Two leg spin ladders under magnetic field

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Collaboration

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The antiferromagnetic spin-1/2 two-leg ladder

\[ H = \sum_{n} J_{\parallel} (\vec{S}_{n,1} \cdot \vec{S}_{n+1,1} + \vec{S}_{n,2} \cdot \vec{S}_{n+1,2}) + J_{\perp} \vec{S}_{n,1} \cdot \vec{S}_{n,2} \]

Ground state is a spin singlet. Excited states are spin 1 triplets, separated from the ground state by a gap.
Experimental realizations


The \( \text{Cu}^{2+} \) ions are carrying the spins 1/2. Superexchange is mediated through the \( \text{O}^{2–} \) or \( \text{Br}^{–} \) ions.
Structure of the ladder material $\text{SrCu}_2\text{O}_3$

Dots : $\text{Cu}^{2+}$ ions ; Corners of squares : $\text{O}^{2-}$ ions.
Magnetic susceptibility of the ladder material \( \text{SrCu}_2\text{O}_3 \)


\[
\chi(T) = \frac{\alpha}{\sqrt{T}} e^{-\Delta/(k_B T)} \quad \text{with} \quad \Delta/k_B = 420 \text{ K}.
\]
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Case $J_{||} = 0$

$S^z = -1$

$S^z = 0$

$S^z = 1$

⇒ Beyond a critical field, ground state = triplets with $S^z = 1$.

$J_{||} \neq 0$ ⇒ broadening of the transition
Pseudospin description of the low energy subspace

- $\tau_z = -1/2 \Rightarrow$ singlet on the rung
- $\tau_z = +1/2 \Rightarrow$ triplet $S^z = 1$ on the rung

\[
S_{n,p}^+ \rightarrow \frac{(-)^p}{\sqrt{2}} \tau_n^+; \quad S_{n,p}^z \rightarrow (1 + 2\tau_n^z)/4
\]
Effective Hamiltonian

\[
H_{\text{eff.}} = \frac{J_{\|}}{2} \sum_n (\tau^+_n \tau^+_n + \tau^-_n \tau^-_n + \tau^z_n \tau^z_{n+1}) \\
- (h - J_{\perp} - J_{\|}) \sum_n \tau^z_n
\]

XXZ spin-1/2 chain ⇒ Luttinger liquid state.
Exact magnetization curve can be obtained.
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Theoretical magnetization curve at zero temperature

\[ M \propto (H - H_{c1})^{1/2} \]

\[ H_{c2} \]

\[ H_{c1} \]

\[ M \]

\[ \frac{1}{2} \]
Luttinger liquid description of the magnetized state

\( \mathcal{H} = \int \frac{dx}{2\pi} \left[ u(H)K(H)(\pi \Pi)^2 + \frac{u(H)}{K(H)}(\partial_x \phi)^2 \right] \),

\([\phi(x), \Pi(x')] = i\delta(x - x')\)

\(S_{n,p} = (-)^p \lambda_x(H) e^{i \pi \int_{x-2\pi n}^{x} dx' \Pi(x')}\)

\(S_{n,p} = -\frac{a}{\pi} \partial_x \phi + \lambda_z(H) \cos(2\phi(2\pi n) - (1 - 2m(H))\pi n)\)
Magnetization curve for \((5\text{IAP})_2\text{CuBr}_4 \cdot 2\text{H}_2\text{O}\)

What is the effect of interladder coupling?

1. on the zero field spin gap state?
2. on the Luttinger liquid state for applied field?
Coupled ladders

\[
H = \sum_{n,m} J_{\perp} \vec{S}_{n,m,1} \cdot \vec{S}_{n,m,2} + J_{\parallel} (\vec{S}_{n+1,m,1} \cdot \vec{S}_{n,m,1} + \vec{S}_{n+1,m,2} \cdot \vec{S}_{n,m,2})
+ J'_{\perp} \sum_{n,m} S_{n,m,2} \cdot S_{n,m+1,1}
\]

\[J_{\perp}, J_{\parallel} \gg J'_{\perp}.\]
Stability of the gapful state

A Bond Operator Theory mean field treatment lead to a stable spin gap for $J'_\perp < 0.12J_\perp$.


More generally, the presence of the spin gap in the isolated ladder implies that its ground state is stable in the presence of weak perturbations.
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Stability of the magnetized state

Interladder coupling gives a term:

\[ \frac{J'_\perp}{4} \sum_{n,m} \left( -\tau^+_n m \tau^-_{n,m-1} - \tau^-_{n,m} \tau^+_n m_{-1} + \tau^z_{n,m} \tau^z_{n,m-1} \right) \]

This term can stabilize a phase with \( \langle \tau^+_n m \rangle \neq 0 \) i.e. an antiferromagnetic order in the plane perpendicular to the applied magnetic field.
Another interpretation of antiferromagnetic order

\[ b_i \leftrightarrow \tau_i^- \; ; \; n_i = b_i^\dagger b_i \rightarrow \tau_i^z \quad \text{with constraint} \quad (b_i^\dagger)^2 = 0 \]

