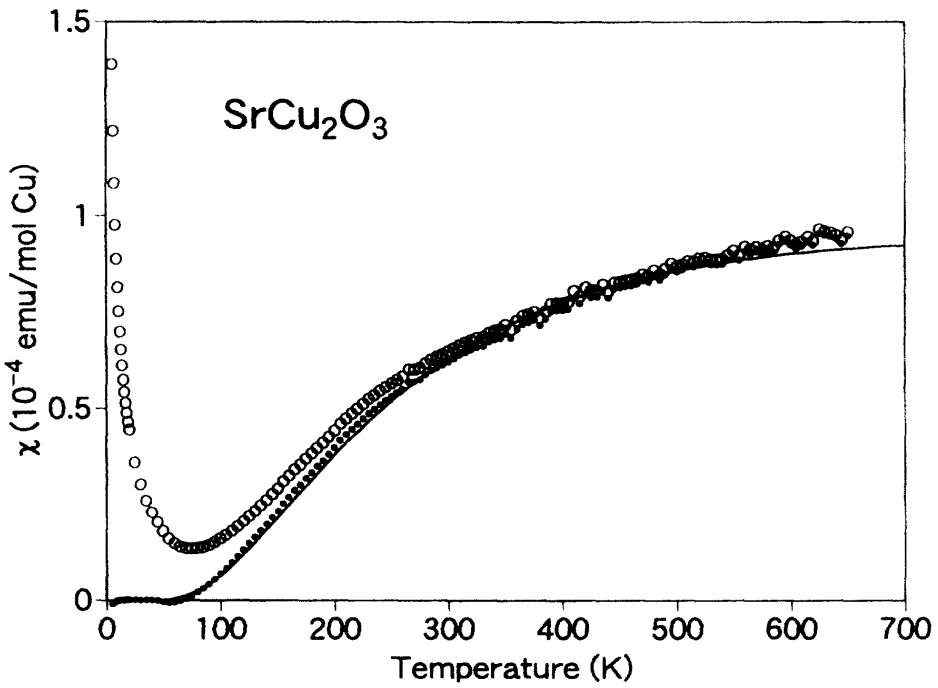
The Cu^{2+} ions are carrying the spins $1/2$.
Superexchange is mediated through the O^{2-} or Br^- ions.

Structure of the ladder material $SrCu_2O_3$



From M. Azuma et al., Phys. Rev. Lett. **73**, 3463 (1994).
Dots : Cu^{2+} ions ; Corners of squares : O^{2-} ions.

Magnetic susceptibility of the ladder material $SrCu_2O_3$

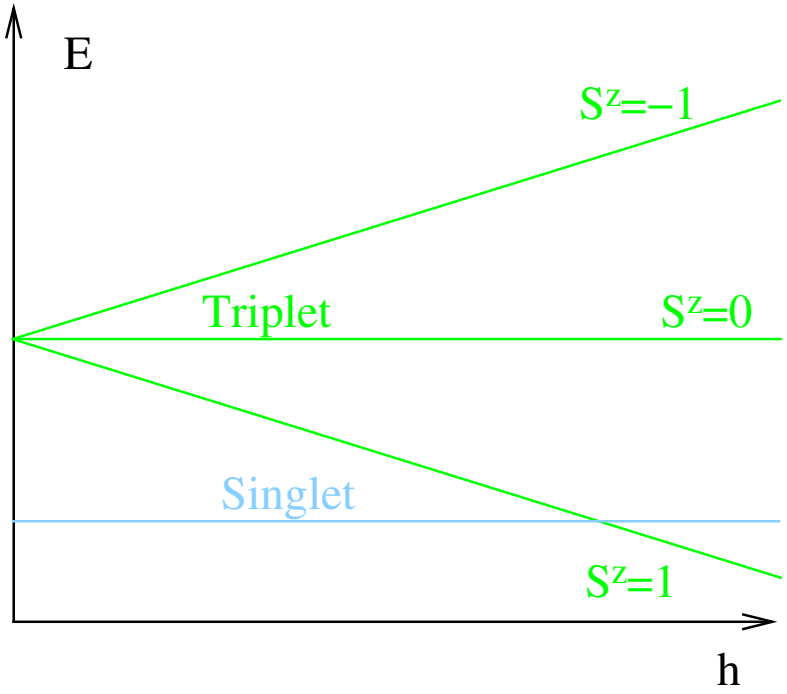


From M. Azuma et al., Phys. Rev. Lett. **73**, 3463 (1994).

$\chi(T) = \frac{\alpha}{\sqrt{T}} e^{-\Delta/(k_B T)}$ with $\Delta/k_B = 420$ K.

Two leg ladder under a magnetic field

Case $J_{\|} = 0$



$\Rightarrow$ Beyond a critical field, ground state = triplets with $S^z = 1$.

$J_{\|} \neq 0 \Rightarrow$ broadening of the transition

Effective theory for $J_\perp \gg J_\parallel$

Pseudospin description of the low energy subspace

- $\tau_z = -1/2 \Rightarrow$ singlet on the rung
- $\tau_z = +1/2 \Rightarrow$ triplet $S^z = 1$ on the rung

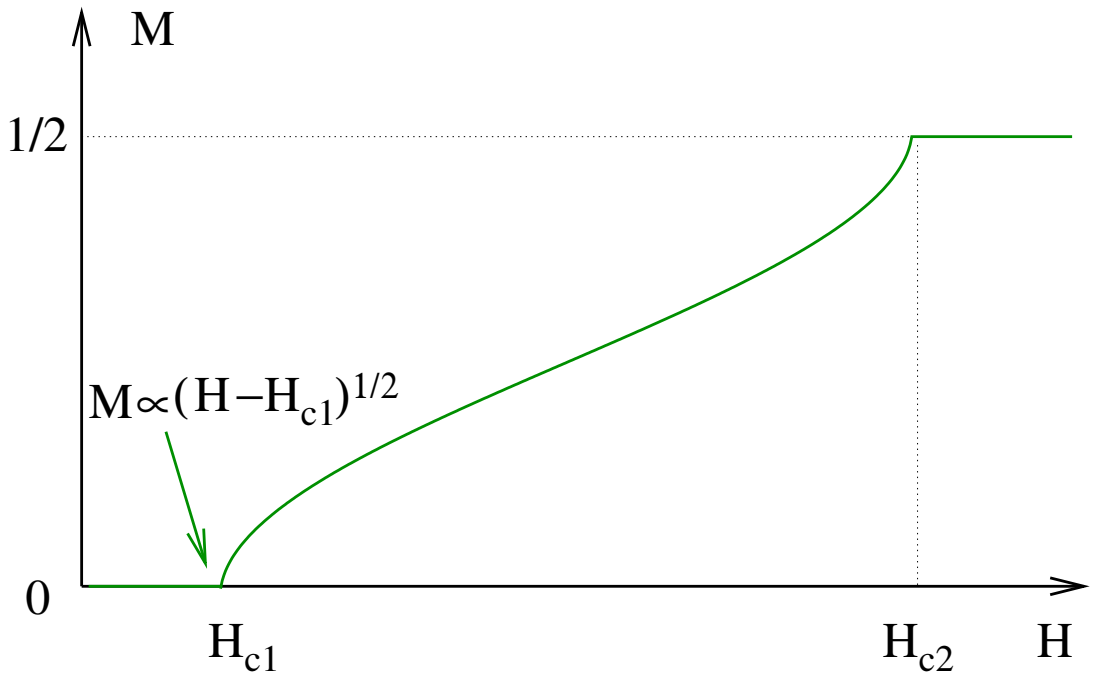
$$S^+_{n,p} \rightarrow \frac{(-)^p}{\sqrt{2}}\tau^+_n; S^z_{n,p} \rightarrow (1 + 2\tau^z_n)/4$$

Effective Hamiltonian

$$
\begin{aligned}
H_{\text{eff.}} &= \frac{J_{\parallel}}{2}\sum_n (\tau_n^+\tau_{n+1}^- + \tau_n^-\tau_{n+1}^+ + \tau_n^z\tau_{n+1}^z) \\
&- (h - J_{\perp} - J_{\parallel})\sum_n \tau_n^z
\end{aligned}
$$

XXZ spin-1/2 chain $\Rightarrow$ Luttinger liquid state.
Exact magnetization curve can be obtained.