(forbidden double occupancy).

\[ \mathcal{H} = -\sum_{i,j} t_{ij} b_i^\dagger b_j + \sum_{i,j} V_{ij} n_i n_j, \]

⇒ Bose Einstein condensation of magnons.
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Phase diagram in T-H plane

- Critical
- Quantum
- Spin Gap
- BEC

$T_N$

$T$

$h_c$

$h$

$T_{N}$

Gap

Spin BEC
Experiments

Materials:
- TiCuCl$_3$, KCuCl$_3$ (3D dimer network)
- BaCuSi$_2$O$_6$ (2D dimer network)
- BPCP (coupled ladders)

Probes:
- magnetization measurements
- Neutron scattering
- NMR, ESR
Coupled ladders with $J_\perp \gg J_\parallel$

Each ladder is the Luttinger liquid state

$$\mathcal{H} = \int dx \sum_m \left[ u(H) K(H) (\pi \Pi_m)^2 + \frac{u(H)}{K(H)} (\partial_x \phi_m)^2 \right]$$

$$- J_\perp \lambda_x^2 \sum_{\langle m, m' \rangle} \cos(\theta_m - \theta_{m'})$$

$\theta_m = \pi \int x \Pi(x')$

Mean field theory:

$\cos(\theta_n - \theta_{n'}) \rightarrow \langle \cos \theta_n \rangle \cos \theta_{n'} + \cos \theta_n \langle \cos \theta_{n'} \rangle$

Transition temperature from mean field theory

$$1 = zJ' \lambda_x^2 \chi \cos \theta(\vec{Q}, H, T_N(H))$$

Camel-Shaped $T_N(H)$ for $J_\perp \gg J_\parallel$. 

-3.00 -1.00 1.00 3.00
0.02 0.03 0.04 0.05 0.06 0.07
$\Gamma_c$

$\tilde{h}$

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$J_\perp/k_B = 12.9 K$, $J_\parallel/k_B = 3.6 K$. 
More complete theory needed

\[ J_\perp \gg J_\parallel \] not satisfied. Mapping on effective XXZ chain not applicable, but Luttinger liquid description still valid.

- accurate determination of \( u(H), K(H), \lambda_x(H) \) from DMRG
- determination of order parameter at \( T = 0 \) from sine-Gordon model.
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NMR measurements on BPCP

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NMR measurements on BPCP

Dzyaloshinskii-Moriya interaction

\[ H_{DM} = \vec{D} \cdot (\vec{S}_1 \times \vec{S}_2) \]

results from relativistic spin-orbit interactions.
A transverse staggered magnetization that does not break the lattice symmetry is seen in NMR measurements.

Dzyaloshinskii-Moriya interactions seem to explain this observation.
Ladder with Dzyaloshinskii-Moriya interactions

\[ H = \sum_n \left[ J_\parallel (\vec{S}_{n,1} \cdot \vec{S}_{n+1,1} + \vec{S}_{n,2} \cdot \vec{S}_{n+1,2}) + J_\perp \vec{S}_{n,1} \cdot \vec{S}_{n,2} \right. \\
- h(S_{n,1}^z + S_{n,2}^z) \left] + H_{DM} \right. \\
H_{DM} = \sum_n (-)^n \vec{D} \cdot (\vec{S}_{n,1} \times \vec{S}_{n,2}) \\
\]

where \( \vec{D} = D\hat{y} \).
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Numerical study


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Strong coupling approach

\[ H_{\text{DM,eff.}} = \frac{D}{\sqrt{2}} \sum_n (-1)^n \tau_x^n. \]

Can be mapped to the quantum sine-Gordon model.

\[ H = H_{\text{LL}} - \frac{D\lambda_x}{a\sqrt{2}} \int dx \cos \theta \]

Solitons and breathers in the classical sine-Gordon

The equations of motions of the classical sine-Gordon model possess soliton and antisoliton solutions.

solitons and antisolitons can form bound states called breathers.
Breather energies are quantized:

\[ M_n = 2M_0 \sin \left( \frac{\pi n}{2 \cdot 8K(H) - 1} \right), \]

where \( M_0 \) is soliton energy, \( n \) is integer and \( 1 \leq n < 8K - 1 \). Breathers excitations will give peaks in Raman scattering measurements.
Conclusions

- Two leg spin-1/2 ladders under magnetic field behave as a Luttinger liquid for field larger than the spin gap.
- At sufficiently low temperature, the Luttinger liquid is unstable to antiferromagnetic ordering.
- A *quantitative* comparison of theory with experiment is possible in the case of the organic ladder BPCP.
- Dzyaloshinskii-Moriya interactions can spoil the Luttinger liquid state under field.
- A signature of Dzyaloshinskii-Moriya interaction in the ladder is the presence of breathers in the partially magnetized state.
Perspectives

- Other 1D spin gap materials (Haldane gap, dimerized chains...)
- Field induced ordering in 2D/3D materials (TlCuCl$_3$...)
- Effect of disorder (Mg$^{2+}$, Zn$^{2+}$ non-magnetic impurities)