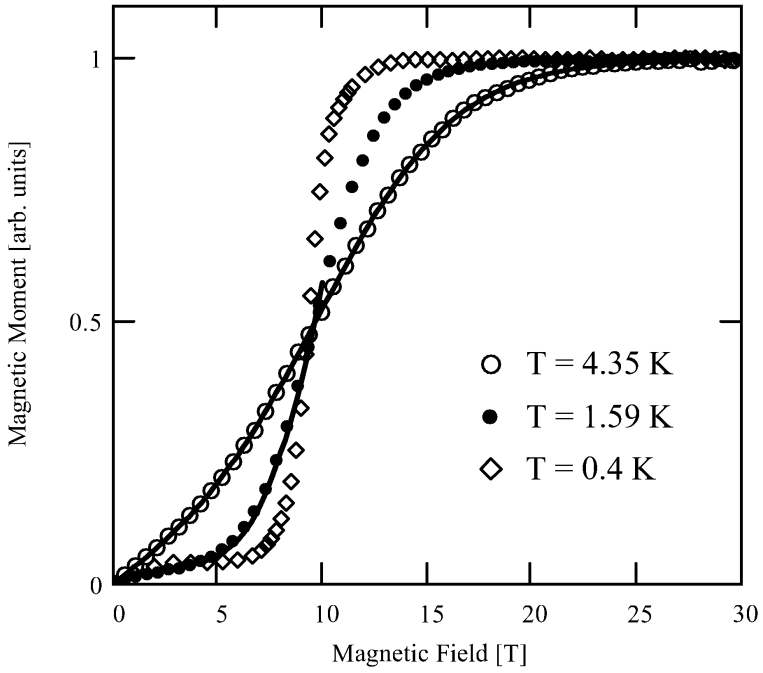
Theoretical magnetization curve at zero temperature



Luttinger liquid description of the magnetized state

$$
\begin{aligned}
\mathcal{H} &= \int \frac{dx}{2\pi}\left[u(H)K(H)(\pi\Pi)^2 + \frac{u(H)}{K(H)}(\partial_x\phi)^2\right], \\
[\phi(x),\Pi(x')] &= i\delta(x-x') \\
S_{n,p} &= (-)^p\lambda_x(H)e^{i\pi\int^{x=na}dx'\Pi(x')} \\
S_{n,p} &= -\frac{a}{\pi}\partial_x\phi + \lambda_z(H)\cos(2\phi(na)-(1-2m(H))\pi n)
\end{aligned}
$$

Magnetization curve for $(5IAP)_2CuBr_4 \cdot 2H_2O$



From Landee et al. Phys. Rev. B **63**, 100402 (2001)

What is the effect of interladder coupling ?

1. on the zero field spin gap state ?
2. on the Luttinger liquid state for applied field ?

Coupled ladders

$$
\begin{aligned}
H &= \sum_{n,m} J_\perp \vec{S}_{n,m,1} \cdot \vec{S}_{n,m,2} + J_\parallel (\vec{S}_{n+1,m,1} \cdot \vec{S}_{n,m,1} + \vec{S}_{n+1,m,2} \cdot \vec{S}_{n,m} \\
&+ J'_\perp \sum_{n,m} S_{n,m,2} \cdot S_{n,m+1,1}
\end{aligned}
$$

$J_\perp, J_\parallel \gg J'_\perp$.

Stability of the gapful state

A Bond Operator Theory mean field treatment lead to a stable spin gap for $J'_\perp < 0.12 J_\perp$.
B. Normand, T. M. Rice Phys. Rev. B **56**, 8760 (1997).
More generally, the presence of the spin gap in the isolated ladder implies that its ground state is stable in the presence of weak perturbations.

Stability of the magnetized state

Interladder coupling gives a term :

$$\frac{J'_\perp}{4}\sum_{n,m}(-\tau^+_{n,m}\tau^-_{n,m-1}-\tau^-_{n,m}\tau^+_{n,m-1}+\tau^z_{n,m}\tau^z_{n,m-1})$$

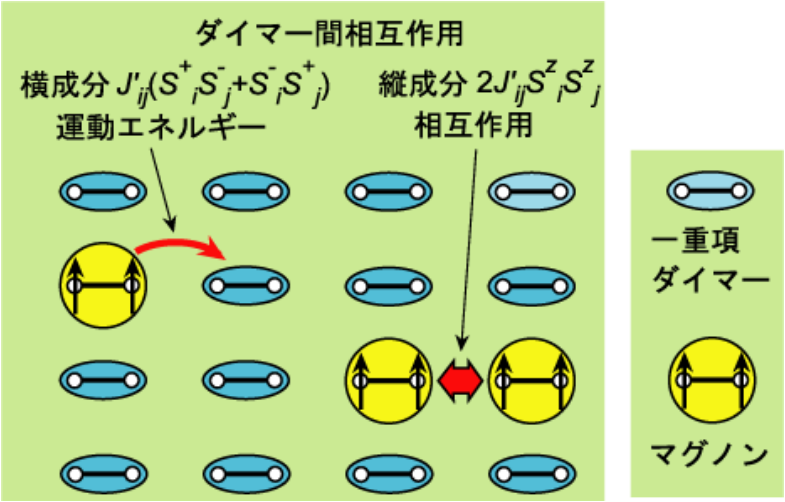
This term can stabilize a phase with $\langle\tau^+_{n,m}\rangle\neq 0$ i.e. an antiferromagnetic order in the plane perpendicular to the applied magnetic field.

Another interpretation of antiferromagnetic order

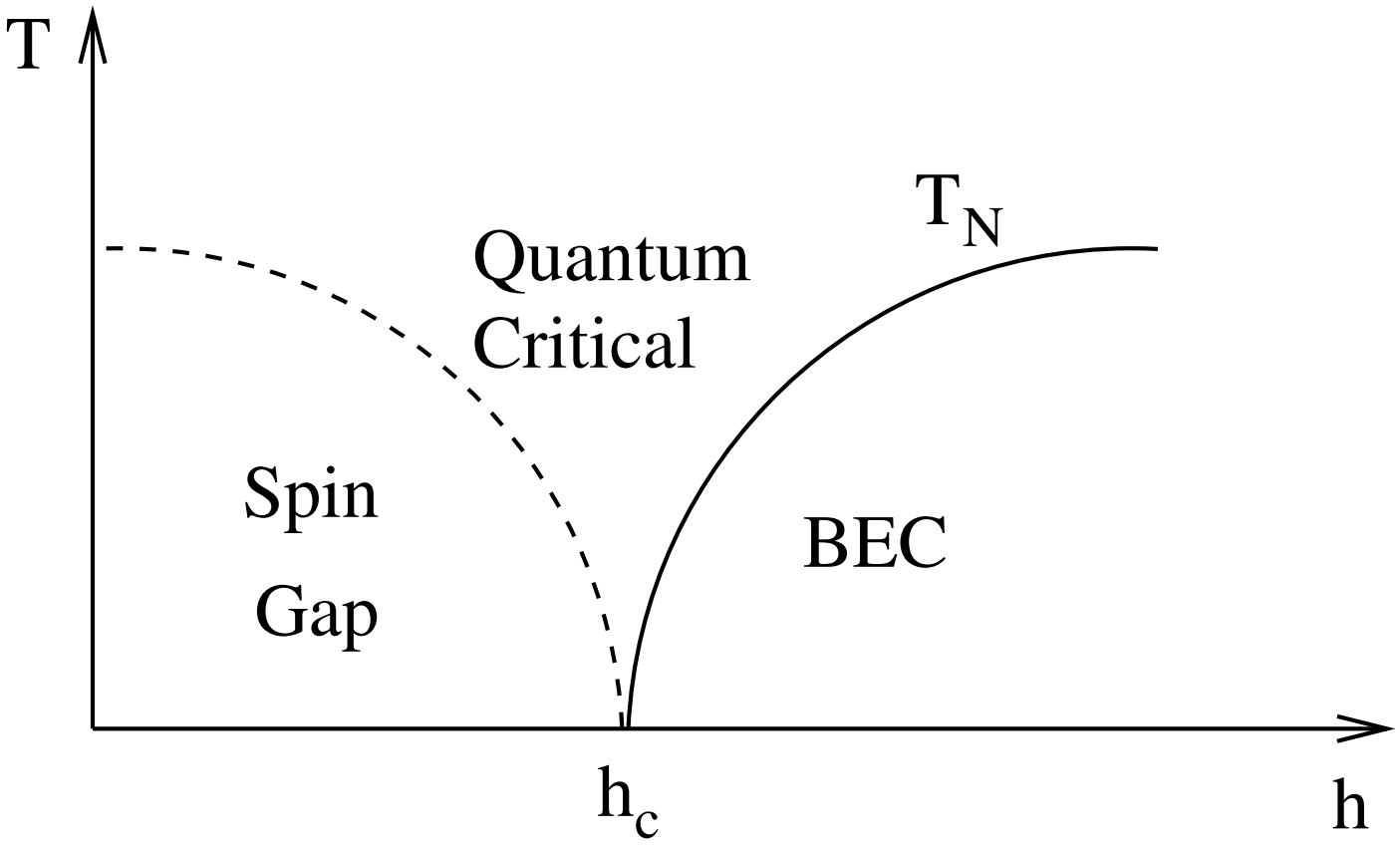
$b_i \leftrightarrow \tau_i^- \; ; \; n_i = b_i^\dagger b_i \rightarrow \tau_i^z$ with constraint $(b_i^\dagger)^2 = 0$ (forbidden double occupancy).

$$\mathcal{H} = -\sum_{i,j} t_{ij} b_i^\dagger b_j + \sum_{i,j} V_{ij} n_i n_j,$$

$\Rightarrow$ Bose Einstein condensation of magnons.



Phase diagram in T-H plane



Experiments

Materials :

- $TlCuCl_3$, $KCuCl_3$ (3D dimer network)
- $BaCuSi_2O_6$ (2D dimer network)
- BPCP (coupled ladders)

Probes :

- magnetization measurements
- Neutron scattering
- NMR, ESR

Coupled ladders with $J_\perp \gg J_\parallel$

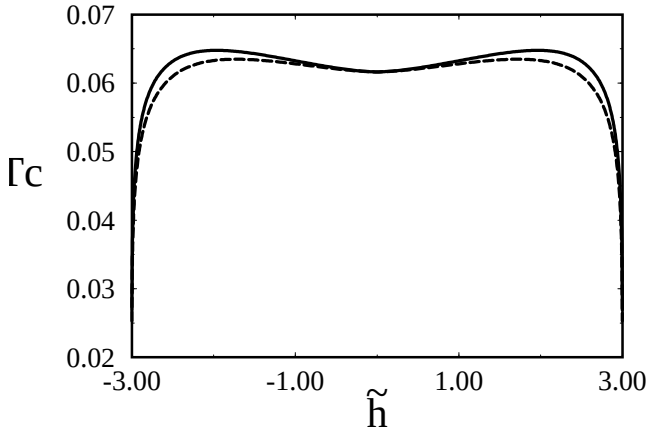
Each ladder is the Luttinger liquid state

$$\begin{aligned}\mathcal{H} &= \int dx \sum_m \left[u(H)K(H)(\pi\Pi_m)^2 + \frac{u(H)}{K(H)}(\partial_x\phi_m)^2 \right] \\ &- J'_\perp \lambda_x^2 \sum_{\langle m,m'\rangle} \cos(\theta_m - \theta_{m'})\end{aligned}$$

$\theta_m = \pi \int^x dx' \Pi(x')$
Mean field theory :
$\cos(\theta_n - \theta_{n'}) \rightarrow \langle \cos\theta_n \rangle \cos\theta_{n'} + \cos\theta_n \langle \cos\theta_{n'} \rangle$
T. Giamarchi, A. M. Tsvelik Phys. Rev. B **59** (1999).

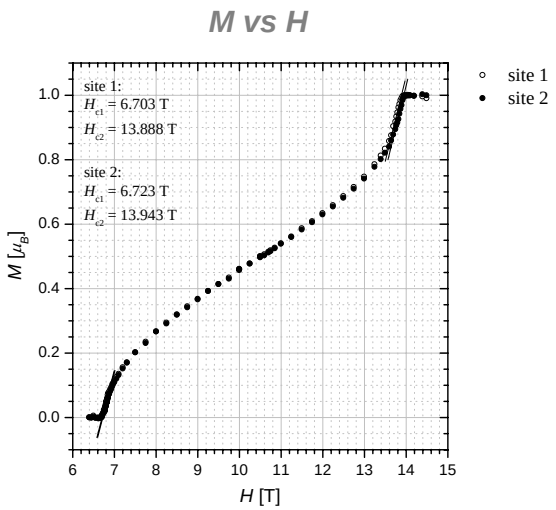
Transition temperature from mean field theory

$$1 = zJ'_{\perp}\lambda_x^2\chi_{cos\theta}(\vec{Q}, H, T_N(H))$$



Camel-Shaped $T_N(H)$ for $J_{\perp} \gg J_{\parallel}$.

Magnetization curve for BPCP



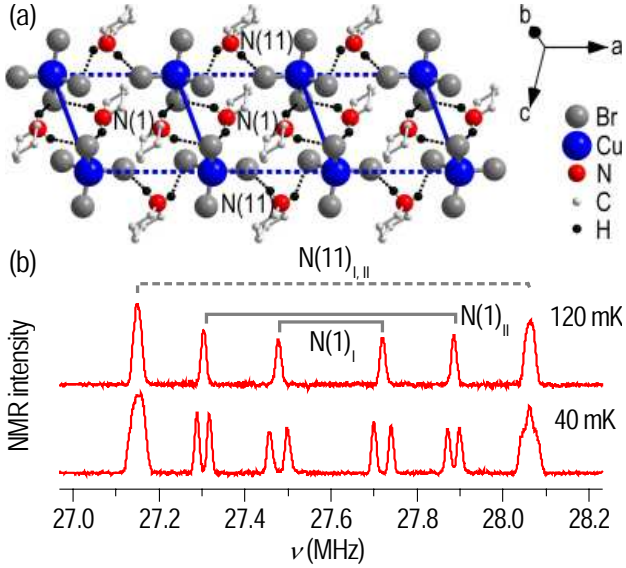
$$J_\perp/k_B = 12.9K, J_{||}/k_B = 3.6K.$$

More complete theory needed

$J_\perp \gg J_\parallel$ not satisfied. Mapping on effective XXZ chain not applicable, but Luttinger liquid description still valid.

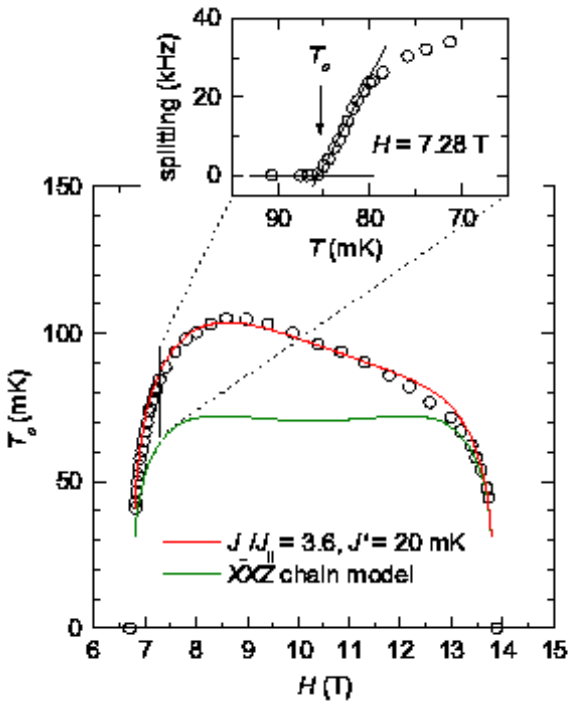
- accurate determination of $u(H), K(H), \lambda_x(H)$ from DMRG
- determination of order parameter at $T = 0$ from sine-Gordon model.

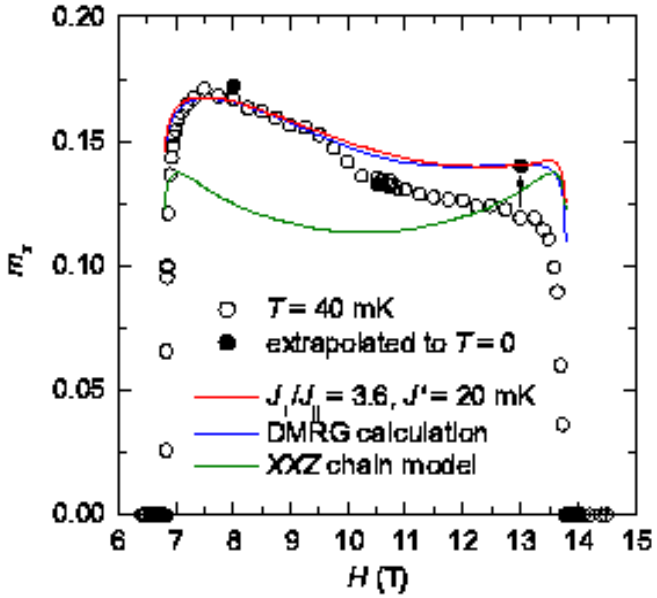
NMR measurements on BPCP



From Klanjsek et al. Phys. Rev. Lett. **100**, (2008).

NMR measurements on BPCP





From Klanjsek et al. Phys. Rev. Lett. **100**, (2008).

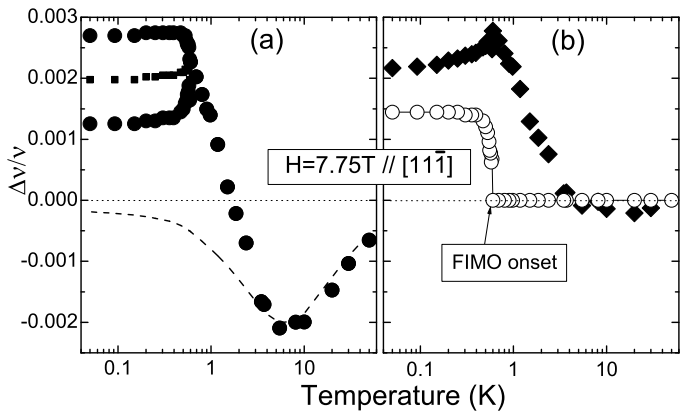
Dzyaloshinskii-Moriya interaction

$$H_{DM} = \vec{D} \cdot (\vec{S}_1 \times \vec{S}_2)$$

results from relativistic spin-orbit interactions.

Transverse staggered magnetization in $Cu_2(C_5H_{12}N_2)_2Cl_4$

Clémancey et al. Physical Review Letters **97**, 167204 (2006)
A transverse staggered magnetization that does not break the lattice symmetry is seen in NMR measurements.



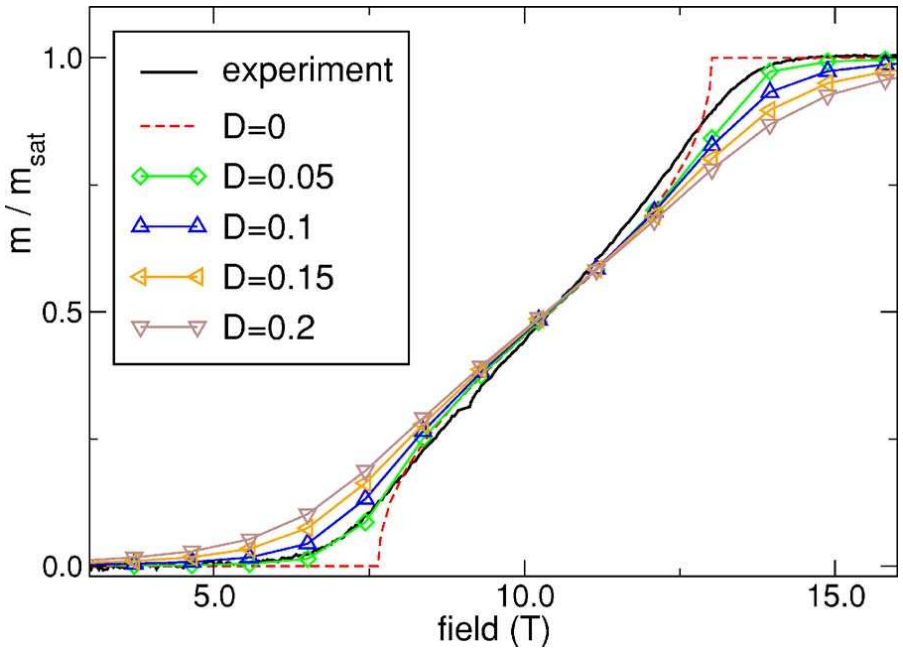
Dzyaloshinskii-Moriya interactions seem to explain this observation.

Ladder with Dzyaloshinskii-Moriya interactions

$$
\begin{aligned}
H &= \sum_n \Big[J_{\parallel}(\vec{S}_{n,1}\cdot\vec{S}_{n+1,1} + \vec{S}_{n,2}\cdot\vec{S}_{n+1,2}) + J_{\perp}\vec{S}_{n,1}\cdot\vec{S}_{n,2} \\
&\quad - h(S^z_{n,1} + S^z_{n,2})\Big] + H_{\mathrm{DM}} \\
H_{\mathrm{DM}} &= \sum_n (-)^n \vec{D}\cdot(\vec{S}_{n,1}\times\vec{S}_{n,2})
\end{aligned}
$$

where $\vec{D} = D\hat{y}$.

Numerical study



S. Capponi, D. Poilblanc Phys. Rev. B, **75**, 092406 (2007).

Strong coupling approach

$$H_{\mathrm{DM,eff.}} = \frac{D}{\sqrt{2}} \sum_n (-1)^n \tau_n^x.$$

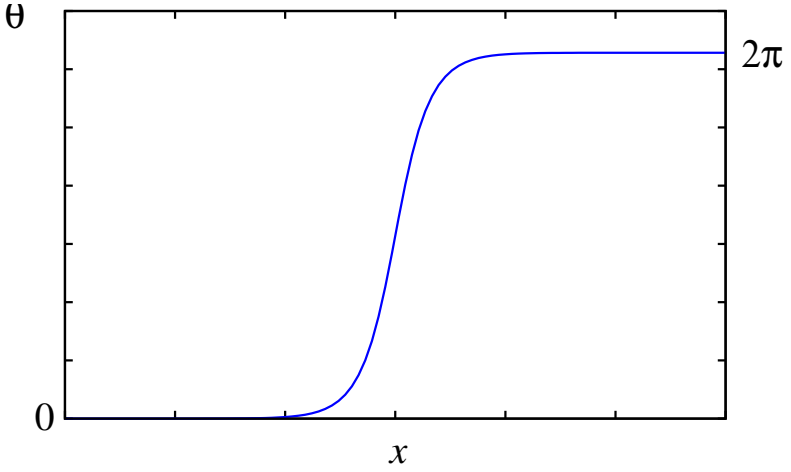
Can be mapped to the quantum sine-Gordon model.

$$H = H_{LL} - \frac{D\lambda_x}{a\sqrt{2}} \int dx \cos\theta$$

E. Orignac et al. Phys. Rev. B, **76**, 144422 (2007).

Solitons and breathers in the classical sine-Gordon

The equations of motions of the classical sine-Gordon model possess soliton and antisoliton solutions.



solitons and antisolitons can form bound states called breathers.

Quantum case

Breather energies are quantized :

$$M_n = 2M_0 \sin\left(\frac{\pi}{2}\frac{n}{8K(H)-1}\right),$$

where M_0 is soliton energy, n is integer and $1 \leq n < 8K - 1$. Breathers excitations will give peaks in Raman scattering measurements.

Conclusions

- Two leg spin-1/2 ladders under magnetic field behave as a Luttinger liquid for field larger than the spin gap.
- At sufficiently low temperature, the Luttinger liquid is unstable to antiferromagnetic ordering.
- A *quantitative* comparison of theory with experiment is possible in the case of the organic ladder BPCP.
- Dzyaloshinskii-Moriya interactions can spoil the Luttinger liquid state under field.
- A signature of Dzyaloshinskii-Moriya interaction in the ladder is the presence of breathers in the partially magnetized state.

Perspectives

- Other 1D spin gap materials (Haldane gap, dimerized chains...)
- Field induced ordering in 2D/3D materials ($\mathrm{TlCuCl_3}$...)
- effect of disorder ($\mathrm{Mg}^{2+}, \mathrm{Zn}^{2+}$ non-magnetic impurities)